Fiscal Consolidation in Heavily Indebted Economies*

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Abstract

In this paper, I build a dynamic general equilibrium model calibrated to the U.S. economy to study the macroeconomic effects of alternative fiscal consolidation strategies in a context where the private sector is heavily indebted. Fiscal consolidation is defined as a permanent reduction of the public debt-to-GDP ratio through government spending cuts or tax hikes. I show that in the long run, fiscal consolidation entails output benefits that are dampened when private debt is high. This effect occurs independently of the fiscal instrument used to stabilize the debt. In the short run, I find that a fiscal policy that rises labor or capital tax rates induces deleveraging in the private sector, which amplifies temporary output losses due to fiscal consolidation policies. By contrast, a fiscal consolidation achieved by government spending cuts or consumption tax hikes facilitates the repayment of private debt, thereby mitigating the negative output effect associated with a public debt reduction. Finally, regarding social welfare, I find that a fiscal consolidation brings higher welfare gains when government spending or consumption tax rates adjust in an environment of high private debt. However, it increases the social welfare loss when capital or labor tax rates adjust to meet the public debt target.

Keywords: Fiscal Policy, Fiscal Consolidation, Private Debt, Great Recession

JEL classification numbers: E32, E44, E61, E62, H63

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1 Introduction

In many advanced countries, the global financial crisis of 2008-09 generates large increases in public debt to-GDP ratios to levels never seen before since the aftermath of World War II. Unfortunately, these countries are more than likely to experience a sharp contraction in growth, run large fiscal deficits, and accumulate debt to fight the COVID-19 outbreak.[1] Against this background, fiscal authorities are planning to design fiscal consolidation plans once the crisis is over to reduce financial vulnerabilities and bring public debt to sustainable levels.

Recent policy debates and research on fiscal policy during the Great Recession and the Eurozone crisis have focused on the macroeconomic effects of fiscal consolidations.[2] However, the financial crisis also generates large increases in private sector debt levels meaning that fiscal consolidation will occur in an unprecedented environment of high private debt (IMF (2016)). In the United States, household debt-to-GDP remains high, and the Federal Reserve remarks that non-financial corporation debt as a percent of GDP has increased since 2010 and now is higher than its pre-crisis level (Kaplan et al. (2019)). Surprisingly, the literature has paid little attention to the interactions between private and public sector debt dynamics, which might have relevant implications for the effectiveness of fiscal policy in reducing public debt ratios. In this paper, I shed light on the effectiveness of alternative fiscal consolidation strategies when private debt is high, considering both household and non-financial entrepreneurial debt. I consider different types of fiscal instruments on both the revenue and expenditure side and examine the effects of implementing one at a time to permanently reduce the public debt-to-output ratio to meet a particular debt target, holding the other fiscal instruments constant. The purpose of this exercise is to offer a systematic analysis allowing to differentiate the effects of alternative fiscal consolidation strategies on private indebtedness. In particular, I investigate whether a fiscal consolidation strategy forces a private debt reduction (deleveraging) or, conversely, enables easy access to private credit.[3] The key insight is that if the fiscal tightening forces agents to reduce their high debt levels through a decrease in private spending, this can create problems to achieve the desired public debt target reducing the effectiveness of the fiscal consolidation strategy relative to a case of lower private debt.

To analyse the effects of alternative fiscal consolidation strategies when private debt is high, I build a dynamic general equilibrium model, featuring borrowing constraints in the spirit of Kiyotaki and Moore

[1] The International Monetary Fund (IMF) in the *World Economic Outlook, April 2020* predicted that the global economy would contract by -3 percent in 2020 and projected additional public debt increases in baselines of already high public debt.

[2] See Alesina and Ardagna (2013) for a survey of the literature.

[3] As in Batini, Melina, and Villa (2019) deleveraging means a reduction in liabilities achieved through cuts to borrowing.

(1997) and Iacoviello (2005). The model is enriched with a detailed fiscal sector block and incorporates government debt. In the model, households exhibit a heterogeneous willingness to save which generates borrowing and lending among them, and hence a role for household debt. In addition to household borrowers, firms can also borrow from household savers. Household borrowers and firms are financially constrained; they can borrow up to a limit given by a fraction of the expected discounted future value of their houses and commercial real estate holdings, respectively. As a consequence, housing prices, inflation, and interest rates play a crucial role in the dynamics of private debt. Fiscal consolidation strategies will change households' and firms' ability to borrow through its impact on collateral values.

To explore how fiscal consolidation strategies impact private debt dynamics and affect the fiscal transmission, I calibrate the model for the United States matching some key statistics regarding private (households and non-financial corporations) debt levels. The United States and the Euro area economies face the same problem of reducing public debt in a private debt overhang. Thus, the effects of different fiscal consolidation strategies found can have implications beyond the United States. In the paper, I model fiscal consolidation as a permanent reduction in the government debt-to-GDP ratio from its value of 76% in 2016 to the average pre-crisis value of 60% and analyse expenditure-based consolidations strategies, consisting of spending cuts, and revenue-based consolidation strategies, consisting on tax hikes on consumption, labour, and capital income.[4] I run simulations under perfect-foresight and compute the initial and final steady state as well as the related transition path. I compare the results found with an economy in which private debt is lower addressing the counterfactual question of what would have been the effects of fiscal consolidation in a much less indebted economy. The purpose of this exercise is to analyse how effective are different consolidation strategies when private debt is higher. The results can be summarized in the following main findings.

First, I find that a fiscal consolidation has positive long-run effects on key macroeconomic variables such as output regardless of the fiscal policy instrument used to reduce public debt. The mechanism behind these positive long-run effects is the fact that a reduction in the government debt-to-GDP ratio generates lower-interest-rate payments on government debt, creating additional fiscal space to increase government spending or reduce the level of distortionary taxes on consumption, labour and capital income.[5] Thus, stimulating economic activity in the long run. However, the long-run output increase associated with fiscal consolidation is reduced in an environment of high private debt for all fiscal instruments considered in the analysis because the benefits from the fiscal space created are lowered.

[4] In the paper, I use the publicly held debt-to-GDP ratio; see the calibration section for details.

[5] Fiscal space is defined as the difference between the current debt level and the debt limit. See Ghosh, Kim, Mendoza, Ostry, and Qureshi (2013).

Second, I show that a fiscal consolidation has negative short-run effects in terms of output for all fiscal consolidation strategies considered. Interestingly, I find an asymmetric effect of fiscal consolidation on private debt dynamics depending on the fiscal instrument used to stabilize debt around the new target. While instruments defined on labour or capital income tax rates induce a private sector deleveraging, amplifying the output losses associated with a fiscal consolidation process relative to an environment in which private debt is lower; government spending or consumption tax rate induces to ease the access to credit through an increase in borrowers' collateral values. Therefore, mitigating the output losses associated with the public debt reduction as borrowers increase their consumption of goods and investment, thus stimulating total consumption and investment in private capital.

Finally, I compute the impact of the alternative fiscal consolidation strategies on welfare measured in terms of consumption equivalent units. I find that private debt amplifies the welfare losses associated with a consolidation based on labour and capital income tax rates, but it also amplifies the welfare gains associated with a consolidation based on government spending and consumption tax rate. To sum up, my findings suggest that private debt plays an important role in the effectiveness of different fiscal consolidation strategies and the results depend crucially on the fiscal policy instrument used to stabilize the public debt-to-output ratio around the new desired target.

Related literature. The Great Recession led to a renewed interest in academia about the effectiveness of economic austerity due to the need of reducing large budget deficits and growing debt, key issues that have come to the forefront of policy debates.[6] This paper brings together two branches of the literature: one that has focused on the macroeconomic effects of fiscal consolidations and another on the relationship between private debt dynamics and the transmission mechanism of fiscal policy.

Since the eruption of the financial crisis, the literature on the macroeconomic effects of fiscal consolidations has focused on two important points, although the results are not conclusive. First, the literature has intensively debated the effects of fiscal consolidation on economic activity and growth. Coenen, Mohr, and Straub (2008) suggest that fiscal consolidation plans have contractionary effects in the short run while generating sizable increases in output in the long run, in particular when the improvement in the budgetary position is used to reduce distortionary taxation (see also Almeida, Castro, Félix, Maria, et al. (2013)). In contrast, some other authors find that fiscal consolidation might even be expansionary in the short run. For example, Cogan, Taylor, Wieland, and Wolters (2013) evaluates the impact of a fiscal consolidation plan

[6] See for example Blanchard and Leigh (2013); Guajardo, Leigh, and Pescatori (2014); Alesina and Ardagna (2010), and Alesina and Ardagna (2013).

on the U.S. Economy that combines a reduction in government spending and distortionary taxes on labour. They find a positive effect on output in the short and long-run relative to a baseline of no consolidation.[7] Second, several studies have assessed the relative effectiveness of tax increases and spending cuts in reducing budget deficits and public debt-to-GDP ratios. Most of the studies find that spending cuts are much more effective than tax hikes to achieve a permanent reduction in public debt and less costly in terms of output (see Alesina, Barbiero, Favero, Giavazzi, and Paradisi (2017)).[8] My analysis revisits the macroeconomic effects of alternative fiscal consolidation measures on key macroeconomic aggregates when household and non-financial entrepreneurial debt are present.

A large body of recent literature on the effects of fiscal policy has also focused on the interaction between fiscal and monetary policy since interest rates were near zero during the financial crisis. In particular, Christiano, Eichenbaum, and Rebelo (2011), Eggertsson (2011), and Woodford (2011) study the effects of fiscal policy when monetary policy is constrained by the zero lower bound on nominal interest rates and show that fiscal multipliers can be large when the nominal interest rate hit the zero lower bound. Relatedly, Erceg and Lindé (2013) find that tax-based consolidation tends to have smaller adverse effects on output than expenditure-based consolidation in the short run if the zero lower bound is binding.

The financial crisis also leaves heavily indebted households and firms in most advanced economies. Thus, the study of the interaction between public and private debt dynamics has become an issue of policy relevance. Batini, Melina, and Villa (2019) finds that in recessions, governments should provide financial assistance to the private sector to mitigate the recessionary consequences of a private deleveraging even in the face of higher public debt. Recent contributions to the literature show that the effectiveness of fiscal stimulus depends on the levels of private debt. Burriel, Checherita-Westphal, Jacquinot, Schonlau, and Stahler (2020), using a DSGE model calibrated for core and periphery countries in Europe, show that economies with high public and private debt levels are less resilient to demand-based recessions due to a crowding out of private debt in the short and long run. Eggertsson and Krugman (2012), Kaplan and Violante (2014), and Andrés, Boscá, and Ferri (2016) find that fiscal multipliers are larger in high household debt states.[9] In this paper, I analyse the effects of implementing alternative fiscal consolidation measures to permanently reduce the public debt-to-GDP ratio in the context of private debt overhang.[10] It would be unfair to consider that the effects of fiscal consolidation programs and fiscal stimulus packages are equal, but in the opposite sign, because fiscal

[7] In a similar vein, Forni, Gerali, and Pisani (2010) find that a reduction in both expenditures and tax rates to permanently reduce the public debt to GDP ratio in euro area countries generates growth in GDP and investment.

[8] Pappa, Sajedi, and Vella (2015) show that tax hikes are more costly in terms of welfare, and produces deep recessions than spending cuts in the presence of tax evasion and corruption.

[9] See also Bernardini and Peersman (2018).

[10] I define private debt as the sum of household and non-financial entrepreneurial debt.

stimulus measures are temporary policy changes aimed at stimulating the economy that might not have the purpose of reducing public debt and budget deficits.

My paper is strongly related to the work of Andrés, Arce, Thaler, and Thomas (2020). In particular, they analyse the interaction between private deleveraging and the macroeconomic cost of alternative fiscal consolidation strategies in a DSGE model of a small open economy within a monetary union. They find that fiscal consolidations increase the length of private deleveraging and make the recession deeper and more prolonged. In my paper, I do not impose that private agents are involved in a deleveraging process. Instead, I implement alternative fiscal consolidation strategies and study their effects on private debt dynamics analysing whether it generates a private deleveraging, or contrarily, facilitate access to credit.

To the best of my knowledge, this is the first theoretical paper investigating the role of household and non-financial entrepreneurial debt on the effectiveness of fiscal consolidation. This study sheds some light on how to design fiscal consolidation strategies when private debt is high to minimize the output cost associated with the fiscal consolidation process.

The rest of the paper is organized as follows. Section 2 provides a detailed description of the model. Section 3 explains the calibration procedure. Section 4 shows the quantitative results of the model, where section 4.1 and 4.2 presents the long-run and the short-run effects of fiscal consolidation when private debt is high, and section 4.3 presents the welfare effects. Section 5 examines some extensions of the model. Finally, section 6 concludes.

2 The model

The model economy is populated by three types of consumers, patient households, impatient households, and entrepreneurs, who differ by the rate at which they discount the future. Patient households have a higher propensity to save, meaning that in equilibrium, they are the ones saving and lending in the economy. Patient households and impatient households are in measures $(1-\omega)$ and ω, respectively. Patient households work, consume non-durables, buy houses, and invest in private bonds and in government bond holdings. Impatient households work, consume non-durables, buy houses and borrow from patient households against the value of their housing stock.[11] Entrepreneurs also borrow from patient households subject to a collateral constraint, produce an intermediate good combining household labour, commercial real estate, and capital, and sell it to

[11] The only asset that serves as collateral is real estate properties. This feature is motivated by the fact that mortgages are the main component of U.S. household liabilities. See Justiniano, Primiceri, and Tambalotti (2015).

a sector of retailers that operate in a monopolistically competitive market.[12] The economy is also populated by a government that collects a mix of lump-sum and distortionary taxes on consumption, capital, and labour income, and finances its fiscal deficit by issuing one-period government debt. Monetary policy is implemented by a central bank that follows a Taylor-type interest rate rule, while fiscal policy follows a rule to meet the government's debt target.[13]

2.1 Households

Households supply labour and derive utility from the consumption of non-durable goods, $c_{i,t}$, and housing services, $h_{i,t}$. They maximize a lifetime utility function given by:

$$E_0 \sum_{t=0}^{\infty} \beta_i^t \left\{ \ln(c_{i,t} - \zeta\ c_{i,t-1}) + \vartheta\ \ln(h_{i,t}) - \varphi \frac{(l_{i,t})^{\ 1+\eta}}{1+\eta} \right\}, \tag{1}$$

where $i = \{P, I\}$ denotes the household's type. Patient and impatient households are denoted by P and I, respectively.[14] Impatient households' discount factor is $\beta_I < \beta_P$, where β_P is the discount factor of the patient households.[15] The parameter η measures the elasticity of labour with respect to the real wage, and ϑ is the housing weight in the utility.[16] The parameter $\zeta \in (0, 1)$ is the degree of habit formation in the consumption of non-durable goods. I consider external habit formation, which simplifies the optimization problem of households because they take $t-1$ average consumption as exogenous.[17] All variables denoted with subscript P or I are expressed per patient or impatient household, respectively.

2.1.1 Patient households

Patient households accumulate residential housing, $h_{P,t}$, invest in one-period private bonds, $b_{P,t}$ and in one-period government bonds, $b_{G,t}$, both in real terms. They also pay taxes on private consumption, τ_t^c, earned labour income, τ_t^w, and lump-sum taxes, $\tau_{P,t}^l$. They maximize their expected utility given by function (1)

[12] Following Iacoviello (2005), I assume that firms only use commercial real estate as collateral. For models in which firms' borrowing capacity depends on the stock of physical capital that they own, see, e.g., Gerali, Neri, Sessa, and Signoretti (2010) and Liu, Wang, and Zha (2013).

[13] Appendix A contains all the equilibrium conditions and provides more details on the model.

[14] The utility function of households is standard with the exception that goods are separable between non-durable and durable goods (see Bernanke (1984)).

[15] All households face the same optimization problem, but since $\beta_I < \beta_P$, it is suboptimal for patient households to hold debt in equilibrium.

[16] As in the literature, housing services are assumed to be a fraction of the housing stock owned by households; see Iacoviello (2005).

[17] For simplicity, I do not differentiate between individual and average consumption given that they are the same in the symmetric equilibrium.

for $i = P$ subject to the following budget constraint in real terms:

$$(1+\tau_t^c)c_{P,t} + p_t^h(h_{P,t} - h_{P,t-1}) + b_{P,t} + b_{G,t} = (1-\tau_t^w)w_{P,t}l_{P,t} + \frac{R_{t-1}}{\Pi_t}(b_{P,t-1} + b_{G,t-1}) - \tau_{P,t}^l, \quad (2)$$

where p_t^h is the relative price of housing, $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, and $w_{P,t}$ is the real wage. R_{t-1} is the gross nominal return on private and public bond holdings between $t-1$ and t.

As in Iacoviello (2005), the gross inflation rate in the term $\frac{R_{t-1}}{\Pi_t}(b_{P,t-1} + b_{G,t-1})$ reflects the assumption that debt contracts are set in nominal terms, meaning that price variations between $t-1$ and t can affect the realized real interest rate.[18]

2.1.2 Impatient households

Impatient households maximize their expected utility given by function (1) for $i = I$ subject to:

$$(1+\tau_t^c)c_{I,t} + p_t^h(h_{I,t} - h_{I,t-1}) + \frac{R_{t-1}}{\Pi_t}b_{I,t-1} = (1-\tau_t^w)w_{I,t}l_{I,t} + b_{I,t} - \tau_{I,t}^l, \quad (3)$$

$$b_{I,t} \leq \rho_c \frac{b_{I,t-1}}{\Pi_t} + (1-\rho_c)\, m_I E_t\left[\frac{p_{t+1}^h \Pi_{t+1} h_{I,t}}{R_t}\right], \quad (4)$$

where equation (3) and (4) denotes the budget constraint and the borrowing constraint respectively. The budget constraint for impatient households is similar to that of patient households, except that impatient households borrow $b_{I,t}$ at the gross nominal interest rate R_t.[19] The borrowing constraint is specific for impatient households. As in Kiyotaki and Moore (1997) and Iacoviello (2005) borrowing is limited to a fraction of the value of the borrowers' housing stock. Thus, outstanding debt $b_{I,t}$ cannot exceed a fraction m_I of the expected discounted value of the impatient household's stock of housing. The parameter m_I in equation (3) can be interpreted as the loan-to-value (LTV) ratio. The term ρ_c allows for a slow adjustment over time of the borrowing constraint.[20] The assumption that $\beta_I < \beta_P$ guarantees that the borrowing constraint holds with equality.[21]

[18] This assumption is based on the empirical evidence that, in low-inflation countries, the vast majority of debt contracts are set in nominal terms (see Iacoviello (2005)).

[19] Note that although the lump-sum tax has different subscript for patient and impatient households, I consider $\tau_{I,t}^l = \tau_{P,t}^l = \tau_t^l$.

[20] This formulation is to capture the idea that in practice lenders do not readjust borrowing limits every quarter, as in Iacoviello (2015).

[21] As shown in Iacoviello (2005), without uncertainty it is possible to show that, in the present framework, impatient households and entrepreneurs borrow up to the maximum in the neighbourhood of the deterministic steady state. Consider for instance the Euler equation of the impatient household evaluated at the deterministic steady state $\mu_I = \frac{\beta_P - \beta_I}{(1+\tau^c)(1-\zeta)c_I(1-\rho_c\beta_I)} > 0$, where

2.2 Production

In the production sector, entrepreneurs produce an intermediate good using a Cobb-Douglas constant returns-to-scale technology that uses household labour, capital, and collateralizable real estate as input.[22] Retailers purchase the intermediate good from entrepreneurs in a competitive flexible price market and transform it into a composite final good.

2.2.1 Entrepreneurs

The entrepreneur maximizes their lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta_E^t \ln(C_{E,t} - \zeta\ C_{E,t-1}), \tag{5}$$

where $\beta_E < \beta_P$, subject to:

$$\begin{aligned}(1+\tau_t^c)C_{E,t} + p_t^h(H_{E,t} - H_{E,t-1}) + I_t =& (1-\tau_t^k)(mc_t Y_t - w_{P,t}L_{P,t} - w_{I,t}L_{I,t}) + B_{E,t} - \frac{R_{t-1}}{\Pi_t}B_{E,t-1} \\ &+ \tau_t^k \delta^k K_{t-1} + (1-\tau_t^k)\Pi_t^r - \xi_{k,t},\end{aligned} \tag{6}$$

$$Y_t = K_{t-1}^{\mu}(H_{e,t-1})^{\nu} {L_{P,t}}^{\alpha(1-\mu-\nu)} L_{I,t}^{(1-\alpha)(1-\mu-\nu)} \tag{7}$$

$$I_t = K_t - (1-\delta^k)K_{t-1} \tag{8}$$

$$B_{E,t} \leq \rho_c \frac{B_{E,t-1}}{\Pi_t} + (1-\rho_c)\ m_E E_t\left[\frac{p_{t+1}^h \Pi_{t+1} H_{E,t}}{R_t}\right]. \tag{9}$$

Entrepreneurs are borrowing-constrained agents that produce a homogeneous intermediate good using a Cobb-Douglas production function given by equation (7), using housing, $H_{E,t}$, labour provided by patient and impatient households $L_{P,t}$ and $L_{I,t}$, respectively, and capital, K_t.[23] Equation (8) is the law of motion

μ_I is the Lagrange multiplier associated with the borrowing constraint of impatient households. An analogous proof holds for entrepreneurs.

[22] Commercial real estate plays a dual role for entrepreneurs. It is an input of the production function and it also serves as collateral in the entrepreneurs' loan contracts.

[23] Labour inputs enter the production function in a Cobb-Douglas fashion following Iacoviello (2005). This assumption implies complementarity between labour types and allows obtaining closed-form solutions for the steady state of the model. The formulation in which hours are substitutes produces similar results but is analytically less tractable (see Iacoviello and Neri (2010)).

for capital goods in which capital depreciates at rate δ^k. I consider that capital installation entails a cost:

$$\xi_{k,t} = \frac{\psi}{2\delta^k}\left(\frac{I_t}{K_{t-1}} - \delta^k\right)^2 K_{t-1}.^{24} \tag{10}$$

Entrepreneurs' budget constraint is given by equation (6). On the right-hand-side, entrepreneurs receive revenues from selling the intermediate good to retailers at price mc_t, discounted at the real wage rates paid to patient and impatient households, $w_{P,t}$ and $w_{I,t}$ respectively. I assume that entrepreneurs receive profits Π_t^r from the retail sector. As impatient households, they borrow from patient ones $b_{E,t}$ in real terms at the interest rate R_t, so that their net borrowing is $b_{E,t} - \frac{R_{t-1}}{\Pi_t} b_{E,t-1}$. Equation (9) is the entrepreneurs' borrowing constraint, loans cannot exceed a fraction m_E of the expected discounted value of their commercial real estate. Entrepreneurs need to pay taxes on consumption τ_t^c and capital income τ_t^k.[25]

2.2.2 Retailers

There exists a measure-one continuum of retailers that purchase the intermediate good from entrepreneurs and transform it one-for-one into differentiated final good varieties at no cost. For these firms, the real price of the intermediate good, mc_t, represents the real marginal cost. Cost minimization by consumers implies that each final good producer $z \in [0, 1]$ faces the following demand curve for its product variety:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t^d, \tag{11}$$

where $P_t(z)$ is the firm's nominal price, $\epsilon > 1$ is the elasticity of substitution between final good varieties and Y_t^d is the aggregate demand for final goods. I assume staggered nominal price adjustment à la Calvo (1983). Letting θ_p denote the constant probability of not adjusting the price, the optimal price decision of price-setting firms is given by:

$$\sum_{k=0}^{\infty} \theta_p^k E_t\left\{\Lambda_{t,k}\left(\frac{P_t^*(z)}{P_{t+k}} - \frac{\epsilon}{\epsilon - 1} mc_{t+k}\right) Y_{t+k}^*(z)\right\} = 0, \tag{12}$$

where $\Lambda_{t.k} = \beta^k(\lambda_{e,t}/\lambda_{e,t+k})$ is the entrepreneurs relevant discount factor.[26] This condition tells that P_t^* equates expected discounted marginal revenue to expected discounted marginal cost. As a fraction θ_p of prices

[24] The specific functional form for capital adjustment costs ensures that the costs are zero in steady state; See Iacoviello (2005).

[25] Capital income taxes are levied on capital income net-of-depreciation.

[26] The parameter $\lambda_{e,t}$ is the Lagrange multiplier associated with entrepreneurs' budget constraint. See appendix A.3 for more details.

stays unchanged, the aggregate price level evolves according to: $P_t = (\theta_p P_{t-1}^{1-\epsilon} + (1-\theta_p)(P_t^*)^{1-\varepsilon})^{1/(1-\varepsilon)}$.

2.3 Monetary policy

The central bank sets the nominal interest rate according to the following Taylor-type interest-rate rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi(1-\rho_R)}, \tag{13}$$

where R and Π are the steady-state values of the nominal gross interest rate and inflation, respectively. In this specification, the nominal interest rate is adjusted in response to deviations of inflation from its steady-state value, where ρ_Π measures the sensitivity of the interest rate to current inflation, and $\rho_R > 0$ is the parameter associated with the interest rate inertia.[27]

2.4 Fiscal policy

The government finances its expenditures, G_t, by levying taxes on consumption, τ_t^c, labour income, τ_t^w, and capital income, τ_t^k, and by issuing one-period government bonds $B_{G,t}$. The government budget constraint in real terms is given by:

$$B_{G,t} = \frac{R_{t-1}}{\Pi_t} B_{G,t-1} + G_t - Tax_t, \tag{14}$$

where Tax_t are government tax revenues given by:

$$\begin{aligned} Tax_t = \; & \tau_t^c(C_{P,t} + C_{I,t} + C_{E,t}) + \tau_t^w(w_{P,t}L_{P,t} + w_{I,t}L_{I,t}) \\ & + \tau_t^k(mc_t Y_t - w_{P,t}L_{P,t} - w_{I,t}L_{I,t} + \delta^k K_{t-1} + \Pi_t^r) + \tau_t^l. \end{aligned} \tag{15}$$

I consider that the government adjusts spending or taxes to keep the government debt-to-GDP ratio close to a target path. If government spending is the fiscal instrument used to stabilize debt around the new desired target, I assume that spending adjusts endogenously according to the following rule:

$$\bar{G}_t = \rho_f \; \bar{G}_{t-1} - (1-\rho_f)\Big[\phi_1^g\Big(B_{t-1}^{gy} - B^{gy^*}\Big) + \phi_2^g\Big(B_t^{gy} - B_{t-1}^{gy}\Big)\Big], \tag{16}$$

[27] In this Taylor-type interest-rate rule, monetary policy does not react to output deviations. As emphasized in Iacoviello (2005), a monetary policy that only reacts to inflation amplifies the financial accelerator mechanism since the central bank does not react when output fall. See Fernández-Villaverde (2010) for a similar Taylor-type interest rate rule specification.

where $\bar{G}_t$ is the deviation of government spending from its steady-state level, B_t^{gy} is the government debt-to-GDP ratio and B^{gy^*} is the government debt-to-GDP target. When the government follows this rule, the other fiscal instruments are assumed to be constant at their steady-state values.

On the revenue side, if distortionary taxes are used as fiscal instruments, the fiscal rule takes the following form:

$$\bar{\tau}_t^x = \rho_f \bar{\tau}_{t-1}^x + (1-\rho_f)\left[\phi_1^{\tau^x}\left(B_{t-1}^{gy} - B^{gy^*}\right) + \phi_2^{\tau^x}\left(B_t^{gy} - B_{t-1}^{gy}\right)\right], \tag{17}$$

where $\bar{\tau}_t^x = \{\bar{\tau}_t^c, \bar{\tau}_t^w, \bar{\tau}_t^k\}$ are the deviations of taxes from its steady-state level. When the government adopts as fiscal instrument one tax, government spending as well as the other taxes that are not used to stabilize government debt remain constant at their steady-state levels. Under either fiscal rule, the lump-sum tax, τ_t^l, remains fixed at its steady-state value. The parameters ϕ_1 and ϕ_2 control for the size and the smoothness of the adjustment, respectively.

2.5 Market clearing conditions and equilibrium

The goods market, the debt market, and the housing market clear. This is summarized by the following equilibrium conditions:

$$Y_t = C_{P,t} + C_{I,t} + C_{E,t} + I_t + G_t + \xi_{k,t}, \tag{18}$$

$$B_{P,t} = B_{I,t} + B_{E,t}, \tag{19}$$

$$H_{E,t} + H_{P,t} + H_{I,t} = 1, \tag{20}$$

where uppercase letters denote aggregate variables for patient and impatient households.[28] The last condition implies that real estate is fixed in supply and normalized to one, which means that its price varies endogenously.

Definition 1 *The (competitive) equilibrium is defined as a sequence of allocations* $\{H_{P,t}, H_{I,t}, H_{E,t}, L_{P,t}, L_{I,t}, Y_t, C_{P,t}, C_{I,t}, C_{E,t}, B_{P,t}, B_{I,t}, B_{E,t}, B_{G,t}\}_{t=0}^{\infty}$ *together with a sequence of prices* $\{w_{P,t}, w_{I,t}, mc_t, p_t^h, R_t, P_t, P_t^*\}_{t=0}^{\infty}$ *such that, given initial conditions, each agent, i.e patient households, impatient households, entrepreneurs and retailers, satisfy their first order conditions, the private and public sector budget*

[28] A more detailed description of the aggregation procedure can be found in Appendix A.5.

constraints, and market clearing conditions for goods, labour, bond holdings and real estate together with the transversality conditions.

3 Calibration

I calibrate the model to the U.S. economy matching some key statistics for 2016, a period characterized by high ratios of both households' and non-financial corporations' debt.[29] Consequently, the model's steady state should be interpreted as the starting initial conditions to analyse the effects of a public debt reduction when private debt is high. The time period in the model is a quarter.

Table 1 summarizes the calibration procedure and some steady-state values. I describe first the set of parameters calibrated using steady-state targets. The discount factor for patient households, β_P, is set to 0.995, which implies an annualized nominal interest rate of about 2%. The housing weight in the utility, ϑ, is calibrated to match household debt to annual GDP (77%), which I compute using data from the U.S. Federal Reserve Board (FRB) and NIPA data from the U.S. Bureau of Economic Analysis (BEA). The elasticity of output to entrepreneurial real estate, ν, is calibrated to match non-financial entrepreneurial debt to annual GDP, which I compute using FRB data and GDP (71%).

Tax rates and government spending are key fiscal policy parameters in the paper. I estimate the consumption tax, $\bar{\tau}^c$, labour income tax, $\bar{\tau}^w$, and capital income tax, $\bar{\tau}^k$, following the methodology of Mendoza, Razin, and Tesar (1994), and using data from the OECD for 2016.[30] According to the estimates, the consumption tax rate is set to 5%, the labour tax rate to 26%, and the capital tax rate to 29.5% to match the values in the data. The estimates are in line with those found by Trabandt and Uhlig (2011). I set spending on government consumption, $\bar{G}$, to target a ratio of government expenditure to output equal to 14.3%, to match U.S. data in 2016. I set the initial target of government debt to quarterly GDP, $\bar{B}^{gy*} = 4 \times 76\%$, to match the publicly held debt-to-GDP ratio in the U.S. economy.[31] The steady-state value of the lump-sum tax, $\bar{\tau}^l$, is treated as a residual to calibrate government debt at the desired steady-state level.

I borrow the remaining parameters from the existing literature. In particular, I borrow from Iacoviello (2005) the discount factors for impatient households and entrepreneurs, β_I and β_E, which are set to 0.95 and 0.98, the adjustment cost parameter, ψ, which is set to 2, and the Taylor rule parameters, $\rho_\Pi = 0.27$ and

[29] I choose 2016 as the starting point for the analysis due to data limitations in obtaining some time series. See Appendix B for a detailed description of the data and sources used.

[30] See Appendix B for a detailed explanation of the methodology used.

[31] Total public debt over GDP in 2016 was 101.7%. In the paper, I use the publicly held debt-to-GDP because it is the measure of debt used by the Congressional Budget Office to examine the future path of budget deficits and debt under different tax policies. As an alternative, I also use the gross debt-to-GDP ratio but none of the results change noticeably.

Table 1: Baseline calibration

Description	Parameter	Value	Source/Target
A. Preferences			
Patient households' discount factor	β_P	0.995	Match U.S. data
Impatient households' discount factor	β_I	0.95	
Entrepreneurs' discount factor	β_E	0.98	
labour supply elasticity	η	2.00	Standard value
labour preference parameter	φ	1.00	Standard value
Habits in consumption	ζ	0.46	Iacoviello (2015)
Share of households borrowers	ω	0.61	Justiniano, Primiceri, and Tambalotti (2015)
Weight on housing services	ϑ	0.33	Match HHs debt-to-annual GDP
B. Technology			
Patient households' wage share	α	0.64	
Capital adjustment cost	ψ	2.00	
Capital share	μ	0.30	
Capital depreciation rate	δ^k	0.03	
Housing share	ν	0.03	Match NFCs debt-to-annual GDP
C. Borrowing constraint			
LTV for impatient households	m_I	0.90	Iacoviello (2015)
LTV for entrepreneurs	m_E	0.90	Iacoviello (2015)
Inertia in borrowing constraints	ρ_c	0.25	Iacoviello (2015)
C. Price setting			
Elasticity of substitution in goods	ϵ	21	
Probability of non-adjusting prices	θ_p	0.75	
D. Monetary policy			
Response to past inflation	ρ_Π	0.27	
Response to past interest rate	ρ_R	0.73	
E. Fiscal policy			
Inertia in fiscal instruments	ρ_f	0.50	Erceg and Lindé (2013)
Tax rate on consumption	$\bar{\tau}^c$	0.05	Match U.S. data
Tax rate on labour income	$\bar{\tau}^w$	0.26	Match U.S. data
Tax rate on capital income	$\bar{\tau}^k$	0.295	Match U.S. data
Government spending-to-GDP ratio	$\bar{G}/\bar{Y}$	0.143	Match U.S. data
Government debt-to-GDP ratio	$\bar{B}^{gy}/4\bar{Y}$	0.76	Match U.S. data

Notes: Unless otherwise stated, parameters are calibrated as in Iacoviello (2005). The abbreviations HHs and NFCs refer to households and non-financial corporations (entrepreneurs), respectively.

$\rho_R = 0.73$. I set ϵ equal to 21, to target an initial net markup equal to 5 percent, and the Calvo parameter, θ_p, is set to 0.75. I also borrow from Iacoviello (2005) the capital share in production, μ, which is equal to

0.30, the depreciation of productive capital, $\delta^k = 0.03$ and the patient household wage share, $\alpha = 0.64$.[32] Since the labour disutility parameter, φ, only affects the scale of the economy, I normalize it to one. The parameter η is set to 2 such that the Frisch elasticity of labour supply equals 1. In the fiscal rules, the parameter ρ_f is set to 0.5 to allow for a small degree of inertia, as in Erceg and Lindé (2013). The habit in consumption coefficient, $\zeta = 0.46$, the inertia parameter in the borrowing constraints, $\rho_c = 0.25$ and the LTV ratios for impatient households and entrepreneurs, $m_I = 0.9$ and $m_E = 0.9$, are set in line with Iacoviello (2015) estimates. The proportion of impatient households, ω, is set to 0.61 as in Justiniano, Primiceri, and Tambalotti (2015). Table 2 reports some model-implied steady-state ratios and their data counterparts. This table shows that the model matches quite well some key macroeconomic ratios and interest rates.[33]

Table 2: Model steady-state ratios versus U.S. data

Description	Variable	Model	Data
Private consumption-to-GDP ratio	$\bar{C}/\bar{Y}$	0.67	0.69
Private investment-to-GDP ratio	$\bar{I}/\bar{Y}$	0.15	0.16
Annual rate of return (percent)	$4 \times \bar{R}$	4.02	4.07
Labour share in total income	$(\bar{w}_P\bar{L}_P + \bar{w}_I\bar{L}_I)/\bar{Y}$	0.63	0.58
Residential wealth-to-GDP ratio	$(\bar{q}\bar{H}_P + \bar{q}\bar{H}_I)/4\bar{Y}$	4.16	1.40
Commercial real estate-to-GDP ratio	$\bar{q}\bar{H}_E/4\bar{Y}$	0.79	0.65
Household debt-to-GDP ratio	$\bar{B}_I/4\bar{Y}$	0.77	0.77
Non-financial entrepreneurial debt-to-GDP ratio	$\bar{B}_E/4\bar{Y}$	0.71	0.71
Household debt-to total credit	$\bar{B}_I/\bar{B}$	0.52	0.52
Non-financial entrepreneurial debt-to total credit	$\bar{B}_E/\bar{B}$	0.48	0.48

Notes: Data sources are the U.S. Bureau of Economic Analysis, the U.S. Bureau of labour Statistics, the Board of Governors of the Federal Reserve System and the Organization for Economic Co-operation and Development.

4 Private debt dynamics and fiscal consolidation

Through this section, I simulate the effects of alternative fiscal consolidation strategies when private debt is high. First, I define how I model fiscal consolidation. Second, I describe the quantitative exercise aiming to shed light on how the presence of private debt can amplify the transmission mechanisms of alternative fiscal consolidation strategies.

Fiscal consolidation is modeled as a permanent decrease in the initial target of government debt-to-annual GDP ratio (76%), through an expenditure reduction or an increase in fiscal revenues, using the fiscal rules

[32] Iacoviello (2005) estimates this parameter and finds that the wage share of constrained agents is 36%. Similarly, Campbell and Mankiw (1989) estimate a 44.9% of constrained agents in the economy.

[33] The residential housing-to-GDP ratio in the model (3.97) is about the empirical moment (1.40). This reflects the fact that part of the household debt can be other than mortgages.

previously defined. I analyse four different consolidation strategies, one for each fiscal instrument (G, τ^c, τ^w, τ^k) and implement one at a time i.e. when one fiscal rule in equation (16) or (17) is active the other fiscal instruments remain constant at their steady-state levels. To achieve the desired public debt reduction, the debt target is decreased to 60%, which is the average government debt-to-GDP ratio from 1995 to 2007, over a 10-year horizon.[34] I consider fully credible fiscal plans and run perfect-foresight simulations in which households and firms fully anticipate the consolidation strategy and make decisions in a forward-looking manner taking into account the future usage of additional long-run fiscal space to increase expenditures or reduce distortionary taxation.

The best way to assess how the transmission mechanisms of alternative fiscal consolidation strategies changes in an environment of high private debt are to set up a counterfactual experiment, such that we can compare two economies that are identical to each other and differ only in their initial level of private indebtedness. To accomplish this exercise, I simulate a public debt reduction in the calibrated model for the U.S. economy, which features high private debt, and compare the macroeconomic effects of alternative fiscal consolidation strategies with an economy in which private indebtedness is lower.[35] This allows us to assess the macroeconomic effects of fiscal consolidation when private debt is high in the long run, and the short run, as well as implement social welfare comparisons.

4.1 What are the long-run effects of reducing public debt?

This section analyses the long-run effects of a permanent reduction in the government debt-to-GDP ratio from 76% to 60% achieved by the alternative fiscal consolidation strategies. First, some key steady-state equations are introduced to understand the long-run effects when private debt is added to the analysis. Next, I compare the long-run effects of a permanent reduction in the government debt-to-GDP ratio in different scenarios regarding the levels of private debt.

4.1.1 Key steady-state equations

To study the long-run mechanisms at work of a permanent public debt reduction in economies with high private indebtedness levels, I rewrite the steady-state budget constraint for the different agents in aggregate terms. The aim is to examine how total spending capacity depends on private indebtedness.

[34] Following Pappa, Sajedi, and Vella (2015), I adjust the parameters of the policy rules for each case to ensure that the debt target is met after 10 years to compare the effects of the alternative fiscal instruments.

[35] In the model, the share of private debt in GDP is an increasing function of the share of household borrowers and the loan-to-value ratio. As a result, lower steady-state private debt levels translate into a lower share of household borrowers and loan-to-value ratios.

For patient households (savers), their total spending capacity depends on after-tax labour income, lump-sum taxes, and net income from the repayment of government bonds and private bonds holdings:

$$(1+\tau^c)C_P = (1-\tau^w)W_P L_P - \tau_P^l + (R-1)(B_G + B_P) \quad \text{with} \quad B_P = B_E + B_I, \tag{21}$$

for impatient households (borrowers), their total spending capacity depends on after-tax labour income, lump-sum taxes, and net cost from private debt issued:

$$(1+\tau^c)C_I = (1-\tau^w)W_I L_I - \tau_I^l - (R-1)B_I \quad \text{with} \quad B_I = \beta_I m_I p^h H_I. \tag{22}$$

Finally, entrepreneurs' (borrowers) spending capacity depends on after-tax profits minus the net cost from issuing private debt:[36]

$$(1+\tau^c)C_E + I = (1-\tau^k)\Pi_E - (R-1)B_E \quad \text{with} \quad B_E = \beta_I m_E p^h H_E. \tag{23}$$

In steady-state, when interest rates and inflation dynamics do not ease the private debt burden, private debt benefits patient households by enhancing their income from debt repayment, but it entails a cost for both types of borrowers, impatient households and entrepreneurs.[37] Consequently, in the long run, when the economy converges to a new steady state with reduced public debt, if the fiscal consolidation strategy involves a private debt increase with respect to the initial steady-state, *ceteris paribus*, borrowers will lose spending capacity against savers, which implies lower consumption and investment, so that aggregate demand decreases.

4.1.2 The long-run effects of the fiscal consolidation

Table 3 compares the long-run effects of reducing the government debt-to-GDP ratio from 76% to 60% for each alternative fiscal instrument in the baseline economy displaying high private debt levels (column 2), and alternatively, in an economy in which there is no access to credit (column 1).[38] The effects are shown as percentage changes relative to the initial steady state levels for the main macroeconomic variables, except for the effects on the fiscal instruments, which are reported as percentage point changes.

[36] In equation (23), steady-state investment is defined as $I = \delta^k(1-\tau^k)k$ and entrepreneurial profits are defined as $\Pi_E = mcY - w_P L_P - w_I L_I + \Pi^r$.

[37] In steady-state $R = 1/\beta_P$.

[38] In this economy, I exogenously set the loan-to-value ratio to zero, i.e. $m_I = m_E = 0$, and the proportion of borrowers is maintained as in the baseline.

In the long run, the economy converges to a new steady state with a smaller government debt-to-GDP ratio. The fall in the ratio implies lower interest rate payments for the government, which creates an additional fiscal space that leads to an increase in government consumption expenditures or a decrease in distortionary tax rates compared to the initial steady state as shown in the last row. These steady-state changes in fiscal instruments are the causes that generate long-run benefits of fiscal consolidation policies in terms of output. This result is in line with Coenen, Mohr, and Straub (2008) and Cogan, Taylor, Wieland, and Wolters (2013), who analyse the impact of different consolidation strategies in the Euro area and the United States, respectively. However, I find that, if the fiscal consolidation is implemented in a private debt overhang the output benefits are shift downward due to a private debt increase. Because of this last effect, the spending capacity of borrowers is lowered as shown by equations (22) and (23), decreasing aggregate consumption and investment compared with a no-debt scenario. Consequently, this effect dampens the long-run benefits in terms of output. The amplification mechanism occurs for all fiscal consolidation strategies considered, and the strength of the effects depends on the loan-to-value ratios and the share of household borrowers.

Now, I analyse in detail the long-run effects for the alternative fiscal consolidation strategies highlighting the role of private debt in shifting the steady-state results. I begin by analyzing the long-run effects of fiscal consolidation when government spending is used as fiscal instrument. This strategy has a direct positive long-run impact on aggregate demand, as the improvement in the fiscal position results in an increase in government spending, which directly affects the resource constraint. The increase in government spending absorbs part of the economy's resources, without providing utility to agents and generating a negative wealth effect on part of them, leading to a crowding-out of aggregate consumption and residential housing and higher labour supply. As result, housing prices decrease and hours worked rises in equilibrium. The latter leads to an increase in the marginal product of capital, which implies an increase in factor inputs, capital and commercial real estate. Consequently, higher labour and factor inputs lead to an increase in output. The crowding-out of aggregate consumption and residential housing is solely caused by patient households. The reason is that the lower government debt-to-GDP ratio leads to lower interest income from holding government bonds as shown by equation (21), so that their consumption and residential housing are reduced. For impatient households labour income increases and so does consumption and residential housing. The increase in commercial and residential real estate implies a rise in non-financial entrepreneurial debt and household debt, respectively. As noted before, the increase in private debt lowers the spending capacity of debtors. This leads to smaller aggregate consumption and investment in the high-private debt scenario

Table 3: Long-run effects of the fiscal consolidation for the alternative fiscal strategies

Private indebtedness:	**G**		τ^c		τ^w		τ^k	
	Zero (1)	High (2)	Zero (1)	High (2)	Zero (1)	High (2)	Zero (1)	High (2)
Output	0.35	0.31	0.35	0.31	0.38	0.36	1.34	1.18
Consumption	-0.14	-0.17	0.44	0.39	0.48	0.45	1.30	1.12
Patient Households	-0.62	-0.58	-0.04	-0.01	0.19	0.20	0.29	0.35
Impatient Households	0.27	0.25	0.85	0.81	1.01	0.98	1.06	0.94
Entrepreneurs	0.35	0.31	0.93	0.88	0.38	0.36	3.88	3.45
Investment	0.35	0.31	0.35	0.31	0.38	0.36	3.18	2.91
After tax labour income	0.35	0.31	0.35	0.31	1.27	1.24	1.34	1.18
Housing prices	-0.50	-0.29	-0.50	-0.29	0.25	0.36	0.69	1.16
Real estate								
Patient households	-0.12	-0.29	-0.12	-0.29	-0.06	-0.16	-0.40	-0.80
Impatient households	0.78	0.54	0.78	0.54	0.76	0.62	0.37	-0.22
Entrepreneurs	0.85	0.60	0.85	0.60	0.13	-0.01	4.43	3.56
Private debt								
Impatient households	–	0.25	–	0.25	–	0.98	–	0.94
Entrepreneurs	–	0.31	–	0.31	–	0.36	–	4.76
Fiscal instrument	2.39	2.36	-0.61	-0.58	-0.66	-0.64	-2.65	-2.49

Notes: The first column for each fiscal instrument refers to a non-indebted economy in which the loan-to value ratio for impatient households and entrepreneurs are exogenously set to $m_I = m_E = 0$. The second column for each fiscal instrument refers to the baseline economy that features high private debt. All macroeconomic variables are denoted in percentage changes from initial steady-state levels, except for the effects on the fiscal instruments, which are denoted in percentage point changes.

compared with a no-debt scenario, which implies a smaller increase in output.[39]

When consumption or labour income taxes are used as fiscal instrument, I find qualitatively similar results. A long-run decrease of consumption or labour income taxes makes leisure relative to consumption more expensive and hence households increase their labour supply. Aggregate consumption increases under both strategies, despite the fall in consumption of patient households when consumption tax is used as fiscal instrument. In this case, the decrease of the consumption tax rate does not fully compensate patient households for the lower interest income from holding government bonds. For impatient households, labour

[39]Note that, although in the high-private debt scenario, the fall in consumption for patient households is smaller compared with a no-debt scenario, due to the fact that the lower interest income from holding government bonds is partly compensated with higher income from holding private debt, aggregate consumption decreases more because impatient borrowers represent a higher proportion in the economy.

income increases and so does residential housing, thus increasing household debt. The factor inputs, capital and commercial real estate, increase because of the larger demand and this implies an increase in output. As before, the increase in commercial real estate and residential housing implies a rise in private debt leading to smaller aggregate consumption and investment in an scenario of high private debt compared with the no-private debt scenario, which implies a smaller increase in output.

Finally, the long-run decrease in the capital income tax rate leads to an increase of capital in production and higher after-tax profits to entrepreneurs. This increases the marginal product of capital. Entrepreneurs increase factor inputs and the marginal product of labour increases rising aggregate wages. Then output, consumption, investment and hours worked increase. Housing prices experiment a high increase due to the demand increase in commercial real estate for entrepreneurs. This, implies a private debt increase that reduces the long-run gains associated with this fiscal consolidation strategy.

In table 4, I report the long-run effects of reducing the government debt-to-GDP ratio for each alternative fiscal instrument in the baseline economy (column 2), and alternatively, in an economy in which I exogenously decrease the loan-to-value ratio by 50%, i.e. $m_I = m_E = 0.45$, and the proportion of household borrowers is the same as household savers ($\omega = 0.5$). I refer to this economy as "low indebted economy" (column 1).[40] As before, the effects are shown as percentage changes relative to the initial steady state levels for the main macroeconomic variables, except for the effects on the fiscal instruments, which are reported as percentage point changes. Table 4 shows that high levels of private debt imply that the increase in the fiscal space is more limited, i.e. the room of the government to raise spending or lower taxes relative to the initial steady-state, compared with an economy characterized by low levels of private debt, thereby reducing the rise in output.

The long-run analysis suggests that a fiscal consolidation would have been more effective in terms of output in a much less indebted economy. In a recent paper, Mian, Sufi, and Verner (2017) using an empirical analysis show that an increase in private debt tends to lower growth in the long run. Therefore, my results corroborate their findings since the alternative fiscal consolidation strategies entail a rise in private debt that reduces the long-run output benefits. This effect occurs independently of the fiscal instrument used to stabilize private debt. However, this amplification mechanism is only at work in the long run. In the next section, I compute the transition dynamics between the initial and the final steady-state for different indebt-

[40] The average loan-to-value ratio for households and non-financial corporations, measured by mortgages-to-real estate ratio, was about 45% during the pre-crisis period. Iacoviello and Neri (2010) estimate a proportion of borrowers to 0.21. I consider a counterfactual value of 0.5 according to the rule-of-thumb literature that sets the proportion of non-Ricardian agents to this value.

Table 4: Long-run effects of the fiscal consolidation for the alternative fiscal strategies

Private indebtedness:	**G**		$\boldsymbol{\tau^c}$		$\boldsymbol{\tau^w}$		$\boldsymbol{\tau^k}$	
	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)
Output	0.33	0.31	0.33	0.31	0.37	0.36	1.29	1.18
Consumption	-0.16	-0.17	0.42	0.39	0.46	0.45	1.24	1.12
Patient Households	-0.60	-0.58	-0.03	-0.01	0.18	0.20	0.29	0.35
Impatient Households	0.27	0.25	0.85	0.81	1.03	0.98	1.06	0.94
Entrepreneurs	0.33	0.31	0.91	0.88	0.37	0.36	3.75	3.45
Investment	0.33	0.31	0.33	0.31	0.37	0.36	3.09	2.91
After tax labour income	0.33	0.31	0.33	0.31	1.26	1.24	1.29	1.18
Housing prices	-0.44	-0.29	-0.44	-0.29	0.27	0.36	0.83	1.16
Real estate								
Patient households	-0.16	-0.29	-0.16	-0.29	-0.08	-0.16	-0.54	-0.80
Impatient households	0.71	0.54	0.71	0.54	0.77	0.62	0.23	-0.22
Entrepreneurs	0.77	0.60	0.77	0.60	0.10	-0.01	4.16	3.56
Private debt								
Impatient households	0.33	0.31	0.33	0.31	0.37	0.36	5.02	4.76
Entrepreneurs	0.27	0.25	0.27	0.25	1.03	0.98	1.06	0.94
Fiscal instrument	2.38	2.36	-0.59	-0.58	-0.65	-0.664	-2.60	-2.49

Notes: The first column for each fiscal instrument refers to a non-indebted economy in which the loan-to value ratio for impatient households and entrepreneurs are exogenously set to $m_I = m_E = 0.45$ and the proportion of borrowers to $\omega = 0.5$. The second column for each fiscal instrument refers to the baseline economy that features high private debt. All macroeconomic variables are denoted in percentage changes from initial steady-state levels, except for the effects on the fiscal instruments, which are denoted in percentage point changes.

edness scenarios and alternative fiscal instruments. The aim is to determine how the short-run transmission of fiscal consolidation changes when private debt is added into the analysis.

4.2 What are the short-run effects of reducing public debt?

While in the long run a reduction in the level of government debt creates additional fiscal space due to lower interest payments that can be used to either increase expenditures or reduce distortionary taxes on consumption, labour or capital income, in the short run, a fiscal consolidation requires government expenditure cuts or significant tax hikes generating costs in terms of output.

To offer a quantitative representation of the adjustment in the output dynamics when the government

debt reduction is produced in an environment of high private debt, I compute in Table 5 the cumulated output losses as the cumulative percentage changes in output relative to the initial steady-state for the alternative fiscal consolidation strategies, aimed at reducing government debt-to-GDP ratio from 76% to 60% in 10 years. I compare the "high indebted economy" outcomes, which corresponds with the calibrated model that features both high household and entrepreneurial debt, with the alternative indebtedness scenario, in which I set the proportion of household borrowers in the economy and also the loan-to-value ratio for household borrowers and entrepreneurs to lower values. As before, I consider the same proportion of patient and impatient households and I reduced the LTV ratio for impatient households and entrepreneurs by 50%.

Table 5: Impact and cumulative change in response of a fiscal consolidation

Private indebtedness:	**Impact**		**1 year**		**2 years**		**5 years**	
	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)
G	-2.08	-1.37	-10.64	-7.93	-19.19	-15.16	-28.87	-24.78
$\boldsymbol{\tau^c}$	-1.07	-0.41	-8.76	-6.31	-18.84	-15.22	-30.08	-26.22
$\boldsymbol{\tau^w}$	-2.58	-3.65	-15.91	-20.98	-34.12	-42.83	-57.89	-64.38
$\boldsymbol{\tau^k}$	-1.30	-2.32	-6.60	-10.71	-17.18	-26.73	-55.47	-74.78

Notes: The first column for each fiscal instrument refers to a low indebted economy in which the loan-to value ratio for impatient households and entrepreneurs are exogenously set to $m_I = m_E = 0.45$ and the proportion of household borrowers is set to $\omega = 0.5$. The second column for each fiscal instrument refers to the baseline economy that features high private debt. Accumulated output losses are denoted in cumulative percentage changes from initial steady state.

Table 5 reveals that the presence of high private indebtedness shifts the cumulated output losses for the alternative fiscal instruments in two different ways. When the fiscal consolidation is carried out through labour or capital income tax rates, the cumulative output losses are always much higher in an scenario of high private debt levels. However, when government consumption or consumption tax rates adjust to stabilize the government debt-to-GDP ratio around the new target, the output losses are higher in a lower indebted economy. Thereby, the results suggest that government consumption expenditures and consumption tax rates are more effective in reducing government debt in terms of output when implemented in high private debt economies.

4.2.1 Amplification mechanism

Before going further into the analysis of the transitional dynamics generated by the alternative fiscal consolidation strategies, I highlight the main economic mechanisms that shift the effects of fiscal consolidation strategies when private debt is high. A more detailed discussion is provided in the next two sections.

In Table 6, I compute the cumulative percentage change, after 8 quarters from the beginning of the fiscal consolidation, for output, aggregate consumption, investment and private debt. Interestingly, I find an asymmetric effect of fiscal consolidation strategies on the dynamics of private debt. If government spending or consumption tax rate adjust to stabilize debt around the new target, private debt increases which allows borrowers to mitigate the short-run costs of fiscal consolidation since they can consume and invest more. However, if labour or capital income tax rates are used as fiscal instruments, the public debt deleveraging has associated a private debt deleveraging, forcing borrowers to a rapid debt repayment. Thereby, lowering consumption and investment more intensively in a high-private debt scenario.

Table 6: Cumulative change in response of a fiscal consolidation after 12 quarters

Private indebtedness:	**G**		τ^c		τ^w		τ^k	
	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)
Output	-24.27	-20.04	-25.04	-21.23	-46.38	-55.96	-30.31	-46.56
Consumption	36.01	39.94	-39.26	-33.73	-59.77	-71.56	-32.10	-43.57
Investment	11.78	13.17	10.91	11.30	-41.08	-52.82	-96.41	-165.63
Private debt-to-GDP	15.11	202.70	12.69	192.67	-27.91	-175.18	-46.24	-146.57

Notes: The first column for each fiscal instrument refers to a low indebted economy in which the loan-to value ratio for impatient households and entrepreneurs are exogenously set to $m_I = m_E = 0.45$ and the proportion of household borrowers is set to $\omega = 0.5$. The second column for each fiscal instrument refers to the baseline economy that features high private debt. All variables are denoted in cumulative percentage changes from initial steady state, except for the private debt-to-GDP ratio which is denoted in cumulative percentage point changes.

Private debt dynamics depends on how interest rates, inflation and house prices evolve. First, fiscal consolidation strategies that generate an increase in interest rates affect borrowers by higher interest payments and lower collateral values, tightening the borrowing constraint. Second, fiscal consolidation strategies that generate an increase in inflation decreases borrowers' cost of private debt financing and increases the expected value of the collateral, facilitating the access to credit. Finally, fiscal consolidation strategies that generate a rise in house prices increase borrowers' collateral values relaxing the borrowing constraints. As

a result, the effectiveness of fiscal consolidation strategies depends on how it indirectly affects private debt dynamics, through changes in the previous macroeconomic variables.

4.2.2 Government spending and consumption tax based consolidation

In Figure 1, I report the adjustment dynamics of a government spending (first column) and a consumption tax based consolidation (second column) across both indebtedness scenarios to study the short- to medium-run effects of fiscal consolidation when private debt is added into the analysis.

Figure 1 shows that the crowding in (crowding out) consumption and in investment is stronger (weaker) in an scenario of high private debt when spending on government consumption adjust to stabilize debt around the new desired target (consumption tax rate). Thus, in this case the responses of private consumption and investment are the reasons why the output losses are smaller in a high indebted economy. Next, I analyse in detail a government spending based consolidation since the amplification mechanism at work is similar to a consumption tax based consolidation.

Considering the adjustment dynamics of a government spending based consolidation, I find a persistent fall in output across both indebtedness scenarios. This fall in output is less pronounced in a high private debt scenario and is caused by the responses of aggregate consumption and investment. Patient and impatient households rise consumption as government spending decreases due to a wealth effect, as they want to smooth their consumption. The wealth effect also decreases hours worked for both types of agents lowering labour income. Patient households anticipate an income increase because of the inflow of funds stemming from the repayment of government debt holdings. They use debt repayment that they do not use for consumption to buy residential real estate. The increase in their housing stock produces a rise in housing prices which leads to a direct housing wealth effect for impatient households and entrepreneurs through an increase in the value of the residential ($p_t^h H_{I,t-1}$) and commercial real estate ($p_t^h H_{E,t-1}$).[41] The rise in housing prices also increases the expected value of the collateral for impatient households ($E_t p_{t+1}^h H_{I,t}$) and entrepreneurs ($E_t p_{t+1}^h H_{E,t}$) increasing their access to new credit (increase in private debt) despite the rise in real interest rates (R_t/Π_{t+1}). The increase in the value of housing combined with an increase in the access to new credit allow borrowers to increase consumption and investment. This effect is stronger in an economy where the LTV ratios and the proportion of household borrowers are higher and led to higher consumption and investment in an scenario of high private debt. Note that because of private debt increases, patient households, instead, consume less

[41] The higher increase in housing prices in the low private debt scenario is due the strong response of residential housing of patient households.

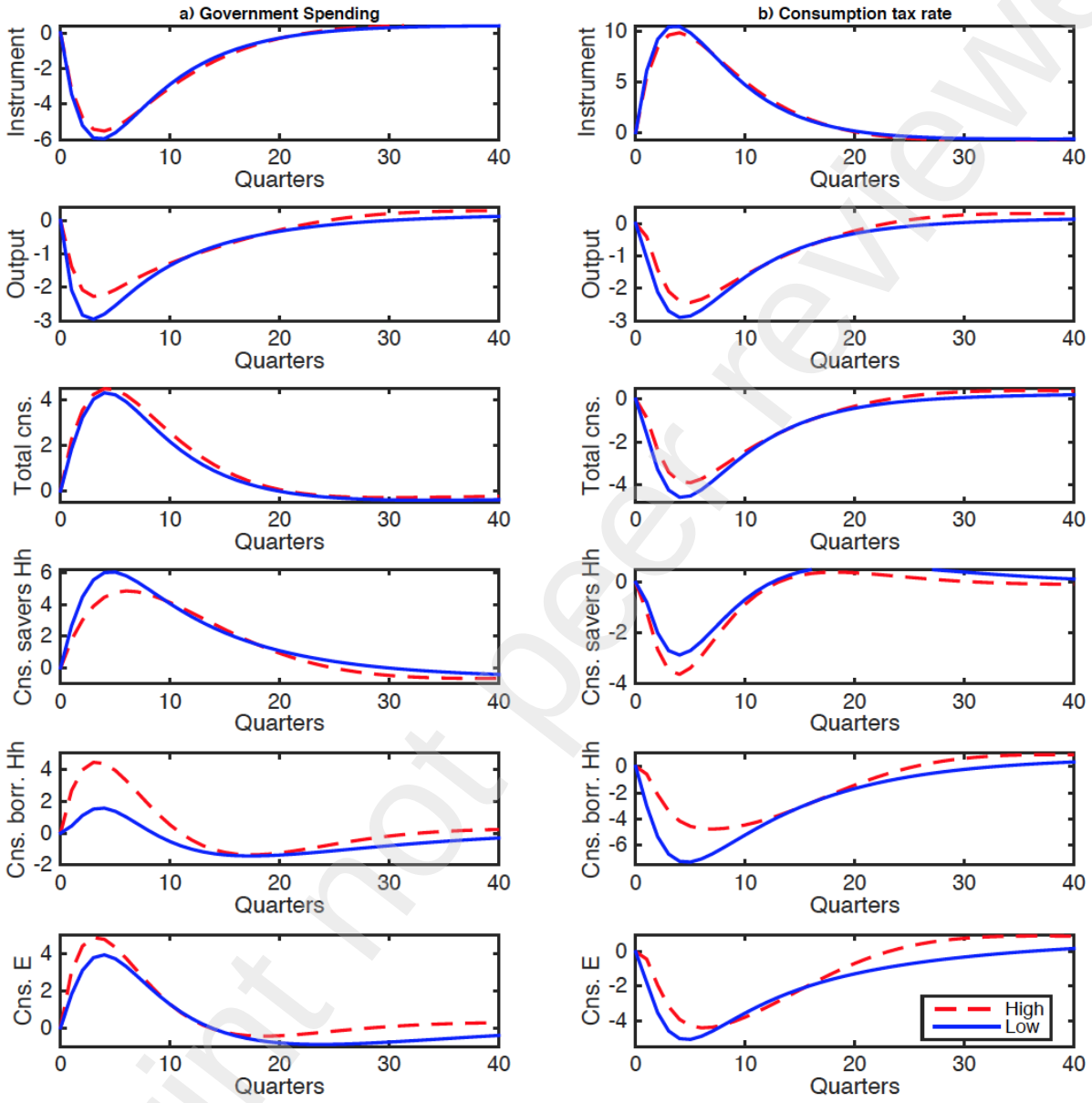

Figure 1: Transitional dynamics of Government spending and consumption tax based consolidation

Notes: The dashed red line corresponds with the high private debt scenario (baseline) and the solid blue line with low private debt scenario ($m_I = m_E = 0.45$ and $\omega = 0.5$). Time on the x-axis is measured in quarters. The percentage change from initial steady-state is measured in the y-axis.

in an scenario of high private debt, but the decline in patient households' consumption does not fully offset the rise in borrowers' consumption, and aggregate consumption rises.

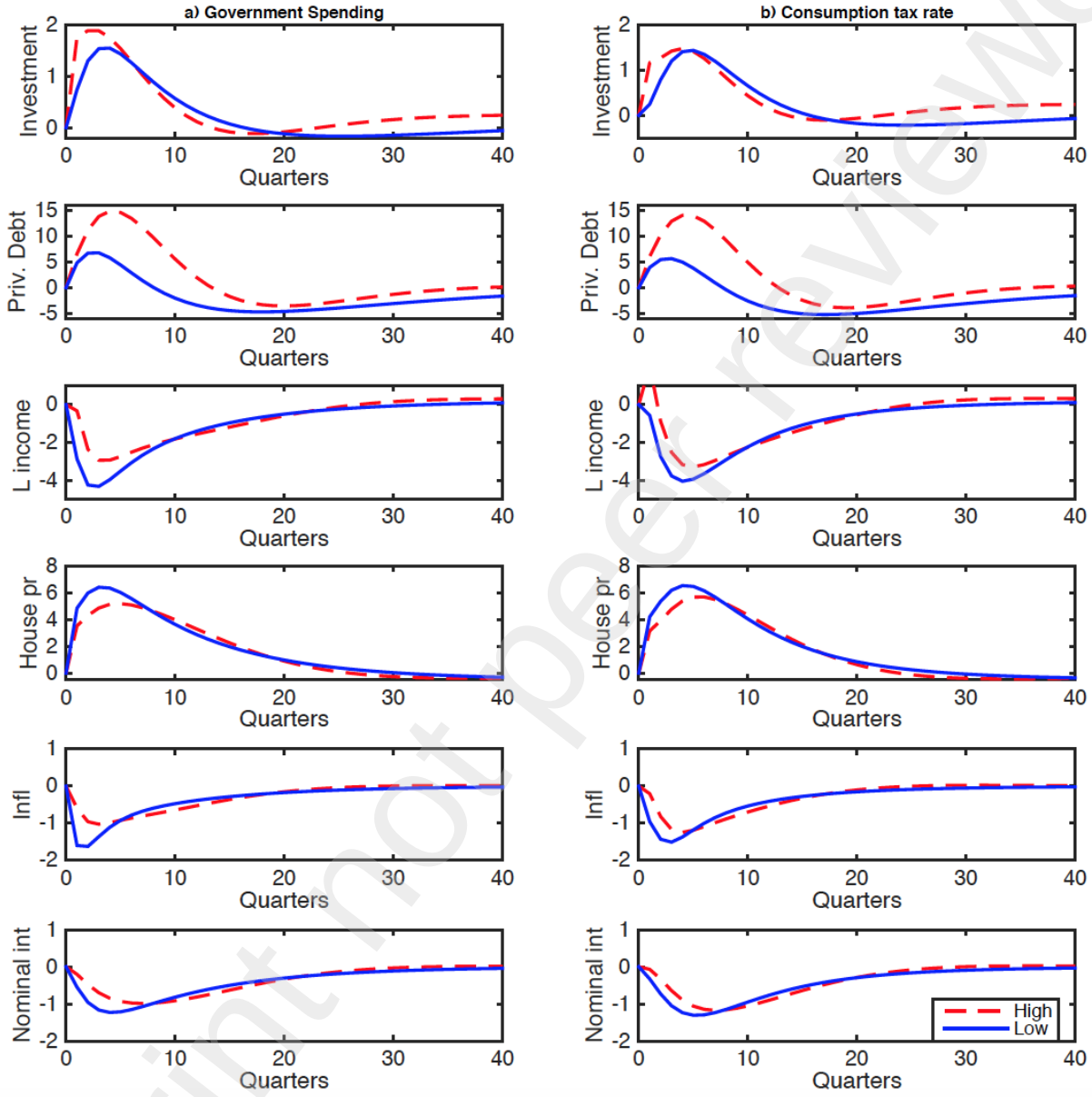

Figure 1 (Cont.): Transitional dynamics of government spending and consumption tax based consolidation

Notes: The dashed red line corresponds with the high private debt scenario (baseline) and the solid blue line with low private debt scenario ($m_I = m_E = 0.45$ and $\omega = 0.5$). Time on the x-axis is measured in quarters. The percentage change from initial steady-state is measured in the y-axis.

4.2.3 Labour income and capital income tax based consolidation

In Figure 2, I report the adjustment dynamics of a labour income tax based consolidation (first column) and a capital income tax based consolidation (second column) and compare the effects across both indebtedness

scenarios. Figure 2 shows that the crowding out in consumption and in investment is stronger in an scenario of high private debt for both fiscal instruments. Thus, in this case the response of private consumption and investment are the reasons why the output losses are higher in a high indebted economy. As before, I analyse in detail a labour tax based consolidation since the amplification mechanism at work is similar to a capital tax based consolidation.

When using the labour income tax as fiscal instrument, the initial increase in the tax rate on labour income leads to a consumption fall and a decline in investment. The negative response of both consumption and investment translate into a persistent negative response of output. Comparing the output response in both indebtedness scenarios, I find that the output fall is amplified in an scenario of high private debt through a more persistent drop in consumption and investment. The increase in labour income taxes reduce households' labour supply. Households' labour income decreases because of the higher labour tax rate and the reduction in hours worked. This implies a reduction in consumption and residential housing for both types of households lowering housing prices. The increase in labour tax rates generates a rise in the real wage because households want to be partly compensated for the implied reduction in the after tax real wage. The rise in real wages increases the marginal cost. As a result of higher marginal cost, inflation and nominal interest rate rises. The fall in output and the rise in real wages decrease entrepreneurial profits and consequently entrepreneurs' consumption, commercial real estate and investment. In an scenario of high private debt, the fiscal consolidation induces to a longer and a deeper recession. The amplification mechanism works through the reduction in the availability of new credit and the deterioration in the net financial wealth defined as housing wealth ($p_t^h H_{x,t-1}$) net of private debt ($R_{t-1}/\Pi_t B_{x,t-1}$, $x = \{I, E\}$). The fall in housing prices decreases the expected value of the collateral for impatient households ($E_t p_{t+1}^h H_{I,t}$) and entrepreneurs ($E_t p_{t+1}^h H_{E,t}$) decreasing their access to new credit (decrease in private debt) despite the fall in real interest rates (R_t/Π_{t+1}). Thus, the spending capacity of borrowers decrease lowering consumption and investment. The net financial wealth for borrowers is also reduced due to the fall in housing prices. Note that the rise in inflation, Π_t, decreases the cost of debt repayment, but the net effect is negative, decreasing more the spending capacity of borrowers and hence reducing even further consumption and investment. In a high private debt scenario, in which the LTV ratios are higher and also the proportion of borrowers, the negative effects are amplified. In the case of a capital income tax based consolidation, the higher decrease in private debt is not so clearly observed because although non-financial business debt decreases, household debt increases (See Appendix C Figure 4).

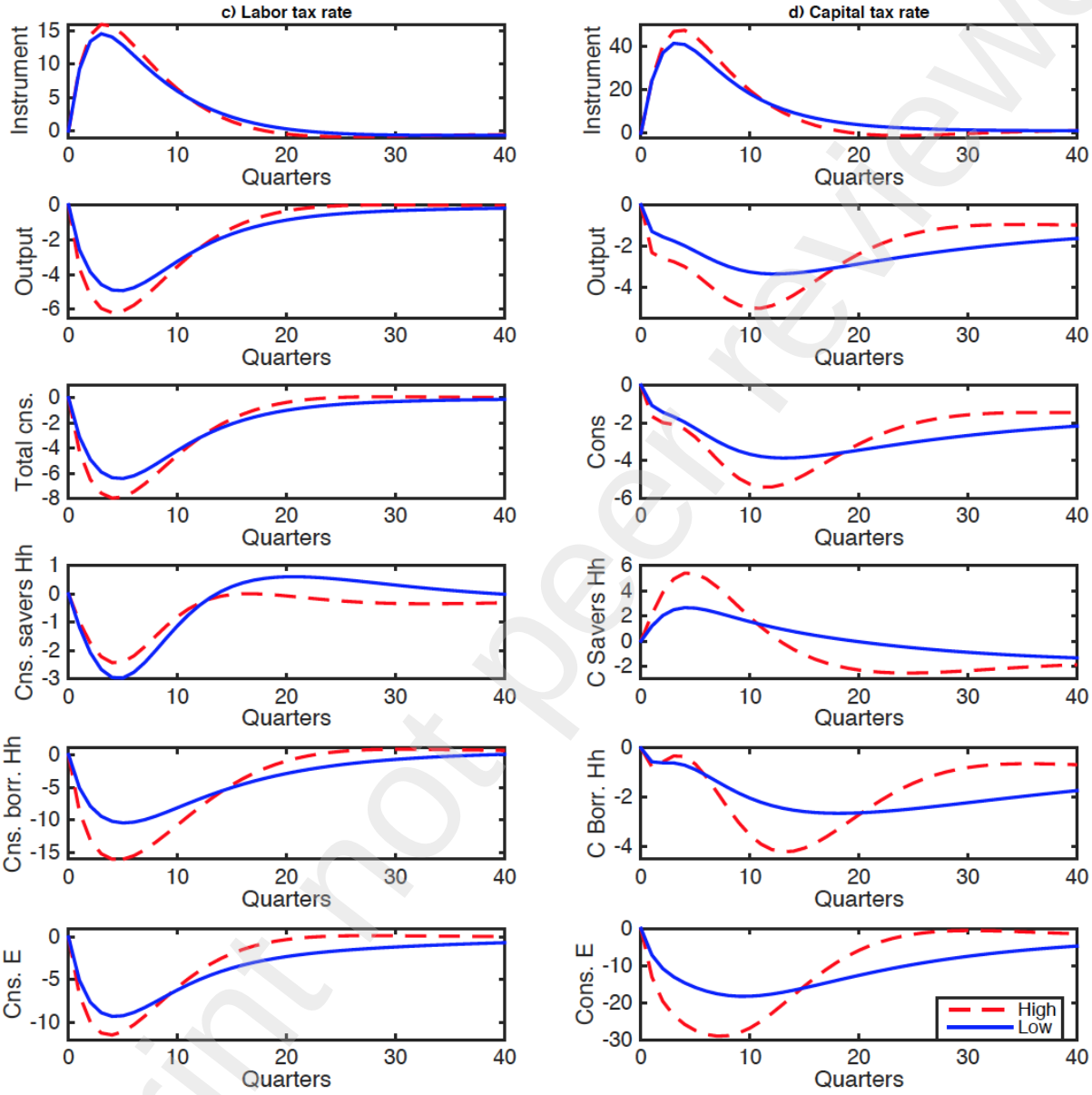

Figure 2: Transitional dynamics of labour and capital tax based consolidation

Notes: The dashed red line corresponds with the high private debt scenario (baseline) and the solid blue line with low private debt scenario ($m_I = m_E = 0.45$ and $\omega = 0.5$). Time on the x-axis is measured in quarters. The percentage change from initial steady-state is measured in the y-axis.

4.3 Who are the winners and losers of reducing public debt?

In this section, I analyse the implications of a fiscal consolidation in a context of high private debt from a welfare perspective. For each fiscal consolidation strategy, I compute the welfare gains and losses incurred

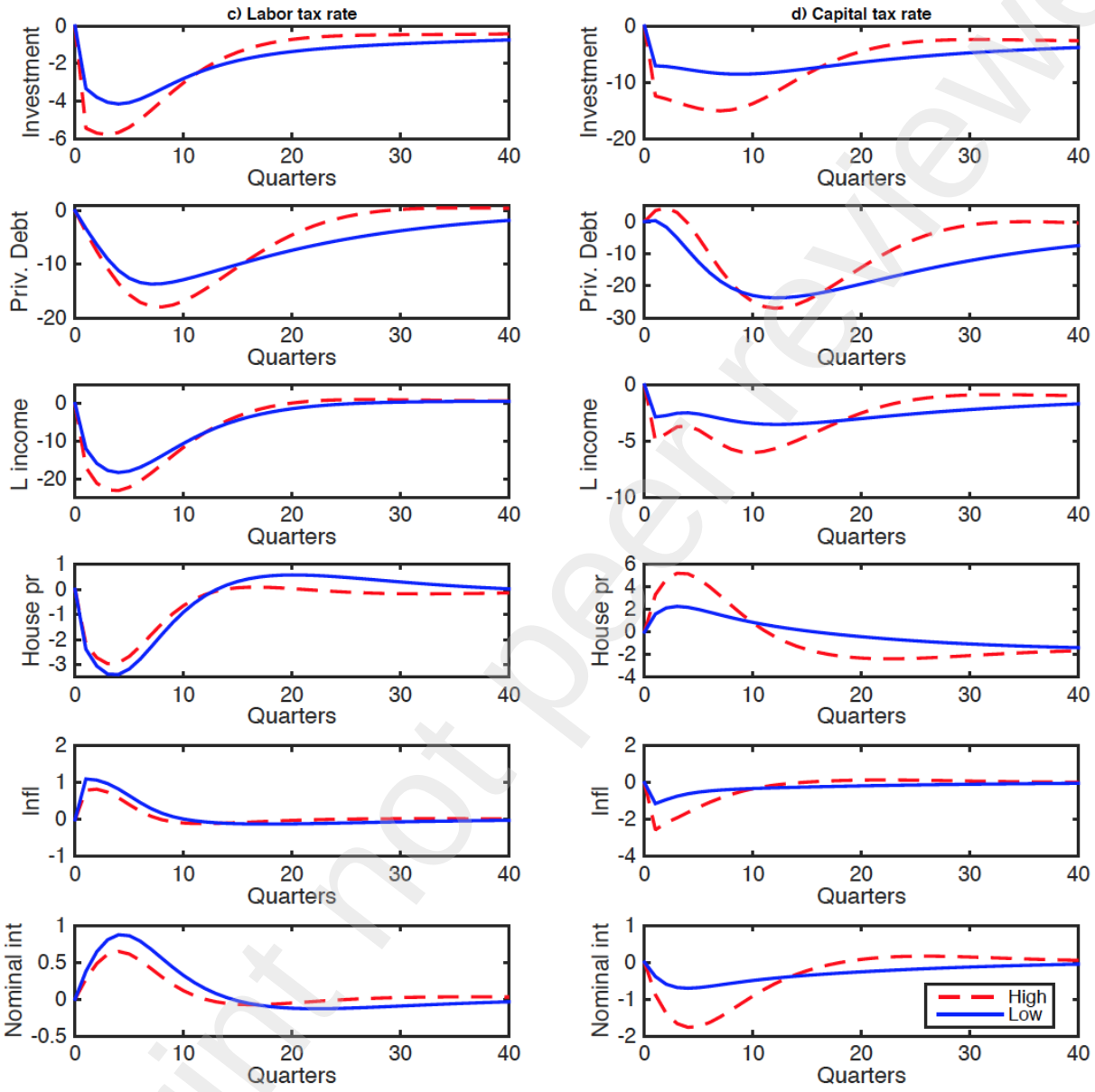

Figure 2 (Cont.): Transitional dynamics of labour and capital tax based consolidation

Notes: The dashed red line corresponds with the high private debt scenario (baseline) and the solid blue line with low private debt scenario ($m_I = m_E = 0.45$ and $\omega = 0.5$). Time on the x-axis is measured in quarters. The percentage change from initial steady-state is measured in the y-axis.

by patient and impatient households and entrepreneurs, and compare the effects across the two different indebtedness scenarios. This will allow to investigate whether high private debt levels help improving agents'

welfare.

Individual welfare for patient households, $W_{P,t}$, impatient households, $W_{I,t}$, and entrepreneurs, $W_{E,t}$, is defined as the discounted lifetime utility function evaluated at the optimally chosen sequences of consumption, labour and housing for the alternative fiscal consolidation strategies:

$$W_{P,t} = \sum_{t=0}^{\infty} \beta_P^t \left\{ \ln(c^*_{P,t} - \zeta \; c^*_{P,t-1}) + \vartheta \;\; \ln(h^*_{P,t}) - \varphi \frac{(l^*_{P,t})^{\;1+\eta}}{1+\eta} \right\}, \tag{24}$$

$$W_{I,t} = \sum_{t=0}^{\infty} \beta_I^t \left\{ \ln(c^*_{I,t} - \zeta \; c^*_{I,t-1}) + \vartheta \;\; \ln(h^*_{I,t}) - \varphi \frac{(l^*_{I,t})^{\;1+\eta}}{1+\eta} \right\}, \tag{25}$$

$$W_{E,t} = \sum_{t=0}^{\infty} \beta_E^t \left\{ \ln(C^*_{E,t} - \zeta \; C^*_{E,t-1}) \right\}. \tag{26}$$

where variables with start denote the optimally chosen paths for consumption, leisure and housing under each fiscal consolidation strategy previously defined.

Following Mendicino, Pescatori, et al. (2004), I define aggregate social welfare, W_t, as the weighted sum of individual welfare for the different types of agents:

$$W_t = \eta_P W_{P,t} + \eta_I W_{PI,t} + \eta_E W_{E,t}, \tag{27}$$

where $\eta_P = (1-\omega)(1-\beta_P)$, $\eta_I = \omega(1-\beta_I)$ and $\eta_E = (1-\beta_E)$. In the above expression households' welfare is weighted with their respective population share and corrected for the subjective discount rate implying that each agent deliver the same level of life-time utility given a constant consumption stream.[42]

I define the welfare effects in consumption equivalents as in Schmitt-Grohé and Uribe (2005), i.e. the variation in consumption that would be necessary to achieve the same welfare as through the impact of the fiscal consolidation. I calculate the compensating variation in consumption, Γ, for each agent evaluating the welfare at the initial steady state, and then comparing it with the welfare obtained when the fiscal consolidation is implemented:

$$W_{P,t} = \sum_{t=0}^{\infty} \beta_P^t \left\{ \ln\left[(1-\zeta)(1+\Gamma_P)\bar{C}_P\right] + \vartheta \;\; \ln(\bar{h}_P) - \varphi \frac{(\bar{l}_P)^{\;1+\eta}}{1+\eta} \right\}, \tag{28}$$

[42] See Mendicino, Pescatori, et al. (2004) and Andrés, Boscá, and Ferri (2016).

$$W_{i,t} = \sum_{t=0}^{\infty} \beta_i^t \left\{ \ln\left[(1-\zeta)(1+\Gamma_I)\bar{C}_I\right] + \vartheta \ \ln(\bar{h}_I) - \varphi \frac{(\bar{l}_I)}{1+\eta}^{1+\eta} \right\}, \tag{29}$$

$$W_{E,t} = \sum_{t=0}^{\infty} \beta_E^t \left\{ \ln\left[(1-\zeta)(1+\Gamma_E)\bar{C}_E\right] \right\}, \tag{30}$$

where variables with a bar denote steady-state values.

The analytical solutions for Γ_P, Γ_I and Γ_E are:

$$\Gamma_i = \exp\left[(1-\beta_i)(W_{i,t} - \bar{W}_i)\right] - 1, \ i = \{P, I\} \tag{31}$$

$$\Gamma_E = \exp\left[(1-\beta_E)(W_{E,t} - \bar{W}_E)\right] - 1, \tag{32}$$

where $\bar{W}_P$, $\bar{W}_I$ and $\bar{W}_E$ are the discounted lifetime utility functions evaluated at the initial steady state. The social welfare gain is given by:

$$\Gamma = \eta_P \Gamma_P + \eta_I \Gamma_I + \eta_E \Gamma_E. \tag{33}$$

A positive value for Γ means a welfare gain, that is $\Gamma \times 100$ is the percentage of consumption without the fiscal consolidation that consumers would be willing to give up to implement a given fiscal consolidation strategy.

Table 7 reports the welfare effects for the alternative fiscal consolidation strategies in consumption equivalence units in an scenario of low (column 1) and high private debt (column 2). There are important differences in terms of welfare effects depending on which instrument is used to reduce the public debt-to-GDP ratio around the new desired target. A government spending based consolidation is associated with a social welfare gain which is more intense in a highly indebted economy. The low indebted scenario leads to a 0.19 percent increase in welfare, whereas the high private debt scenario leads to a 0.23 percent increase. The main reason for the welfare differences across both indebtedness scenarios is the higher increase in private debt in the high indebted scenario which leads to a stronger response of consumption and housing for impatient households and entrepreneurs which are the borrowers in the economy. Note that for patient households the welfare gain is reduced more in the high private debt scenario. A consumption tax based consolidation is associated with a positive social welfare which is also amplified in a highly indebted economy. As in the previous case, the

higher welfare increase is due to the higher increase in private debt which reduces the drop in consumption and housing, and hence increases welfare.

Table 7: Welfare effects for the alternative fiscal consolidation strategies

Indebtedness level	$\mathbf{G}$		$\boldsymbol{\tau^c}$		$\boldsymbol{\tau^w}$		$\boldsymbol{\tau^k}$	
	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)
Social Welfare	0.192	0.233	0.087	0.114	0.007	-0.156	-0.015	-0.066
Patient households	0.154	0.090	0.154	0.089	0.155	0.091	0.152	0.089
Impatient households	-0.009	0.088	-0.088	-0.006	-0.140	-0.246	-0.043	-0.025
Entrepreneurs	0.046	0.055	0.021	0.031	-0.009	-0.001	-0.124	-0.130

Notes: Welfare is defined in consumption equivalent units, i.e. the variation in consumption that would be necessary to achieve the same welfare as through the impact of the fiscal consolidation. The first column for each fiscal instrument refers to a low indebted economy. The second column for each fiscal instrument refers to the baseline economy that features high private debt.

A labour income tax based consolidation or a capital income tax based consolidation are associated with a social welfare loss which is more intense in a highly indebted economy. The reason for the amplification in the decrease in welfare for both fiscal instruments is the private debt deleveraging which is more intense in a much leveraged economy. A stronger decrease in private debt leads to a stronger decrease in consumption and housing for impatient households and entrepreneurs amplifying the welfare losses. Note that the welfare losses are amplified for impatient households in the case of a labour tax based consolidation. However, a capital income tax based consolidation amplifies the losses for entrepreneurs in the high private debt scenario.

It is worthwhile to look at the different welfare effects of patient households. For patient households the welfare impact is positive for all fiscal consolidation strategies. However, the positive effects on welfare are reduced when the fiscal consolidation is implemented in a highly indebted economy. Thus, they always lose welfare from reducing public debt in an environment of high private debt while borrowers could gain welfare depending on which fiscal instrument is used to stabilize debt around the new debt target.

5 Robutness

In this section, I first examine the effects of a fiscal consolidation when government spending provides utility to consumers. Next, I study the effects of a government investment based consolidation by adding into the model productive public capital.

5.1 Utility from government consumption

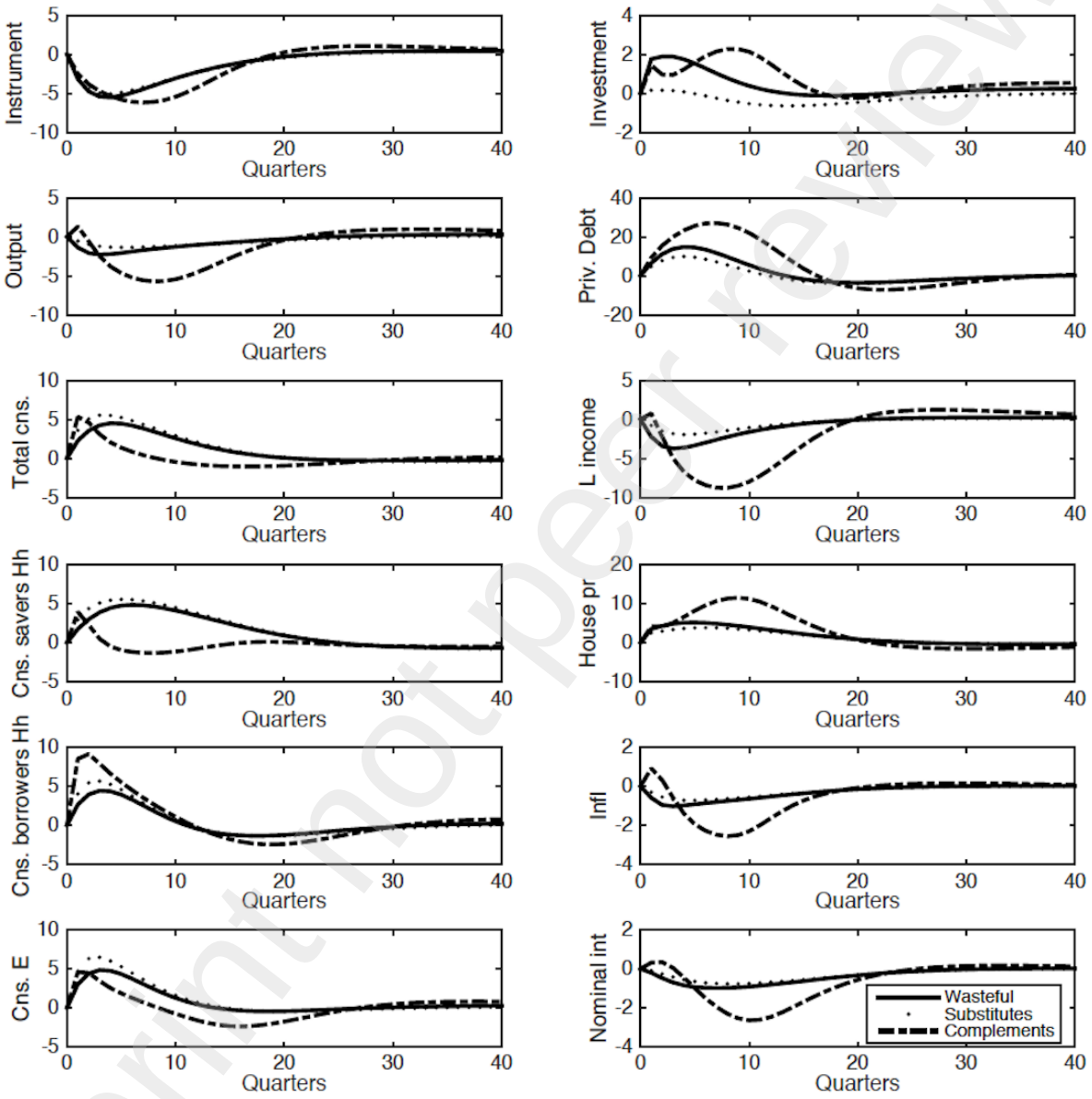

Figure 3: Transitional dynamics of government spending tax based consolidation when agents derive utility from government spending

Notes: The solid black line corresponds with the high private debt scenario (baseline) in which government consumption is complete wasteful. The dotted and dashed black line represent a high private debt scenario in which public and private consumption are substitutes ($\gamma = 0.5$) and complements ($\gamma = -0.5$), respectively. Time on the x-axis is measured in quarters. The percentage change from initial steady-state is measured in the y-axis.

I extend the baseline model to allow government spending to enter the consumer's utility function in a non-separable form. Government consumption in agents' utility function can be interpreted as a public good that households and entrepreneurs consume at no cost. Common examples of public goods include education, health care, national defense and public. Earlier studies that allow government expenditures to provide utility to consumers include Barro (1981), Baxter and King (1993), Ni (1995), Amano and Wirjanto (1998) and Finn (1998).

I define a linearly homogeneous constant-elasticity-of-substitution (CES) consumption bundle of private consumption and consumption of public services as in Linnemann and Schabert (2004) for the three types of consumers in the economy:

$$\tilde{c}_{i,t} = \tilde{c}_{i,t}(c_{i,t}, G_t) = \left[\varrho\ c_{i,t}^{\gamma} + (1-\varrho)G_t^{\gamma}\right]^{1/\gamma}, \quad \gamma \in (-\infty, 1),\ \varrho \in (0,1), \tag{34}$$

$$\tilde{C}_{E,t} = \tilde{C}_{E,t}(C_{E,t}, G_t) = \left[\varrho\ C_{E,t}^{\gamma} + (1-\varrho)G_t^{\gamma}\right]^{1/\gamma}, \quad \gamma \in (-\infty, 1),\ \varrho \in (0,1), \tag{35}$$

where $i = \{P, I\}$ denotes patient and impatient households, respectively.[43] The parameter ϱ denotes the relative weight of private consumption and the parameter γ determines the degree of substitution $\sigma = 1/(1-\gamma)$ between private and government consumption. Note that if $\varrho \to 1$, government consumption drops out of the consumption bundle and the utility simplifies to the standard case in which all government spending is wasteful.

In this case, households seek to maximize a lifetime utility function given by:

$$E_0 \sum_{t=0}^{\infty} \beta_i^t U(\tilde{c}_{i,t}, h_{i,t}, l_{i,t}) = E_0 \sum_{t=0}^{\infty} \beta_i^t \left\{ \ln(\tilde{c}_{i,t} - \zeta\ \tilde{c}_{i,t-1}) + \vartheta \ln(h_{i,t}) - \varphi \frac{(l_{i,t})^{1+\eta}}{1+\eta} \right\},\ i = \{P, I\}, \tag{36}$$

and entrepreneurs:

$$E_0 \sum_{t=0}^{\infty} \beta_E^t U(\tilde{C}_{E,t}) = E_0 \sum_{t=0}^{\infty} \beta_E^t \ln(\tilde{C}_{E,t} - \zeta\ \tilde{C}_{E,t-1}). \tag{37}$$

The existing literature examining the elasticity of substitution between private and public consumption finds ambiguous results. While several empirical studies find evidence of substitutability between private and public consumption (see Kormendi (1983), Aschauer (1985), and Ahmed (1986)), some other studies find complementarity (see Karras (1994), Ni (1995), and Fiorito and Kollintzas (2004)). Therefore, I examine different values for γ. In particular, I discuss the cases where the goods are substitutes (γ =0.5) and

[43] Lowercase letters with subscript P or I are expressed per patient or impatient household, respectively.

complements (γ =-0.5). I set the weight of private consumption $\varrho = 0.8$ as in Bouakez and Rebei (2007).

Figure 3 reports the effects of a fiscal consolidation through a government spending adjustment when public and private consumption are substitutes (dotted line), complements (dashed line) and compares the results when government spending is complete wasteful (solid line) as in the baseline economy displaying high levels of private debt in the three cases. Figure 3 illustrates that, qualitatively, the results do not dramatically change when agents derive utility from government consumption. If private and public consumption are substitutes, a decrease in public consumption raises the marginal utility of the private good. Agents react increasing private consumption and reducing their housing stock, so that private debt increases less compare with the wasteful case in which agents do not derive utility from public consumption. When γ =-0.5, private and public consumption become complements as a decrease in government consumption decreases the marginal utility of private consumption. Figure 3 shows that total consumption decreases in response to the drop in government consumption despite the increase in the first periods. Initially impatient households compensate the decrease in utility increasing their housing stock, which generates a rise in their collateral values and house prices allowing them to consume more. Private debt increases more in this case but does not compensate the drop in total consumption, leading to a higher fall in output.

Table 8 reports the welfare effects in consumption equivalence units when public consumption is wasteful (column 1), and when public and private consumption are substitutes (column 2) and complements (column 3). The welfare gains are reduced when public and private consumption are substitutes and complements compared to the case in which public consumption is wasteful. When public and private consumption are substitutes, the welfare increase for impatient households and entrepreneurs are smaller compared to the wasteful case. This is due to the combined effect of a lower increase in private debt, which reduces their housing stock, with the dynamics of government consumption which generates a drop in private consumption and then reducing the utility of agents. In the case of complementarity the reductions in welfare are caused by entrepreneurs due to a lower consumption.

5.2 Productivity of public capital

I have also examined the effects of a government investment based consolidation. In order to do it, I consider government investment in productive public capital as in Baxter and King (1993). This new feature allows for a positive externality of public capital due to an increase in the productivity of private firms.

I extend the baseline model by introducing a new production function which incorporates public capital

Table 8: Welfare effects when government spending is used as fiscal instrument and provides utility to agents

	Wasteful	Substitutes	Complements
	(1)	(2)	(3)
Social Welfare	0.232	0.114	0.138
Patient households	0.089	0.105	0.104
Impatient households	0.088	-0.018	0.205
Entrepreneurs	0.055	0.028	0.011

Notes: Welfare is defined in consumption equivalent units, i.e. the variation in consumption that would be necessary to achieve the same welfare as through the impact of the fiscal consolidation. The first column for each fiscal instrument refers to the baseline economy that features high private debt. The second and the third column refers to an economy in which private and public consumption goods are substitutes and complements, respectively, and private debt levels are set to the baseline economy displaying high levels of private debt.

as an input:

$$Y_t = A_t K_{t-1}^{\mu} K_{G,t-1}^{\alpha_G} H_{E,t-1}^{\nu} {L_{P,t}}^{\alpha(1-\mu-\nu)} L_{I,t}^{(1-\alpha)(1-\mu-\nu)}, \tag{38}$$

where $K_{G,t-1}$ is the public capital stock in period t and the parameter α_G is the elasticity of output with respect to public capital.[44] The accumulation of public capital is given by:

$$G_{I,t} = K_{G,t} - (1-\delta^G) K_{G,t-1}, \tag{39}$$

where $\delta^G \in (0,1)$ is the depreciation rate. I define a fiscal rule for government investment, $G_{I,t}$:

$$\widehat{G}_{I,t} = \rho_f \, \widehat{G}_{I,t-1} - (1-\rho_f)\Big[\phi_1^{G_I}\Big(B_{t-1}^{gy} - B^{gy^*}\Big) + \phi_2^{G_I}\Big(B_t^{gy} - B_{t-1}^{gy}\Big)\Big]. \tag{40}$$

In this case, the government budget constraint is given by:[45]

$$B_{G,t} = \frac{R_{t-1}}{\Pi_t} B_{G,t-1} + G_t + G_{I,t} - Tax_t, \tag{41}$$

and finally, the new resource constraint is given by:

$$Y_t = C_{P,t} + C_{I,t} + C_{E,t} + I_t + G_t + G_{I,t} + \xi_{k,t}. \tag{42}$$

I set the depreciation rate of public capital equal to the depreciation rate of private capital, $\delta^G = 0.025$.

[44] I assume that there are constant returns to scale over privately provided inputs: $\mu + \nu = 1$.

[45] $Tax_t = \tau_t^c(C_{P,t} + C_{I,t} + C_{E,t}) + \tau_t^w(w_{P,t}L_{P,t} + w_{I,t}L_{I,t}) + \tau_t^k(mc_t Y_t - w_{P,t}L_{P,t} - w_{I,t}L_{I,t} + \delta^k K_{t-1} + \Pi_t^r) + \tau_t^l$.

The productivity parameter is set to $\alpha_G = 0.05$ as in Baxter and King (1993). Government investment spending is calibrated to match government investment-to-output ratio at the value of 3.3%, as in 2016.[46] I consider the common assumption that government spending does not provide utility to consumers ($\varrho = 1$).

Table 9: Steady-state and cumulative changes in response of a fiscal consolidation via government investment

Private indebtedness:	**Steady-state**		**Impact**		**1 years**		**5 years**	
	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)	Low (1)	High (2)
Output	2.03	1.99	-2.43	-1.37	-12.79	-11.68	-65.44	-63.79
Consumption	1.36	1.32	1.49	2.26	11.40	11.96	-0.95	1.44
Investment	2.03	1.99	0.41	1.76	3.77	4.87	-13.25	-14.75
Private debt-to-GDP	-0.02	-0.19	2.11	11.92	10.51	69.96	3.60	104.81

Notes: The first column refers to a low indebted economy in which the loan-to value ratio for impatient households and entrepreneurs are exogenously set to $m_I = m_E = 0.45$ and the proportion of household borrowers is set to $\omega = 0.5$. The second column refers to the baseline economy that features high private debt. Cumulative changes are denoted in cumulative percentage changes from initial steady state, except for the private debt-to-GDP ratio which is denoted in cumulative percentage point changes.

A consolidation via government investment would have similar effects as a cut in government consumption, with the exception that a public investment cut lowers the stock of public capital. Table 9 reports the steady-state and cumulative changes in response of a fiscal consolidation achieved through government investment. In the short term, a fiscal consolidation achieved by cutting public investment produces a decline in GDP, which is smaller in a high private debt scenario because borrowers (impatient households and entrepreneurs) react increasing their stock of private debt to overcome the negative effects associated with the fiscal consolidation. Entrepreneurs invest in private capital and commercial real estate to temporary substitute for public investment. The increase in housing demand triggers a rise in housing prices. Thus borrowers' collateral values increase allowing them to consume and invest more. In a high private debt scenario, this mechanism is reinforced by higher LTV ratios and a larger proportion of household borrowers. As shown in Table 9 the cumulative increase in consumption and investment are higher in the high private debt scenario which generates a higher rise in output.

[46] The data is from the U.S. Bureau of Economic Analysis.

6 Concluding Remarks

In this paper I analyse the macroeconomic effects of different fiscal consolidation plans in which the government gradually reduces its public debt-to-GDP ratio and the private sector is highly indebted.

To this end, I evaluate the effects of a fiscal consolidation as a permanent decrease in the targeted government debt-to-GDP ratio by means of government spending cuts or tax hikes. I find that in the long run, fiscal consolidation entails output benefits that are dampened when private debt is high. This effect occurs independently of the instrument used to stabilize debt and is especially important when distortionary taxation is used to reduce public debt. In the short run, my findings gives rise to the interpretation that effectiveness of fiscal consolidation in an environment of high private debt depends on which fiscal instruments adjust to stabilize debt. Fiscal policies that rise labour or capital tax rates induce to a private sector deleveraging. The simultaneous private and public sector deleveraging amplifies temporary output losses due to fiscal consolidation. Hence, in an economy characterized by a high level of private indebtedness the negative effects of a fiscal consolidation process are amplified. By contrast, fiscal consolidation achieved by government spending cuts or consumption tax hikes ease private debt repayment, thereby mitigating the negative output effect associated with a public debt reduction.

Regarding social welfare, I find that a fiscal consolidation produces a social welfare gain when government spending or consumption tax rates are used as fiscal instruments to stabilize debt. This effect is amplified in an environment of high private debt through the increase in private debt, which allows agents to consume more and increase their housing units. However, it entails a social welfare loss when capital or labour tax rates adjust that is amplified in a heavily indebted economy due to the private sector deleveraging.

In the paper, the firms' borrowing constraint is determined by the liquidation value of physical assets that firms can pledge as collateral as in Hart and Moore (1994), Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). Lian and Ma (2021) show that, in a sample of listed firms, large firms' constraints are determined by cash flows, whereas small firms are more dependent on asset values. Drechsel et al. (2019) in a New Keynesian model considers these two types of borrowing constraints and concludes that they give rise to different transmission mechanisms of macroeconomic shocks. New forms of modelling firms' borrowing constraints will be explored in future research.

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Appendix

A Solving the baseline model

In this appendix, I provide a detailed description of the model developed in section 2.

A.1 Patient households (savers)

$$\max_{c_{P,t},h_{P,t},l_{P,t},b_{P,t},b_{G,t}} \; E_0 \sum_{t=0}^{\infty} \beta_P^t \left\{ \ln(c_{P,t} - \zeta \; c_{P,t-1}) + \vartheta \ln(h_{P,t}) - \varphi \frac{(l_{P,t})}{1+\eta}^{1+\eta} \right\}, \tag{43}$$

subject to:

$$(1+\tau_t^c)c_{P,t} + p_t^h(h_{P,t} - h_{P,t-1}) + b_{P,t} + b_{G,t} = (1-\tau_t^w)w_{P,t}l_{P,t} + \frac{R_{t-1}}{\Pi_t}(b_{P,t-1} + b_{G,t-1}) - \tau_{P,t}^l. \tag{44}$$

Intertemporal maximization yields the following first-order conditions for consumption, debt, housing holdings and labour supply:

$$\lambda_{P,t} = \frac{1}{x_{P,t}(1+\tau_t^c)}, \tag{45}$$

$$\frac{1}{x_{P,t}(1+\tau_t^c)} = \beta_P E_t \left[\frac{R_t}{x_{P,t+1}(1+\tau_{t+1}^c)\Pi_{t+1}} \right], \tag{46}$$

$$\frac{p_t^h}{x_{P,t}(1+\tau_t^c)} = \frac{\vartheta}{h_{P,t}} + \beta_P E_t \left[\frac{p_{t+1}^h}{x_{P,t+1}(1+\tau_{t+1}^c)} \right], \tag{47}$$

$$w_{P,t} = \varphi(l_{P,t})^{\eta} \frac{(1+\tau_t^c)}{(1-\tau_t^w)} x_{P,t}, \tag{48}$$

where $\lambda_{P,t}$ is the Lagrange multiplier associated with equation (44), which is the budget constraint of patient households, $\Pi_t = p_t/p_{t-1}$ represents the gross inflation rate and $x_{P,t} = c_{P,t} - \zeta \; c_{P,t-1}$. Equation (45) states that the current marginal utility of consumption is the inverse of actual consumption. Equation (46) is the Euler equation for patient households. Equation (47) determines the housing demand. Finally, equation (48) is the first order condition for labour supply. This condition pins down hours of work as a function of the wage.

A.2 Impatient households (borrowers)

$$\max_{c_{I,t},h_{I,t},l_{I,t},b_{I,t}} \; E_0 \sum_{t=0}^{\infty} \beta_I^t \left\{ \ln(c_{I,t} - \zeta \; c_{I,t-1}) + \vartheta \ln(h_{I,t}) - \varphi \frac{(l_{I,t})^{\;1+\eta}}{1+\eta} \right\}, \tag{49}$$

subject to:

$$1 + \tau_t^c) c_{I,t} + p_t^h (h_{I,t} - h_{I,t-1}) + \frac{R_{t-1}}{\Pi_t} b_{I,t-1} = (1 - \tau_t^w) w_{I,t} l_{I,t} + b_{I,t} - \tau_{I,t}^l, \tag{50}$$

$$b_{I,t} \leq \rho_c \frac{b_{I,t-1}}{\Pi_t} + (1 - \rho_c) \; m_I E_t \left[\frac{p_{t+1}^h \Pi_{t+1} h_{I,t}}{R_t} \right]. \tag{51}$$

Intertemporal maximization yields the following first-order conditions for consumption, loans, housing holdings and labour supply:

$$\lambda_{I,t} = \frac{1}{x_{I,t}(1 + \tau_t^c)}, \tag{52}$$

$$\frac{1}{x_{I,t}(1 + \tau_t^c)} = \beta_I E_t \left[\frac{R_t}{x_{I,t+1}(1 + \tau_{t+1}^c)\Pi_{t+1}} \right] - \beta_I E_t \left[\rho_c \frac{\mu_{I,t+1} R_{t+1}}{\Pi_{t+1}} \right] + \mu_{I,t} R_t \tag{53}$$

$$\frac{p_t^h}{x_{I,t}(1 + \tau_t^c)} = \frac{\vartheta}{h_{I,t}} + E_t \left[\frac{\beta_I p_{t+1}^h}{x_{I,t+1}(1 + \tau_{t+1}^c)} + (1 - \rho_c) \mu_{I,t} m_I p_{t+1}^h \Pi_{t+1} \right], \tag{54}$$

$$w_{I,t} = \varphi (l_{I,t})^{\eta} \frac{(1 + \tau_t^c)}{(1 - \tau_t^w)} x_{I,t}, \tag{55}$$

where $\lambda_{I,t}$ is the Lagrange multiplier associated with equation (50) the budget constraint of impatient households, $\mu_{I,t}$ is the Lagrange multiplier associated with equation (51) the borrowing constraint and $x_{I,t} = c_{I,t} - \zeta \; c_{I,t-1}$. Equation (52) and equation (55) are identical to the first-order condition for the patient households. Equation (54) is identical to the housing demand equation for the patient households except for the term $(1 - \rho_c)\mu_{I,t} m_I p_{t+1}^h \Pi_{t+1}$. This term captures that when impatient households increase their housing stock it also relaxes their budget constraint and allows them to borrow more against the value of their houses. The Euler equation (53) differs from the Euler equation of patient households in the term $\mu_{b,t} R_t$, which is included because increasing consumption incurs an extra cost by tightening the collateral constraint and also in the term $\beta_I E_t \left[\rho_c \mu_{I,t+1} R_{t+1} \right]$.

A.3 Entrepreneurs (borrowers)

$$\max_{C_{E,t},b_{E,t},I_t,K_t,H_{E,t},L_{P,t},L_{I,t}} \quad E_0 \sum_{t=0}^{\infty} \beta_E^t \ln(C_{E,t} - \zeta\ C_{E,t-1}) \tag{56}$$

subject to:

$$Y_t = K_{t-1}^{\mu}(H_{E,t-1})^{\nu} {L_{P,t}}^{\alpha(1-\mu-\nu)} L_{I,t}^{(1-\alpha)(1-\mu-\nu)} \tag{57}$$

$$\begin{aligned}(1+\tau_t^c)C_{E,t} + p_t^h(H_{E,t} - H_{E,t-1}) + I_t =& (1-\tau_t^k)(mc_t Y_t - w_{P,t}L_{P,t} - w_{I,t}L_{I,t}) + B_{E,t} - \frac{R_{t-1}}{\Pi_t}B_{E,t-1} \\ &+ \tau_t^k \delta^k K_{t-1} + (1-\tau_t^k)\Pi_t^r - \xi_{k,t},\end{aligned} \tag{58}$$

$$B_{E,t} \leq \rho_c \frac{B_{E,t-1}}{\Pi_t} + (1-\rho_c)\ m_E E_t\left[\frac{p_{t+1}^h \Pi_{t+1} H_{E,t}}{R_t}\right]. \tag{59}$$

$$I_t = K_t - (1-\delta^k)K_{t-1} \tag{60}$$

The adjustment cost function is:

$$\xi_{k,t} = \frac{\psi}{2\delta^k}\left(\frac{I_t}{K_{t-1}} - \delta^k\right)^2 K_{t-1}. \tag{61}$$

Intertemporal maximization yields the following first-order conditions for consumption, loans, investment, capital, commercial real estate and labour:

$$\lambda_{E,t} = \frac{1}{x_{E,t}(1+\tau_t^c)}, \tag{62}$$

$$\frac{1}{x_{E,t}(1+\tau_t^c)} = \beta_E E_t\left[\frac{R_t}{x_{E,t+1}(1+\tau_{t+1}^c)\Pi_{t+1}}\right] - \beta_E E_t\left[\rho_c \frac{\mu_{E,t+1}R_{t+1}}{\Pi_{t+1}}\right] + \mu_{E,t}R_t. \tag{63}$$

$$u_t = \frac{1}{x_{E,t}(1+\tau_t^c)}\left[1 + \frac{\psi}{\delta^k}\left(\frac{I_t}{K_{t-1}} - \delta^k\right)\right], \tag{64}$$

$$u_t = \beta_E E_t \left[\frac{1}{x_{E,t+1}(1+\tau^c_{t+1})} \left(\frac{\psi}{\delta^k} \left(\frac{I_{t+1}}{K_t} - \delta^k \right) \frac{I_{t+1}}{K_t} - \frac{\psi}{2\delta^k} \left(\frac{I_{t+1}}{K_t} - \delta^k \right)^2 \right) \right] + \beta_E E_t \left[\frac{1}{x_{E,t+1}(1+\tau^c_{t+1})} \left((1-\tau^k_{t+1}) \mu \frac{mc_{t+1} Y_{t+1}}{K_t} \right) + u_{t+1}(1 - \delta^k(1-\tau^k_{t+1})) \right], \tag{65}$$

$$\frac{p^h_t}{x_{E,t}(1+\tau^c_t)} = E_t \left[\frac{\beta_E}{x_{E,t+1}(1+\tau^c_{t+1})} \left((1-\tau^k_{t+1}) \nu \frac{mc_{t+1} Y_{t+1}}{H_{E,t}} + p^h_{t+1} \right) + (1-\rho_c) \mu_{E,t} m_E p^h_{t+1} \Pi_{t+1} \right], \tag{66}$$

$$w_{P,t} = mc_t \frac{\alpha(1-\mu-\nu) Y_t}{L_{P,t}}, \tag{67}$$

$$w_{I,t} = mc_t \frac{(1-\alpha)(1-\mu-\nu) Y_t}{L_{I,t}}, \tag{68}$$

where $\lambda_{E,t}$ is the Lagrange multiplier associated with equation (58) the budget constraint, $\mu_{E,t}$ is the Lagrange multiplier associated with equation (59) the collateral constraint, u_t is the Tobin's Q and $x_{E,t} = C_{E,t} - \zeta\, C_{E,t-1}$. Equation (63) is the Euler equation, equation (64) and (65) are the investment decisions and equation (66) is the housing demand equation. Finally, equation (67) and (68) determines labour demand.

A.4 Retailers

The retailers' problem is:

$$\max_{P^*_t} \; E_0 \sum_{k=0}^{\infty} (\beta \theta_p)^k \; \Lambda_{t+k} \left[\left(\frac{P^*_t(z)}{P_{t+k}} - mc_{t+k} \right) Y_{t+k}(z) \right] \tag{69}$$

subject to:

$$Y_{t+k}(z) = \left(\frac{P_t(z)}{P_{t+k}} \right)^{-\epsilon} Y^d_{t+k}. \tag{70}$$

The first order condition of this problem is:

$$\sum_{k=0}^{\infty} \theta^k_p E_t \left\{ \Lambda_{t+k} \left(\frac{1}{P_{t+k}} Y_{t+k} - \epsilon \frac{1}{P_{t+k}} Y_{t+k} + \epsilon \frac{mc_{t+k}}{P^*_t} Y_{t+k} \right) \right\} = 0. \tag{71}$$

After some algebra, I derive equation (12).

A.5 Aggregation

Aggregate consumption for patient households and impatient households is given by:

$$C_{P,t} = (1-\omega)c_{P,t},$$
$$C_{I,t} = \omega c_{I,t},$$

Aggregate consumption in the economy is computed as the sum of entrepreneurs' consumption and households' consumption:

$$C_t = C_{E,t} + C_{P,t} + C_{I,t}.$$

Aggregate labour for patient and impatient households are given by:

$$L_{P,t} = (1-\omega)l_{P,t},$$
$$L_{I,t} = \omega l_{I,t}.$$

Aggregate private debt is given by:

$$B_{P,t} = (1-\omega)b_{P,t},$$
$$B_{I,t} = \omega b_{I,t}.$$

Aggregate public debt is given by:

$$B_{G,t} = (1-\omega)b_{G,t},$$

Aggregate housing is given by:

$$H_{P,t} = (1-\omega)h_{P,t},$$
$$H_{I,t} = \omega h_{I,t},$$

Finally, aggregate lump-sum taxes are given by:

$$\tau_t^l = (1-\omega)\tau_{P,t}^l + \omega\tau_{I,t}^l,$$

I assume that patient and impatient households pay the same amount of lump-sum taxes i.e. $\tau_{P,t}^l = \tau_{I,t}^l = \tau_t^l$.

B Data and sources

This appendix presents the data and sources for calibrating the steady state to match key statistics of the U.S. economy in 2016.

- **Household debt:** Total liabilities of households and non-profit organizations. Source: Board of Governors of the Federal Reserve System, Flow of Funds Accounts, Table B.101.
- **Non-financial entrepreneurial debt:** Sum of debt securities and loans of non-financial corporate business and loans of non-financial non-corporate business. Source: Board of Governors of the Federal Reserve System, Flow of Funds Accounts, Table B.103 and B.104.[47]
- **Public debt-to-GDP**: Federal debt held by the public as percentage of gross domestic product. Source: Congressional Budget Office, Historical Budget Data.
- **Government consumption and investment:** Government consumption expenditures and gross government investment. Source: U.S. Bureau of Economic Analysis, Table 3.1. Government Current Receipts and Expenditures.

I construct consumption, labour income and capital income tax rates following the methodology proposed by Mendoza, Razin, and Tesar (1994). I estimate the effective tax rate as the ratios between the tax revenues from particular taxes and the corresponding tax bases using data from the OCDE *National Accounts* and *Revenue Statistics* data from the same source.

- **Effective tax rate on consumption:** The effective tax rate on consumption is computed as the ratio of the revenue from indirect taxation and the pre-tax value of consumption:

$$\tau^c = \frac{T_{5110} + T_{5121}}{C + G - CE - (T_{5110} + T_{5121})} \tag{72}$$

where the numerator comprises T_{5110}=*General taxes on goods and services* and T_{5121}=*Excise taxes*. The denominator is the base of the consumption tax, *C=Private final consumption expenditures*, *G=Government final consumption expenditures* and *CE=Compensation of employees paid by producers of government services*. The formula subtracts revenues from indirect taxation due to the fact that nominal consumption expenditures in national accounts are at post-tax prices.[48]

[47] This data serie is also available at the IMF and BIS.

[48] Government consumption of goods must be included in the denominator because *Revenue Statistics* reports data on indirect tax revenue that includes taxes paid by the government.

- **Effective tax rate on labour income:** As emphasized by Mendoza, Razin, and Tesar (1994) tax revenue sources do not provide a breakdown of individual tax revenue in terms of labour and capital income. In order to decompose tax revenue from labour and capital income, I compute a personal income tax rate on labour and capital:

$$\tau^h = \frac{T_{1100}}{OSMI + PI + W}, \tag{73}$$

where the numerator is the individual income tax revenue and the denominator is the pre-tax household income. T_{1100}=*Taxes on income, profits, and capital gains of individuals*, *OSMI=Operating surplus and mixed income*, *PI=Property income and W=wages and salaries.*

Then, I compute effective tax rate on labour income as the ratio of the revenues of labour income taxes to the labour income of employees:

$$\tau^w = \frac{\tau^h W + T_{2000} + T_{3000}}{W + T_{2200}}, \tag{74}$$

where $\tau^h W$ is the revenue from the income tax on wages and salaries, T_{2000}= *Total social security contributions* and T_{3000}= *Taxes on payroll and taxes.* In the denominator the tax based is enlarged with T_{2200}=*Employers contribution to social security.*

- **Effective tax rate on capital income:** The effective tax rate on capital income is computed as the ratio of the revenues of capital income taxes to the capital income:

$$\tau^k = \frac{\tau^h (OSMI + PI) + T_{1200} + T_{4100} + T_{4400}}{OS}, \tag{75}$$

where T_{1200}=*Taxes on income, profits, and capital gains of corporations*, T_{4100}=*Recurrent taxes on immovable property*, T_{4400}=*Taxes on financial and capital transactions* and OS=*Total operating surplus of the economy.*

C Additional figures

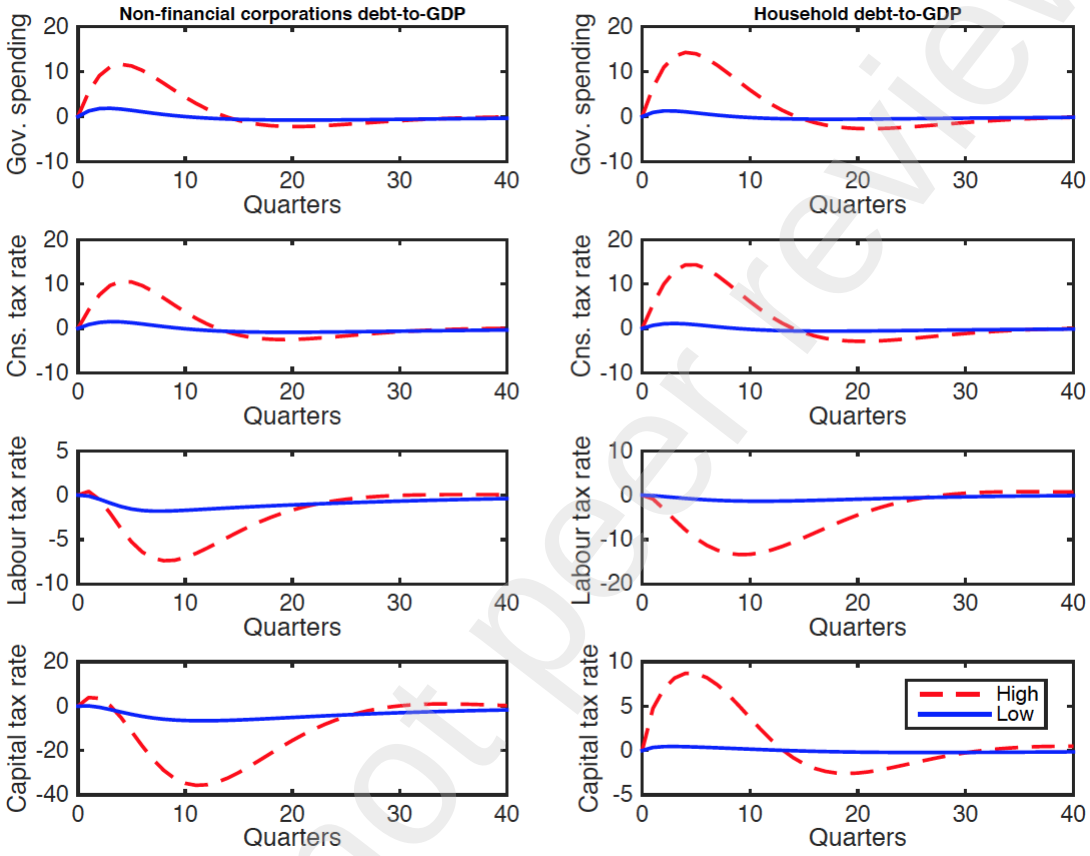

Figure 4: Private debt-to-GDP ratio dynamics

Notes: The dashed red line corresponds with the high private debt scenario (baseline) and the solid blue line with low private debt scenario ($m_I = m_E = 0.45$ and $\omega = 0.5$). Time on the x-axis is measured in quarters. The percentage point change from initial steady-state is measured in the y-axis.