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Article *in* The Journal of the Acoustical Society of America · October 2010

DOI: 10.1121/1.3479756 · Source: PubMed

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On the limitation of a diffusion equation model for acoustic predictions of rooms with homogeneous dimensions (L)

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(Received 9 March 2010; revised 26 July 2010; accepted 26 July 2010)

In recent years a model for predicting sound fields in enclosures has been proposed, based on the mathematical theory of diffusion. This model is held to be valid for predicting the late reverberation component of the impulse response, on the basis that sufficient reflection events must occur [Valeau *et al.*, J. Acoust. Soc. Am. **119**, 1504–1513 (2006)]. The present work determines numerically the extent of reflections necessary for the solution of the diffusion equation model to be accurate in quasi-cubic rooms. Some preliminary numerical experiments have been carried out to determine after how many mean-free times of the impulse response, which is obtained by a geometrical-acoustic approach, gives a similar result to the solution obtained from a diffusion equation model. © *2010 Acoustical Society of America.* [DOI: 10.1121/1.3479756]

PACS number(s): 43.55.Br, 43.55.Ka [NX] Pages: 1586–1589

I. INTRODUCTION

Ollendorff[1] and, more recently, Valeau *et al.*,[2] have proposed a new method for predicting the sound field in arbitrary rooms. This method employs a diffusion equation model and gives good accuracy with low computational expense. Other authors have applied the diffusion equation to varying types of rooms, including coupled spaces.[3,4] The model assumes, however, that a sufficient number of reflection events occur to ensure that there are only small variations in the acoustic energy density and energy flow per mean-free path.[5] This suggests that the acoustic diffusion model is valid mainly for predicting the late part of reverberating sound fields in enclosed spaces.

These assumptions require a certain time delay after direct sound arrival in order to yield a sound energy distribution that can be compared against the solution of a diffusion equation model. Some authors provide tentative values of this time delay.[3,4] No study of the validity of this argument has yet been made, however. The present letter provides preliminary comparisons of several room-acoustic parameters, obtained from the impulse response of a geometrical-acoustic approach and from the energy decay function of the diffusion model. The results are discussed in order to assist architectural acousticians in gaining a coherent understanding of the diffusion equation model.

II. ACOUSTIC DIFFUSION EQUATION

The acoustic diffusion equation model is based on the analogy between sound energy density and a density of *sound particles* traveling at velocity c along straight lines. This method, which has been further developed by Picaut *et al.*[5] and Valeau *et al.*,[2] allows the modeling of acoustics of enclosed spaces which have diffusely reflecting surfaces, similar to the model for the diffusion of particles in a scattering medium.

Consider now the acoustic energy density, $w(\mathbf{r},t)$ as a function of position, $\mathbf{r}$, and time, t defined on a domain V. It satisfies a second order partial differential equation with mixed boundary conditions,[2,6]

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) + cmw(\mathbf{r},t) = P(t)\delta(\mathbf{r}-\mathbf{r}_s) \quad \text{in} \quad V, \tag{1}$$

$$-D\frac{\partial w(\mathbf{r},t)}{\partial \mathbf{n}} = Acw(\mathbf{r},t) \quad \text{on} \quad \partial V. \tag{2}$$

Equation (1) is the inhomogeneous diffusion equation, where ∇^2 is the Laplace operator, $D=\lambda c/3$ is the diffusion coefficient, and c is the speed of sound. The diffusion coefficient is derived from the theory of diffusion for particles in a scattering medium. This theory considers collisions of "particles" at diffusely reflecting boundaries defining the room in terms of the *mean-free path* λ, which corresponds to the average distance that a "sound particle" can travel without impact.[5] According to statistical room-acoustics theory, the mean-free path in a room of volume V and interior surface area S is $\lambda=4V/S$. The *mean-free time* of a sound particle is defined as the average time between collisions, λ/c. The term m accounts for the air absorption.[7] The right hand side of Eq. (1) is a source function $P(t)$ located at position $\mathbf{r}_s$.

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Equation (2) expresses boundary conditions that model the local effects on the sound field of differing degrees of absorption on surfaces. The absorption factor $A = A(\mathbf{r}, \alpha)$ (where α is the absorption coefficient) takes different forms for rooms with low and high absorption:

$$A_S(\mathbf{r}, \alpha) = \frac{\alpha(\mathbf{r})}{4}, \tag{3}$$

$$A_E(\mathbf{r}, \alpha) = \frac{-\log(1 - \alpha(\mathbf{r}))}{4}, \tag{4}$$

$$A_M(\mathbf{r}, \alpha) = \frac{\alpha(\mathbf{r})}{2(2 - \alpha(\mathbf{r}))}. \tag{5}$$

The Sabine absorption factor,[2] $A_S(\mathbf{r}, \alpha)$, has been shown to be accurate only for boundaries with low absorption. To improve the accuracy of the model for boundaries with strong absorption, the Eyring absorption coefficient, $A_E(\mathbf{r}, \alpha)$, has been defined.[3,8] These models have their own limitations, however, and Jing and Xiang proposed a modified absorption factor,[6] $A_M(\mathbf{r}, \alpha)$.

III. SIMULATIONS AND ANALYSIS

In the present work, the time at which the diffusion equation model becomes valid is estimated. Valeau *et al.* assert that this model becomes valid after one mean-free time,[2] but Xiang *et al.* state that at least two mean-free times are needed.[4] Neither claim has yet been validated systematically.

The recent studies on the diffusion equation model have been mainly focused on evaluating the reverberation time (RT). This room-acoustic parameter is usually calculated using Schroeder's backward integration method.[4] The standard method derives the reverberation time from a logarithmic slope regression between −5 dB and −35 dB, so that it gives only very weak dependence of reverberation on the early reflections. However, this room-acoustic parameter does not seem adequate to estimate the time to be disregarded from the impulse response, and therefore the dependence on the first order reflections.

In the present paper, several room-acoustic parameters are evaluated after disregarding a certain initial duration from the impulse response to examine when they equal to those provided by the diffusion equation model. These investigations were performed using ray-tracing software with diffuse reflections.

Four further room-acoustic parameters are calculated: the early decay time (EDT), definition (D_{50}), clarity (C_{80}) and center time (CT). These parameters may be approximately calculated from the diffusion equation model since they have a strong dependence on the energy of the first order reflections.[9] However, they are suitable for use as a reference when analyzing the decay slope function. The approach of these investigations is to disregard an appropriate initial interval from the ray-tracing impulse response, which is proportional to the time for sound to travel the mean-free path, λ / c, and to compare these room-acoustic parameters with those obtained from the diffusion equation model. To

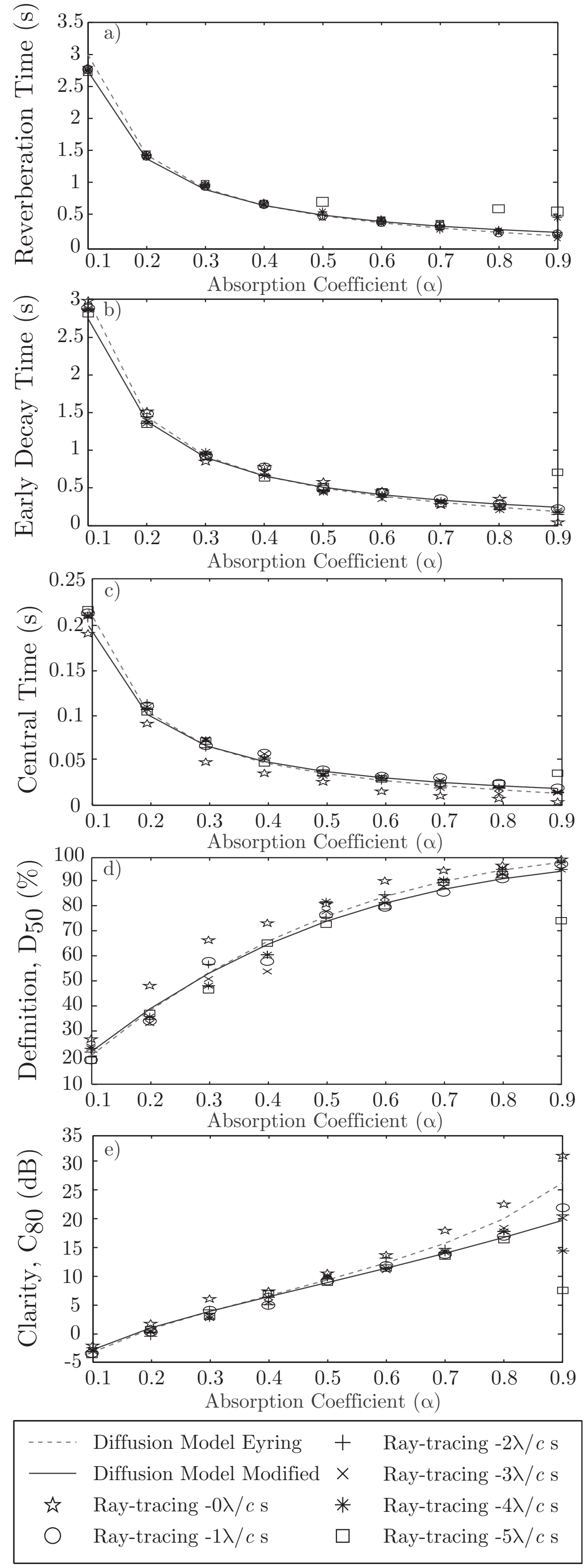

FIG. 1. Comparison of room-acoustic parameters: (a) reverberation time, (b) early decay time, (c) central time, (d) speech definition, and (e) clarity, where the diffusion equation model is compared to the ray-tracing after disregarding the early energy of 0 to 5 mean-free times, as a function of the overall absorption coefficient.

obtain these parameters, all of the integrals involved in the room-acoustic parameters calculation are taken up to time $t = n\lambda / c$, where $t = 0$ is the arrival time of the direct sound and n is the number of mean-free times to be disregarded. In

TABLE I. Average difference and their corresponding standard deviation (indicated inside the brackets) when the early energy of zero to five mean-free times (mft) is disregarded. All the data are computed as a function of the absorption coefficient, ranged to 0.1 to 0.9.

	RT (%)	EDT (%)	D_{50} (%)	C_{80} (dB)	CT (ms)
0 mft	5.2(4.2)	20.3(23.5)	13.8(7.9)	3.4(3.5)	13.4(2.9)
1 mft	5.0(5.4)	7.7(4.9)	6.2(5.3)	1.0(0.6)	4.3(5.3)
2 mft	4.6(5.4)	11.7(10.2)	4.6(2.9)	1.2(0.5)	4.2(3.8)
3 mft	8.3(9.3)	11.2(7.7)	6.9(5.3)	0.9(0.6)	5.3(3.2)
4 mft	16.2(30.5)	6.4(8.3)	5.3(3.7)	1.1(1.7)	4.4(2.9)
5 mft	37.6(55.1)	29.9(63.0)	6.8(7.3)	1.8(4.0)	5.4(7.0)

other words, the parameter D_{50} is calculated by integrating over the early times from $t=n\lambda/c$ to $t=n\lambda/c+50$ ms.

The simulations were made in a shoe-box configuration $(10\times 20\times 10\ \text{m}^3)$, with a uniform absorption coefficient that increases monotonically from 0.1 to 0.9 at all interior surfaces. The diffusion equation model assumes that the overall absorption is not high,[2,6] and the range of absorption coefficients has been chosen for illustrative purposes. The diffusion equation model is implemented by a finite element modeling software, and the room-acoustic parameters are calculated using two distinct boundary conditions: Eyring and modified boundary conditions.

For each configuration, ray-tracing-based simulations were conducted using EASE commercial software running with the AURA module, which incorporates complete diffusion and enables uniform scattering from the interior surfaces of the room.[10] Both simulations were performed at the same source-(2.5,10,5) m-and receiver positions-(7.5,10,5), (5,5,7.5) and (5,15,2.5) m, and the results were evaluated at the octave frequency-band of 1 kHz. To facilitate comparison this study disregards the direct sound and the early reflections from the ray-tracing impulse responses, and starts the calculation of the room-acoustic parameters after one to five mean-free times; in this specific room, this mean-free time corresponds to 23.5 ms. In all configurations, the room-acoustic parameters were obtained by averaging the results at the different positions.

Figure 1 shows the room-acoustic parameters as functions of the absorption coefficients ranging between 0.1 and 0.9. As expected, results of the reverberation time and the early decay time [see Figs. 1(a) and 1(b), respectively] do not show any particular dissimilarity when part of the ray-tracing impulse response is ignored. This shows that the decay of the Schroeder curves is linear, at least according to the reverberation time. Clear differences can be observed in the other room-acoustic parameters, however. The most significant are those obtained from the entire impulse response, since this includes the early reflections. If one or more mean-free times are disregarded, the agreement between ray-tracing and the diffusion equation model is considerably improved. Furthermore, in most cases, differences between the different boundary conditions used in the diffusion equation model are small for the clarity parameter C_{80}; for C_{80}, the modified boundary condition agrees well with most of the ray-tracing results, whereas the corresponding curve to the Eyring boundary condition starts to grow faster from $\alpha=0.6$, with considerable differences. These results further support the choice of the modified boundary condition for the diffusion equation model. Although the results are improved, there are some oscillations around the diffusion equation model solution, and the decision of how many mean-free times to exclude becomes more difficult.

It has been compared the results of the ray-tracing simulations with the results of applying the diffusion model equation using the modified boundary conditions. These differences are also evaluated in terms of *just noticeable differences*. According to ISO/DIS 3382, the subjective threshold for the reverberation time, the early decay time and the definition index is 5%, whereas for the clarity index it is required to be 0.5 dB and the central time is 10 ms. Table I lists the average difference and its corresponding standard deviation in each situation. The mean has been computed as a function of the absorption coefficients in a range from 0.1 to 0.9. As expected, the greatest average differences are found using the entire ray-tracing impulse response, whereas if one mean-free time is disregarded, the average difference approaches the subjective limens. However, the average difference in these terms becomes greater than the subjective threshold for the early decay times, ranging between 7.7% and 11.2% when the early energy of one to three mean-free times are disregarded; the difference for clarity is approximately 1 dB. These results indicate that at least one mean-free time should be disregarded, as proposed by Valeau *et al.*[2]

However, the diffusion equation model assumes that the scattered or reflected energy is greater than that absorbed, so that predictions for surface materials having low-to-medium absorption coefficients should be accurate. For this reason a new averaging process has been undertaken, this time as a function of absorption coefficients varying between 0.1 and 0.5. Table II lists these results. The average error is reduced in all cases, especially for the clarity index C_{80}, for which the results are very close to 0.5 dB. Major differences in the definition parameter D_{50} are concentrated in the absorption coefficient range between 0.2 and 0.4 [see Fig. 1(d)]; however, when the early energy of two mean-free times is disregarded, the average difference becomes nearly the same as the previous scenario. In the range of absorption coefficients between 0.1 and 0.5 it is therefore affirmed that the average difference is minor in most of the room-acoustic parameters.

Table II lists that at least three room acoustic parameters-the reverberation time, the definition index and

TABLE II. Average differences and their corresponding standard deviation (indicated inside the brackets) when the early energy of zero to five mean-free times (mft) is disregarded. All the data are computed as a function of the absorption coefficient, ranged to 0.1 to 0.5.

	RT (%)	EDT (%)	D_{50} (%)	C_{80} (dB)	CT (ms)
0 mft	3.9(1.9)	11.3(5.2)	18.7(7.0)	1.3(0.6)	12.3(3.6)
1 mft	3.1(2.1)	8.1(5.8)	9.9(4.1)	0.6(0.5)	6.6(6.2)
2 mft	2.9(2.3)	7.7(3.4)	5.2(4.0)	0.8(0.6)	5.7(4.7)
3 mft	3.8(2.2)	7.0(4.8)	9.9(5.4)	0.7(0.5)	7.0(2.7)
4 mft	5.2(3.9)	4.6(3.9)	8.1(1.7)	0.6(0.5)	5.9(3.0)
5 mft	12.0(17.6)	3.0(0.7)	6.7(6.3)	0.5(0.3)	5.7(6.9)

the central time–have lowest difference when the early energy of two mean-free times is disregarded from the ray-tracing impulse response. The early decay time and the clarity index do not have their lowest difference in this situation, but their differences when the early energy of one or three mean-free times is excluded can be considered negligible. In consequence, it is indeed necessary to disregard at least the early energy of two mean-free times from the ray-tracing impulse response, consistent with recent works.[4]

The point beyond which the diffusion equation model is valid is somewhat related to the *mixing time*. The latter is defined as the time at which the transition occurs from early reflections to late reverberation. Some authors suggest that it is equal to three mean-free times.[11] This has to be considered as an upper limit, however, because the transition in the impulse response is highly dependent on the geometry and on the diffusion properties of boundaries.[12] The diffusion equation model assumes that the scattering density is high, and that reflection of energy is dominant over absorption, so that after numerous diffuse reflections the energy density becomes nearly isotropic. This does not mean, however, that a perfectly diffuse sound field prevails in the diffusion equation model from the beginning. Also, the late reverberation assumes that the acoustical energy density is distributed uniformly across a room with homogeneous dimensions.

These findings suggest that the diffusion equation models not only the late reverberation but also a certain fraction of early reflections (approximately one mean-free time) prior to the time at which mixing occurs. A more detailed analysis and further experiments should be conducted in order to provide sharper conclusions.[13]

IV. CONCLUSIONS

The diffusion equation model has recently been deployed to study room acoustics; it is accurate mainly in predicting the late part of the decay process. However, no systematic studies have yet been published of the number of discrete early reflections, measured in terms of the mean-free time, after which the diffusion equation model can be considered valid. Comparison of a ray-tracing simulation with complete diffusing boundaries show that there is good agreement between these methods whenever the first order reflections are excluded from the impulse response of the ray-tracing simulations. Several different room-acoustic parameters are employed, after disregarding the early energy of several mean-free times, to estimate the time at which the diffusion equation model becomes accurate. This study confirms the findings of recent studies that the diffusion equation model is valid after two mean-free times.

The present results suggest that further investigations should be performed, in which the diffusion model is applied in more complex situations such as disproportionated or coupled rooms, and the numerical results should be compared with systematic acoustic experiments.

ACKNOWLEDGMENTS

This research is supported by the Spanish Ministry of Science and Technology (MCYT) under Project ref. TEC2009-14414-C03-01 and FEDER funds, and also by the University of Jaén and Caja Rural de Jaén (Spain) under Project UJA2009/12/12.

[1] F. Ollendorff, "Statistical room-acoustics as a problem of diffusion (a proposal)," Acustica **21**, 236–245 (1969).
[2] V. Valeau, J. Picaut, and M. Hodgson, "On the use of a diffusion equation for room-acoustic prediction," J. Acoust. Soc. Am. **119**, 1504–1513 (2006).
[3] A. Billon, V. Valeau, A. Sakout, and J. Picaut, "On the use of a diffusion model for acoustically coupled rooms," J. Acoust. Soc. Am. **120**, 2043–2054 (2006).
[4] N. Xiang, Y. Jing, and A. Bockman, "Investigation of acoustically coupled enclosures using a diffusion-equation model," J. Acoust. Soc. Am. **126**, 1187–1198 (2009).
[5] J. Picaut, L. Simon, and J. D. Polack, "A mathematical model of diffuse sound field based on a diffusion equation," Acta. Acust. Acust. **83**, 614–621 (1997).
[6] Y. Jing and N. Xiang, "On boundary conditions for the diffusion equation in room acoustic prediction: Theory, simulations, and experiments," J. Acoust. Soc. Am. **123**, 145–153 (2008).
[7] A. Billon, J. Picaut, C. Foy, V. Valeau, and A. Sakout, "Introducing atmospheric attenuation within a diffusion model for room-acoustic predictions," J. Acoust. Soc. Am. **123**, 4040–4043 (2008).
[8] Y. Jing and N. Xiang, "A modified diffusion equation for room-acoustic prediction (L)," J. Acoust. Soc. Am. **121**, 3284–3287 (2007).
[9] M. Vörlander, *Auralization: Fundamentals of Acoustics, Modelling, Simulation, Algorithms and Acoustic Virtual Reality* (Springer, New York, 2007), pp. 92–102.
[10] M. Vorländer, "Simulation of the transient and steady state sound propagation in rooms using a new combined sound particle-image source algorithm," J. Acoust. Soc. Am. **86**, 172–178 (1989).
[11] B. Blesser, "An interdisciplinary synthesis of reverberation viewpoints," J. Audio Eng. Soc. **49**, 867–903 (2001).
[12] J.-D. Polack, "Playing billiards in a concert hall: The mathematical foundations of geometrical acoustics," Appl. Acoust. **38**, 235–244 (1993).
[13] G. Defrance and J.-D. Polack, "Estimating the mixing time of concert halls using the extensible Fourier transform," Appl. Acoust. **71**, 777–792 (2010).