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Influence of the scattering and absorption coefficients on homogeneous room simulations that use a diffusion equation model (L)

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The diffusion equation model was used for room acoustic simulations to predict the sound pressure level and the reverberation time. The technical literature states that the diffusion equation method accurately models the late portion of the room impulse response if the energy is sufficiently scattered. This work provides conclusions on the validity of the diffusion equation model for rooms with homogeneous dimensions in relation to the scattering coefficients of the boundaries. A systematic evaluation was conducted out to determine the ranges of the absorption and scattering coefficient values that result in low noticeable differences between the predictions from a geometrical acoustic model and those from the diffusion equation model. © 2013 Acoustical Society of America. [http://dx.doi.org/10.1121/1.4789928]

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I. INTRODUCTION

A diffusion equation for modeling room acoustics has been successfully used to model enclosures with low absorption surfaces and with diffuse reflection surfaces.¹ This model is considered an extension of diffuse sound field theory with the additional advantages of a dependence on the source/receiver positions and a nonhomogeneous absorption distribution.² However, this acoustic diffusion model assumes that the variations in the energy density and in the energy flow are small over one mean-free path.² This assumption implies that the reflected energy must dominate over the absorbed energy and that the reflection events must be predominantly diffuse.³

With regard to the above assumption, it is important to determine when the energy is scattered enough for the diffusion model to be considered accurate. This model was validated for three-dimensional arbitrary rooms with low absorption surfaces,¹ and later extended to diffusely reflecting rooms with higher absorption values.^{4,5} However, it is difficult to fulfill the total scattering assumption in most real-life cases. To use the diffusion equation model for rooms with specular reflections, an adjustment was proposed⁶ to the diffusion constant to extend the applicability range of the model for predicting the sound pressure level to rooms with non-diffusely reflecting surfaces. In an ongoing study, an empirical law for the diffusion constant was derived as a function of the scattering coefficient⁷ and exhibited good

agreement with the sound pressure level attenuations in several geometries for low absorption surfaces. Recently, an investigation on the validity of Fick's law⁸ stated that the reverberant sound field is well-described by a spatially varying diffusion coefficient, especially for long rooms where the increase in the diffusion coefficient depends on the cross-sectional area of the room and on the absorption coefficients of the boundaries. The value of the diffusion coefficient as a function of the room geometry, reflection features and source position is still an ongoing area of research.

This letter presents a systematic evaluation of a diffusion equation model without any modifications; the theoretical diffusion coefficient depends only on the geometrical features of the room. The purpose of this work is to determine over what ranges of absorption and scattering values the diffusion equation model can be used to produce valid room acoustic predictions for homogeneous rooms. This study compares the predictions of the different room acoustics parameters obtained from a geometrical acoustic model with those obtained from the diffusion equation model for a homogeneous room.

II. ACOUSTIC DIFFUSION EQUATION MODEL

The acoustic diffusion equation is an analytical approximation of the acoustic radiative transfer equation obtained by expanding the sound radiation to first order spherical harmonics.³ The diffusion equation model for the sound energy density at a position \mathbf{r} , and a time t , $w(\mathbf{r}, t)$, defined on a domain V with a sound source term $P(t)$, which is located at the position \mathbf{r}_s , consists of a partial differential equation with mixed boundary conditions,^{1,5}

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$$\frac{\partial w(\mathbf{r}, t)}{\partial t} - D\nabla^2 w(\mathbf{r}, t) + cmw(\mathbf{r}, t) = P(t)\delta(\mathbf{r} - \mathbf{r}_s) \text{ in } V, \quad (1)$$

$$-D \frac{\partial w(\mathbf{r}, t)}{\partial \mathbf{n}} = A(\mathbf{r}, \alpha) cw(\mathbf{r}, t) \text{ on } \partial V. \quad (2)$$

∇^2 is the Laplace operator, and $D = \lambda c/3$ is the diffusion coefficient, where c is the speed of sound. This diffusion coefficient takes into account the room geometry with volume V and total interior area S_r , through its mean-free path $\lambda = 4V/S_r$. The term $cmw(\mathbf{r}, t)$ accounts for the atmospheric attenuation within the room; m is the absorption coefficient of air.

Equation (2) is a mixed boundary condition that models the local effects on the sound field that are induced by the different degrees of surfaces absorption. Different equations for this absorption factor have been proposed in the technical literature.⁵ In this paper, a modified absorption factor is used for the simulations because it has been shown to have the widest range of applicability.^{3,5}

III. NUMERICAL RESULTS AND ANALYSIS

A. Simulation environment

In this work, the range of the absorption and scattering coefficients of the boundaries, denoted by α and s , respectively, over which the diffusion equation model becomes consistent with the geometrical acoustic model, which includes diffuse reflections, are analyzed. It has been suggested that, assuming totally diffuse reflections, the mean absorption coefficient of a homogeneous room with uniformly distributed absorption coefficients should be less than 0.6 to obtain accurate predictions using the diffusion equation.⁵ An empirical modification to the diffusion coefficient as a function of the scattering coefficient⁷ extends the diffusion model, taking into account both specular and diffuse mixed reflections. This empirical diffusion model has exhibited good agreement with geometrical acoustic software, mainly in regard to the sound pressure level attenuation in rooms with low absorption surfaces. For reverberation time predictions, the empirical diffusion model produced reliable results when the scattering coefficient used was greater than 0.4, which mostly correspond to disproportionate rooms with low absorption. However, this relative model requires the estimation of the diffusion coefficient empirical values, which can vary with the room acoustic model. Therefore, this investigation attempts to determine the amount of energy impacting the surfaces that must be diffusely reflected to satisfy the assumption of the diffusion approximation that uses a theoretical diffusion coefficient without modification. These investigations are performed using a ray-tracing software with mixed reflections and by comparing the results from this software with the predicted values from the diffusion equation algorithm.

In the simulation, a shoe-box-shaped room $8 \times 4 \times 3 \text{ m}^3$ is modeled. With respect to a corner of this rectangular cuboid, an omnidirectional sound source is located at (2, 2, 1.5) m with a sound pressure level of 84 dB at 1 m away from the source. Three receiver points are situated along the

long axis of the room at \mathbf{r}_1 , (3, 2, 1.5) m, \mathbf{r}_2 , (4, 2, 1.5) m, and \mathbf{r}_3 , (6, 2, 1.5) m. A Du Fort-Frankel finite difference scheme⁹ is used for the diffusion equation model. In this numerical technique, both time and space are discretized. A sampling frequency of 32 kHz and a spatial discretization of 0.2 m for all the Cartesian axes are chosen, in accordance with the conclusions obtained in Ref. 9. These configuration parameters ensure that the predictions converge to a fixed value with small error, compared to the exact solution of the system of equations, Eqs. (1) and (2). It should be noted that, in the diffusion equation model, the air absorption coefficient m [see Eq. (1)] is fixed at $1.202 \times 10^{-3} \text{ m}^{-1}$, which corresponds to the air absorption coefficient at 23 °C, 50% relative humidity, and a normal atmospheric pressure at 1 kHz. The same atmospheric quantities used in the diffusion equation model were inputted into the ray-tracing software, which used a frequency-dependent filter to model the air absorption. In the ray-tracing algorithm, a 100 000 ray emission was used. Moreover, diffuse reflections are modeled using a Lambert's law approximation.

The inner surfaces of the room have uniform absorption and scattering distributions. The mean absorption coefficient of the room is varied from 0.1 to 0.7 to cover the range used by previous authors and to corroborate the conclusions of these authors.⁵ For each absorption coefficient value, several simulations of the ray-tracing algorithm were run, with different scattering coefficients values. The values of the scattering coefficient s used in the simulations ranged from 0 to 1, i.e., from specular reflection to total diffuse reflection. The energy decay functions obtained from the ray-tracing method and from the diffusion equation model were used to calculate the reverberation time (RT) using the RT_{30} value and the early decay time (EDT) using Schroeder backward integration. The sound pressure levels (SPLs) at the receiver positions were also predicted. The percentage error between the results obtained from the diffusion equation model and those obtained from the geometrical method is calculated for each pair of parameter values, including both the absorption and scattering coefficients, and at each position. In this calculation, this error is the difference between the diffusion equation model value and the ray-tracing predicted value as a percentage of the ray-tracing predicted value. This error is also evaluated in terms of the just noticeable differences (JNDs) concept. According to ISO/DIS 3382-1:2009, the subjective threshold for the RT and the EDT is 5%, whereas for the SPL the threshold is 1 dB. In the following sections, some numerical results from these simulations and a discussion of these results are presented.

B. Reverberation time and early decay time

The averages of the percentage errors of RT at the three receiver locations, as defined in Sec. III A, are shown in Fig. 1(a), and provide an overall description of the differences between the simulation methods considered in this study. The data shown in Fig. 1(a) indicate that, for values of $\alpha \geq 0.45$, the errors are higher than one JND, 5% for all of the s values. In the high absorption range ($\alpha \geq 0.45$), the diffusion equation model usually underestimates the ray-tracing

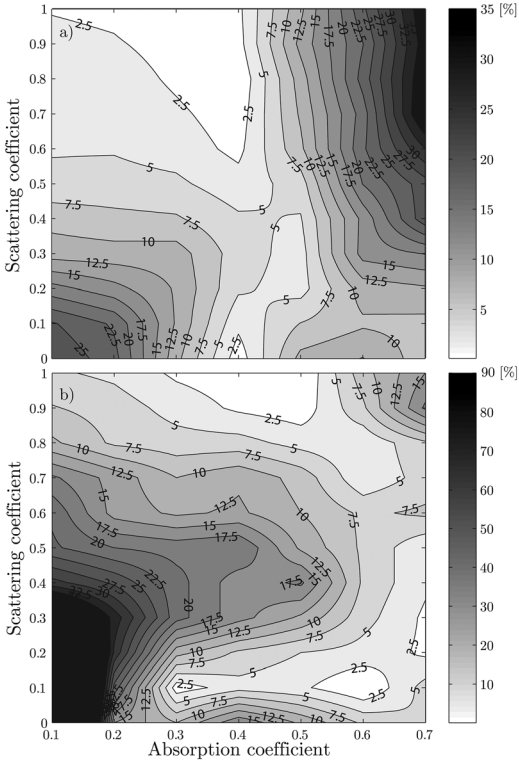


FIG. 1. RT (a) and EDT (b) average difference values between the diffusion and the ray-tracing simulation results in percentage.

predictions. From the ray-tracing predicted data, it is observed that the influence of s on the RT is more significant for smaller α . For $\alpha > 0.45$, the RT is almost independent of s .¹⁰ In the low absorption range ($\alpha < 0.45$), the percentage error decreases with increasing s because the diffuse reflected energy increases and the assumption of the diffusion equation model is fulfilled. Moreover, in this low absorption range, the average error values are less than one JND when $s > 0.6$. For $s > 0.6$, the difference between the values predicted by the two methods increases when α increases. This result may be due to a lower amount of reflected energy, which would lead to the assumption of the diffusion equation model's no longer being fulfilled.³

To study how the predictions change as a function of the distance from the source, the RT predicted from both models with $\alpha = 0.2$ and s between 0 and 1 at positions \mathbf{r}_1 , \mathbf{r}_2 , and \mathbf{r}_3 were determined, as shown in Table I. According to the results, the predicted values of the RT from the ray-tracing model increase when s increases, converging to the predicted values from the diffusion equation model. Thus, the error value decreases when s increases. For $s > 0.6$, the RT predicted data are similar at the three locations. The \mathbf{r}_3 position exhibits the highest increase in the RT values with increasing s . The results of the observations for other α values in the low absorption range ($\alpha < 0.45$) follow the same trend as those when $\alpha = 0.2$.

Similar to the analysis of the RT results, an analysis of the EDT results is presented. In Fig. 1(b), the EDT average percentage errors at all of the receiver locations for different α and s values are shown. There are two different trends, depending on the α value, that can be observed in this figure. For $\alpha < 0.3$, the error decreases when s increases. In this low

TABLE I. RT and EDT values for an absorption coefficient of 0.2 and the scattering coefficient values between 0 and 1 at (\mathbf{r}_1) 1, (\mathbf{r}_2) 2, and (\mathbf{r}_3) 4m from the source.

Model	Scattering coefficient	\mathbf{r}_1 RT/EDT(s)	\mathbf{r}_2 RT/EDT(s)	\mathbf{r}_3 RT/EDT(s)
Diffusion	1	0.49/0.48	0.49/0.49	0.49/0.51
Ray tracing	0	0.42/0.37	0.41/0.37	0.37/0.34
Ray tracing	0.1	0.42/0.38	0.42/0.38	0.38/0.34
Ray tracing	0.2	0.44/0.39	0.44/0.39	0.41/0.35
Ray tracing	0.3	0.45/0.38	0.45/0.37	0.43/0.35
Ray tracing	0.4	0.46/0.38	0.46/0.42	0.45/0.38
Ray tracing	0.5	0.46/0.40	0.47/0.43	0.47/0.42
Ray tracing	0.6	0.47/0.41	0.47/0.45	0.47/0.45
Ray tracing	0.7	0.47/0.42	0.47/0.44	0.48/0.45
Ray tracing	0.8	0.47/0.43	0.48/0.47	0.49/0.50
Ray tracing	0.9	0.48/0.42	0.48/0.48	0.49/0.50
Ray tracing	1	0.48/0.47	0.49/0.49	0.50/0.50

absorption range, the ray-tracing predictions are less than the diffusion predictions. Only predictions for scattering coefficients between 0.9 and 1 have an error less than one JND 5%. In the high absorption range ($\alpha \geq 0.3$), the diffusion equation model underestimates the ray-tracing predictions.

Table I lists the EDT predicted values at positions \mathbf{r}_1 , \mathbf{r}_2 , and \mathbf{r}_3 for $\alpha = 0.2$ and for different values of s . In general, the ray-tracing results increase, i.e., the error values decrease, when s increases. The diffusion model results are only consistent with the ray-tracing data when the reflections are diffuse, i.e., $s = 1$. The \mathbf{r}_3 position has the lowest and highest predicted EDT values, and the data converge with the diffusion predictions faster when $s > 0.8$.

It must be emphasized that several authors have stated that room acoustic quality parameters, such as EDT, cannot be precisely predicted using the diffusion equation model.¹¹ The EDT is used in this investigation to study the applicability limits of the diffusion equation model. These previous comments, as well as other observations that have been performed using the collected data, confirm that the diffusion equation model does not properly model the behavior of early reflection events. However, the diffusion equation model may be used to approximately estimate the EDT parameter at locations far from the source and in rooms where diffuse reflections predominate.

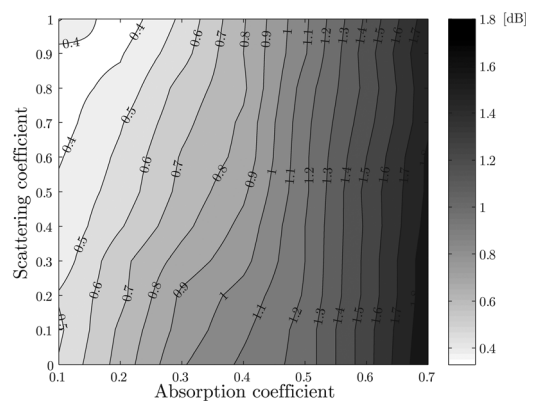


FIG. 2. SPL average difference values between the diffusion and the ray-tracing simulation results in dB.

TABLE II. SPL values and absolute difference values (Δ) for an absorption coefficient of 0.2 and the scattering coefficient values between 0 and 1 at (r_1) 1, (r_2) 2, and (r_3) 4 m from the source.

Model	Scattering coefficient	SPL/ Δ (dB)		
		r_1	r_2	r_3
Diffusion	1	88.99	86.54	84.25
Ray tracing	0	88.06/0.92	86.30/0.24	85.30/1.05
Ray tracing	0.1	88.00/0.98	86.21/0.33	85.11/0.86
Ray tracing	0.2	88.06/0.92	86.21/0.33	85.02/0.77
Ray tracing	0.3	88.06/0.92	86.21/0.33	85.02/0.77
Ray tracing	0.4	88.12/0.86	86.21/0.33	84.83/0.58
Ray tracing	0.5	88.19/0.80	86.30/0.24	84.83/0.58
Ray tracing	0.6	88.25/0.74	86.30/0.24	84.83/0.58
Ray tracing	0.7	88.25/0.74	86.30/0.24	84.73/0.48
Ray tracing	0.8	88.31/0.68	86.30/0.24	84.64/0.39
Ray tracing	0.9	88.37/0.61	86.38/0.16	84.64/0.39
Ray tracing	1	88.37/0.61	86.47/0.07	84.54/0.29

C. Sound pressure level

In this section, SPL prediction results are discussed. The simulations results that were calculated using the ray-tracing software indicate that of all of the parameters studied, scattering has the smallest effect on the SPL. The absolute values of and the difference between the models, denoted Δ , are calculated in dB to determine the predicted distribution of the sound field for each method and to determine the similarities between the two methods.

Figure 2 graphs the SPL absolute average difference values at all of the receiver locations for different α and s values. The difference value increases when α increases. The effect of scattering on this difference is more significant when $\alpha < 0.45$, where the convergence of the predicted values from both models is faster. The sound attenuation is larger for diffuse reflections than for specular reflections in the low absorption range.⁶ The SPL predictions have a weak dependence on scattering if the absorption is high. It is observed that for values of $\alpha \geq 0.45$, the average differences are greater than 1 dB for all of the scattering coefficient values. For $\alpha < 0.3$, all of the difference values are less than one JND. The limiting range that can be used to obtain consistent predictions can be set to $\alpha < 0.45$ and $s > 0.6$.

Table II lists the SPL absolute values and the differences at positions r_1 , r_2 , and r_3 for $\alpha = 0.2$ and for various values of s . It is observed that almost all of the difference values are less than one JND. For the ray-tracing predictions, the SPL slightly increases when s increases at positions close to the source (r_1). However, the SPL decreases when the scattering coefficient increases at remote positions (r_3). By examining the values in detail, it can be observed that the SPL difference is the smallest at position r_2 . The sound attenuation along the distance from the source increases when the scattering increases.

IV. CONCLUSIONS

In this work, several simulations were conducted for a homogeneous room to determine when the energy is scat-

tered enough to fulfill the diffusion equation model assumption and to identify when the predicted values of this model are sufficiently accurate. This study was performed by comparing the different predictions from a ray-tracing software, which incorporated scattering reflections, with predictions from the diffusion equation model.

For absorption coefficients below 0.45 and scattering coefficients above 0.6, the reverberation time comparison provides percentage errors less than one JND of 5%. The simulation results confirmed that the early decay time is not properly modeled by the diffusion equation model. The diffusion equation predictions of the sound pressure level are in good agreement with the ray-tracing model data for low absorption cases, absorption coefficients of less than 0.3, and all scattering coefficient values. A simple rule for the absorption/scattering ranges where the diffusion equation model can be used to predict reverberation time and sound pressure level values that are consistent with the ray-tracing results for homogeneous rooms may be $s > 0.6$ and $\alpha < 0.45$. More simulations that use different room shapes and proportions should be the focus of future research. Additionally, nonhomogeneous absorption distributions should be considered.

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