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During the last 40 years, there has been an increasing interest on performing room acoustic simulations for several purposes; among them, to get a lower sound level, to reduce sound propagation between rooms, to increase speech intelligibility and to adjust reverberance. However, there is no universal method able to deal with every kind of scenario. Recently, a method based on the assumption that the sound energy is propagated in a similar fashion to heat transfer or particles in a gas has been proposed. This method, known as a *acoustic diffusion equation model*, has been successfully applied to several scenarios that may be considered as challenging, such as long rooms, flat rooms, fitted rooms and coupled rooms. The interest of this method lies on its accuracy and efficient implementation. This article presents the current state-of-the-art of this method with special emphasis to its applicability to indoor building simulation.

Keywords: acoustic diffusion equation; room acoustics simulation; sound pressure level; reverberation time; building acoustics

1. Introduction

At the present time, the acoustic comfort in a building is increasing as an important feature in construction. Existing buildings are often exposed to high levels of noise both from the outside or from adjacent housing. These noise levels cause discomfort and even health problems among the inhabitants of the building. On the other hand, the room acoustics of unique facilities, such as restaurants, meeting and conference rooms, and classrooms are sometimes deficient causing decreasing speech intelligibility due to the excessive reverberation.

Buildings are complex enclosures which have several specific acoustic characteristics. A building consists of several volumes of different sizes, usually small or medium, which are coupled among them through openings or through their walls. In these complex scenarios it is difficult to apply simulation techniques based on wave propagation (Kuttruff 2000). Sometimes one dimension, such as its height, is smaller than the rest of dimensions resulting in non-diffuse reverberant sound fields that cannot be estimated by the basic theory of reverberation (Hodgson 1996). Rooms often contain obstacles, such as furniture, which absorb and scatter propagating sound and greatly changes the acoustic behaviour compared with the same empty room.

To simulate these particular characteristics, several methods have been developed in the past few years. Most predictive models focus on the sound pressure level (SPL) at steady state in buildings. The statistical energy analysis method is one of the most relevant (Craik 1996). This prediction model has been used as a reference in the international standard UNE-EN ISO 12354 (ISO 2000) to standardize the estimation of *in situ* sound insulation for building products and elements, hence satisfying the requirements of the European Construction Products Directive (Gerretsen 1998). Numerical methods based on the concept of geometrical acoustics were also applied, such as ray-tracing (Kulowski 1985) and beam-tracing (Funkhouser et al. 2004), for modelling transient sources that allow for an estimate of the time response of the system. However, implementing these geometrical acoustic models in the case of construction has certain limitations (Lehnert 1993) in the simulation of openings between enclosures and transmission between them. In the ray-tracing method some important reflection paths can be missed in configurations composed of multiple connected spaces. Moreover, the number of sources and receivers can also drastically increase the computational cost.

Diffusion processes of the acoustic energy can be used to predict the sound field in arbitrary shape rooms and non-homogeneous distribution of the absorption (Picaut, Simon, and Polack 1997). Recently, a theoretical model was proposed based on the energy radiative transfer equation to generalize the modelling techniques, which make use of the sound particles propagation approach (Navarro et al. 2010). This energy transfer theory allows to formulate the foundations of the geometrical acoustics techniques

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*Corresponding author. Email: jmnavarro@ucam.edu encompassing the diffusion equation model between them. The acoustic diffusion equation model is stated to be accurate mostly in low absorption rooms and to predict the late part of the room impulse response (Valeau, Picaut, and Hodgson 2006). From this estimated room impulse response, it is possible to calculate the SPL and the reverberation time (RT) at every position of the enclosure. The advantages of this method are its low computational cost and easiness of implementation even in rooms with complex geometry.

The scope of this article is to present to the building performance simulation community the current state-of-the-art on the use of the diffusion equation model and how this new simulation paradigm allows the performing of efficient and accurate acoustic simulations in a wide variety of room configurations. This paper reviews some of the most important contributions of the diffusion equation model for room acoustics simulation. The aim of this paper is not to present new results, but to introduce the model features and to summarize what a novel reader should know about this method. The article is divided into the following sections: first, the diffusion theory for room acoustic simulation is reviewed and its equations are presented. Then, the acoustics parameters that can be predicted with this model are described. The following section will explain several applications of this model related with buildings. Finally, the conclusions are stated.

2. The acoustic diffusion equation model

2.1. The diffusion theory for room acoustic simulation

Sound, just as light, is a wave phenomenon. There are several differences between light and sound. In spite of these differences, similarities between light and sound phenomena have made it possible the development of well-established modelling techniques in room acoustics, known as *geometrical acoustics* (Savioja 1999). A large number of simulation methods with certain similarities exists, but their theoretical foundations are often derived separately. Recently, a general foundation for geometrical room acoustics modelling, the acoustic radiative transfer model has been presented (Navarro et al. 2010). Using this energy transfer model, the acoustic diffusion equation can be derived by expanding the sound radiation to spherical harmonics (Navarro et al. 2010) allowing its consideration as an alternative approach to the geometrical acoustics technique (Figure 1).

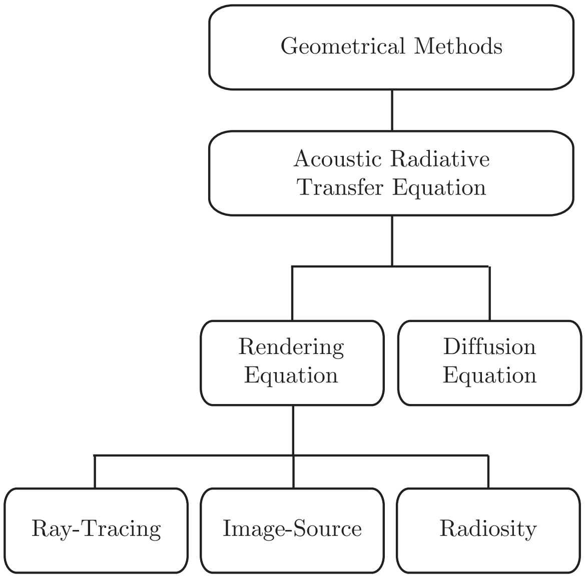

Figure 1. Diagram of different approaches to geometrical room acoustics modelling after the acoustic radiative transfer model (Navarro 2012).

The diffusion equation is a parabolic partial differential equation which describes such physical processes as heat conduction in a solid body, population dispersion and other similar processes. Classical diffusion is an irreversible physical process, in which particles are introduced into a medium that was initially absent, increasing the system entropy formed by the scattered particles and the medium into which they diffuse. Picaut, Simon, and Polack (1997) were the first to propose a mathematical model for describing the sound field in rooms based on a diffusion theory described by Ollendorff (1969). In this model, the field is composed by sound particles (Joyce 1974) with the same constant energy, propagating along straight lines and striking walls or scattering objects. That diffusion model consists on assuming that the sound propagation in rooms with diffusely reflecting boundaries can be compared with the movement of a single particle moving in a gas and hitting scattering objects (Picaut, Simon, and Polack 1997). The model was then generalized for predicting room acoustics in rooms with arbitrary geometry (Valeau, Picaut, and Hodgson 2006).

This approach has been actively studied in last 10 years due to its high efficiency and flexibility. These studies have shown that the diffusion model is accurate for low absorption cases and in predicting the late part of the sound energy decay in a room. Initially, a mathematical model based on the diffusion equation was used to model simple cubic enclosures (Picaut, Simon, and Polack 1997) to show its agreement with the statistical theory. Then, after comparing predicted results in long rooms with a ray-tracing method, the obtained results showed that the diffusion model can be regarded as an extension of the classical theory of reverberation for non-diffused sound fields (Picaut, Simon, and Polack 1999). After that, the acoustic diffusion equation method has been extended to model a wide variety of room types, such as proportionate enclosures and long rooms (Valeau, Picaut, and Hodgson 2006), fitted rooms (Valeau, Hodgson, and Picaut 2007) and even coupled-rooms (Valeau et al. 2004; Billon et al. 2006; Xiang, Jing, and Bockman 2009). This model has also been used for sound field predictions in open spaces like city streets (Picaut, Simon, and Hardy 1999), and in a street canyons (Polls et al. 2004). Although specular reflection cannot be applied, an empirical extension to this model for use in non-diffuse reflections has been developed (Foy et al. 2009). Moreover, to account for sound transmission between two rooms, a modification of the diffusion model for room acoustics has been proposed (Billon et al. 2008a).

In the following sections the set of mathematical expressions, both the governing equation and the boundary conditions, which compose the acoustic diffusion equation model are presented.

2.2. Governing equation

In order to derive the diffusion equation approach for room acoustics, two main assumptions are needed:

1. The scattering density must be high and the reflection of energy must dominate over absorption. This means that after numerous diffuse reflections the radiance becomes nearly isotropic. This assumption is sometimes called *directional broadening*.
2. In a primarily scattering medium, the time for substantial energy flux changes is much longer than the time it takes to traverse one mean free path. Thus, over one transport mean free path, the fractional change in the sound energy flow vector is much less than unity (Morse and Feshbach 1953). This property is sometimes called *temporal broadening*.

In summary, the acoustic diffusion equation model assumes that the reflections on objects and on boundaries of the room are diffuse. Moreover, the variation of sound energy density and energy flow vector must be small. In this case, the sound energy flow vector $\mathbf{J}(\mathbf{r}, t)$ given at position $\mathbf{r}$ and at instant t within a room is related to the gradient of the sound energy density $w(\mathbf{r}, t)$ according to Fick's law (Morse and Feshbach 1953; Visentin et al. 2012)

$$\mathbf{J}(\mathbf{r}, t) = -D\nabla w(\mathbf{r}, t), \tag{1}$$

where D is the *diffusion coefficient* which is expressed as

$$D = \frac{\lambda c}{3}. \tag{2}$$

This is a term that accounts for the room geometry and shape through the mean free path λ, where c is the speed of sound in air. In classical acoustic theory (Pierce 2007; Kosten 1960), the mean free path for empty room is given by

$$\lambda = \frac{4V}{S_t}, \tag{3}$$

for volume V and interior surface area S_t of the enclosure under investigation.

With an omnidirectional sound source that has an energy per volume $q_0(\mathbf{r}, t)$, the sound energy density $w(\mathbf{r}, t)$ distribution can be expressed as (Valeau, Picaut, and Hodgson 2006)

$$\frac{\partial w(\mathbf{r}, t)}{\partial t} = -\nabla \cdot \mathbf{J}(\mathbf{r}, t) = D\nabla^2 w(\mathbf{r}, t) - \sigma w(\mathbf{r}, t) - mcw(\mathbf{r}, t) + q_0(\mathbf{r}, t) \text{ in } V, \tag{4}$$

where $\sigma w(\mathbf{r}, t)$ describes the energy loss per unit volume due to absorption at the room boundaries, with $\sigma = \bar{\alpha} c/\lambda$, $\bar{\alpha}$ being the mean absorption coefficient of the room. The term $cmw(\mathbf{r}, t)$ accounts for air dissipation within the room with m being the absorption coefficient of air (Bass, Bauer, and Evans 1972; Billon et al. 2008b). These absorption losses are frequency dependent for both room boundaries and air.

2.3. Boundary conditions

If the diffusion equation should be used in a room of arbitrary shape, the equivalent scattering medium has to be bounded with the appropriate boundary conditions. In previous works two kinds of boundary conditions have been presented, a *homogeneous Neumann* boundary condition (Valeau, Picaut, and Hodgson 2006) where sound energy cannot escape from the room boundaries (its validity is discussed in Jing and Xiang 2008); and a more realistic *mixed* boundary condition for general situations which allows energy exchanges with the boundaries ∂V.

In order to use the mixed boundary conditions, the term $\sigma w(\mathbf{r}, t)$ in Equation (4) is replaced with an equation in ∂V, because the absorption should occur on room surfaces rather than in the volume. Therefore, to model the local effects on the sound field induced by different absorbing materials on surfaces, the following mixed boundary conditions have been published (Valeau, Picaut, and Hodgson 2006):

$$\mathbf{J}(\mathbf{r}, t) \cdot \hat{\mathbf{n}} = -D\frac{\partial w}{\partial \hat{\mathbf{n}}} = A_x c w(\mathbf{r}, t) \text{ on } \partial V, \tag{5}$$

where A_x is an exchange factor denoted as the *absorption factor* that is dependent of the surface absorption coefficient $\alpha(\mathbf{r})$, see Equations (8)–(10). $\hat{\mathbf{n}}$ is the normal direction of the surface, $\mathbf{J}$ is the energy flux vector and ∂V makes reference to the boundary conditions of the volume V. The latter equation makes possible the assignment of different absorption coefficients to individual walls, i.e. described locally. It is important to note that these absorption coefficient values are usually given in octave or $\frac{1}{3}$ octave frequency bands. Therefore, a change of the evaluated frequency band in the diffusion equation model is carried out by changing the values of the absorption coefficients.

This mixed boundary conditions give rise to the following system of equations which compose the acoustic diffusion equation model (Valeau, Picaut, and Hodgson

2006; Billon et al. 2008b)

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) + mcw(\mathbf{r},t) = q_0(\mathbf{r},t) \text{ in } V, \quad (6)$$

$$D\frac{\partial w(\mathbf{r},t)}{\partial \hat{\mathbf{n}}} + cA_x w(\mathbf{r},t) = 0 \text{ on } \partial V, \quad (7)$$

which is a second-order parabolic partial differential equation with mixed boundary conditions. The absorption factor $A_x = A_x(\mathbf{r},\alpha)$ in Equation (7) can describe rooms with low (Valeau, Picaut, and Hodgson 2006) or high absorption (Jing and Xiang 2007; Billon, Picaut, and Sakout 2008). If the room walls are made of N surfaces with constant absorption α_i, then absorption factor A_x for each surface portion i can be written as

$$A_x = A_S(\mathbf{r},\alpha) = \frac{\alpha(\mathbf{r})}{4}. \quad (8)$$

The subscript S in A_S is used to denote Sabine absorption, which is in accordance with diffuse sound-field theory (Kuttruff 2000). The Sabine absorption factor has been found to be accurate only for modelling rooms with low absorption around mean effective absorption values below 0.2 (Valeau, Picaut, and Hodgson 2006; Picaut, Simon, and Polack 1999; Picaut, Simon, and Hardy 1999). In order to improve the predictions in higher absorbing rooms, where Sabine's theory is no longer valid, the Eyring absorption factor has been defined (Jing and Xiang 2007; Billon, Picaut, and Sakout 2008) as follows:

$$A_E(\mathbf{r},\alpha) = \frac{-\log_e(1-\alpha(\mathbf{r}))}{4}. \quad (9)$$

Another derivation of these boundary conditions based on a boundary condition in light diffusion in media that avoids the singularity within the Eyring factor, when the absorption coefficient becomes 1.0, is called modified absorption factor (Jing and Xiang 2008; Navarro et al. 2010).

$$A_M(\mathbf{r},\alpha) = \frac{\alpha(\mathbf{r})}{2(2-\alpha(\mathbf{r}))}. \quad (10)$$

However, the main advantage of this boundary condition is that it has been demonstrated to be as accurate as other models for low absorption coefficients with values up to 0.4, and more accurate than other models in the high absorption range with values up to 0.7 (Jing and Xiang 2008).

3. Predicting acoustics parameters with a diffusion equation model

An acoustic enclosed space can be considered as a linear time invariant system where the output signal can be obtained by performing the convolution of the input signal with the system's impulse response (Oppenheim, Willsky, and Hamid 1996). It is well known in room acoustics that it is possible to define the acoustics properties of an enclosed space through the room impulse response. There is a single and unique room impulse response for each pair of source–listener positions in an enclosure. In general, room acoustics computer simulation techniques model the sound propagation indoors and outdoors by trying to estimate the room impulse responses. From the room impulse response estimated using the diffusion equation model, it is possible to calculate the SPL and the RT as two of the most important acoustic criteria.

3.1. *Reverberation time*

The diffusion equation method models the time-dependent propagation of the sound energy density $w(\mathbf{r},t)$, and its reflections on walls and objects. Therefore, the predicted result of the diffusion equation model, when the emitted signal by the source is a dirac impulse, is an estimation of the energy density impulse response in the evaluated frequency band that depends on the boundary condition values. After solving the time-dependent problem of the diffusion model using an impulse function as input, the temporal function of the sound energy density $w(\mathbf{r},t)$ is calculated and then the SPL decay curve can be expressed as

$$\mathrm{SPL}(\mathbf{r},t) = 10 \times \log_{10}\left(\frac{w(\mathbf{r},t)\rho c^2}{P_{\mathrm{ref}}^2}\right), \quad (11)$$

where P_{ref} is 2×10^{-5} Pa and ρ is the air density. Remember that the absorption features of the room are frequency dependent and a simulation has to be run in each of the evaluated bands to estimate the SPL decay. An example of the energy decay curve is shown in Figure 4(a) where a typical double-slope decay of two coupled rooms by an aperture is observed.

Then, the RT can be calculated by applying the Schroeder method (Schroeder 1965) to this decay curve, i.e. obtaining the slope of a backwards integration of the impulse response. The RT is defined as the time that would be required for the SPL to decrease by 60 dB after a stationary sound source is turned off. Due to the difficulty of getting the required dynamic range to evaluate 60 dB of decay, the decay rate is usually obtained from −5 to −35 dB of the maximum starting level, and it is called T_{30}.

3.2. *Sound pressure level*

The SPL value is an important index for building acoustics. Estimation of SPL values can be useful for tasks such as noise prediction and control, soundproofing design, insulation materials and noise barriers.

It is possible to estimate the SPL in different locations with the diffusion equation model. This is done by the integration over the energy room impulse response leading to the sound energy density level $w(\mathbf{r})$. As it has been discussed in Section 2.1, the diffusion equation models only the reverberant part of the sound field and the direct sound field must be added at a later stage. Thus, the total SPL at a location **r** can be obtained with (Valeau, Picaut, and Hodgson 2006)

$$\mathrm{SPL_T}(\mathbf{r}) = 10 \times \log_{10}\left(\rho c\left(\frac{P}{4\pi d^2} + cw(\mathbf{r})\right) \times \frac{1}{P_{\mathrm{ref}}^2}\right), \tag{12}$$

where $d = |\mathbf{r} - \mathbf{r}_s|$ is the distance between the listener and the source.

3.3. *Numerical method for the diffusion equation model*

To solve the system of equation of the diffusion equation model, numerical methods of calculus are needed. The first method applied to the acoustic diffusion was the finite elements method (Picaut, Simon, and Polack 1997; Valeau, Picaut, and Hodgson 2006) using several commercially available software. The finite elements method is mainly suitable for the diffusion equation model simulation in the steady-state taking advantage of its capability of modelling irregular shapes. However, the computational requirements increase if the finite elements method is used for a transient analysis of the diffusion equation. Recently, the finite differences method has been presented (Navarro, Escolano, and López 2012) as an alternative technique in the time domain. The proposed finite differences method provides some schemes to solve the diffusion equation in a transitional regime, which are easy to implement and with lower computational costs than the finite elements implementation. It is important to note that the results with the finite differences method are evaluated along all the calculated domains, so there is no need to carry out another simulation if the position of the receiver is moved. That is to say, the room impulse response is predicted in all receiver positions of the discretized room with only one run of the algorithm.

Without entering onto mathematical details, the approach of Navarro, Escolano, and López (2012) deals with a Dufort–Frankel finite-difference approach for Equations (6) and (7). The advantage of this approach is having an unconditionally stable method, for any temporal (Δt) and spatial sampling frequency (Δx, Δy and Δz) and including air absorption. However, in order to provide accurate results, it is stated that an order of magnitude of 10^{-8} for the relation $(\Delta t)^2/(\Delta \nu)^2$ is necessary, where $\Delta \nu = \min\{\Delta x, \Delta y, \Delta z\}$ (Navarro, Escolano, and López 2012). Clearly, this method supposes an advantage compared with other alternative methods such as the *forward-time central-space* approach (Kraszewski 2012), which is a conditionally stable method where the spatial sampling frequency is dependent on the diffusion equation and therefore, on the volume of the room under simulation (Navarro, Escolano, and López 2009). During the following sections, all the simulations are carried out using this approach.

4. Applications of a diffusion equation model for building acoustics simulation

In the following sections, several examples of application of the diffusion equation model are exposed. These examples are of interest since the most popular methods, e.g. ray-tracing, may have difficulties in obtaining accurate results, or the associated computational cost may be prohibitive in some cases. Moreover, the main limitations of the diffusion equation model are described, not only in order to discourage its use but also to promote future research lines on this highly promising simulation model.

4.1. *Single volume rooms*

Buildings are composed of single volume rooms of different shapes and sizes. The most basic application of the diffusion equation model is the simulation of the sound field propagation in rooms composed of one volume. The diffusion equation model assumes that the reflections are totally diffuse, thus the energy must be sufficiently scattered, both in space and time, for the predictions to be valid. Therefore, a time gap has to be considered in the impulse response if it is to be comparable to the solution of using a diffusion equation model. Experimental measurements from a ray-tracing software and the diffusion equation model in a rectangular-shaped room conclude that, at least, two mean-free times, defined as λ/c, need to be disregarded from the original impulse response in order to estimate RT and SPL values with accuracy when compared with the diffusion equation model solution (Escolano, Navarro, and López 2010). Despite of what looks like a limitation of the diffusion equation model when calculating parameters such as RT and SPL distribution, this does not affect to final result (Schroeder 1965). Although it is assumed that the diffusion equation model is only valid for completely diffuse boundary conditions, it has been shown recently that the diffusion equation model is a reliable model for the prediction of RT and SPL values, for absorption coefficient values below 0.45 and scattering coefficient values above 0.6, and therefore the range of application of this paradigm is broader than expected from its definition (Navarro et al. 2013).

In this section, an example of the acoustic diffusion equation model with the additional air absorption term and non-homogeneous absorption distribution is presented. The experiment focuses on comparing predictions of the diffusion equation model, with other geometrical diffuse reflection prediction models, such as radiosity (Hodgson and Nosal 2006) and diffuse ray-tracing (Nosal 2003). In this example, a small shoe-box office is used, whose dimensions are 3.94 m wide, 5.36 m long and 2.71 m high. The real enclosure has a vinyl tile floor on concrete, four drywall walls on 100 mm studs and a suspended acoustical-tile ceiling, which consists of different absorption coefficients (Cox and D'Antonio 1985).

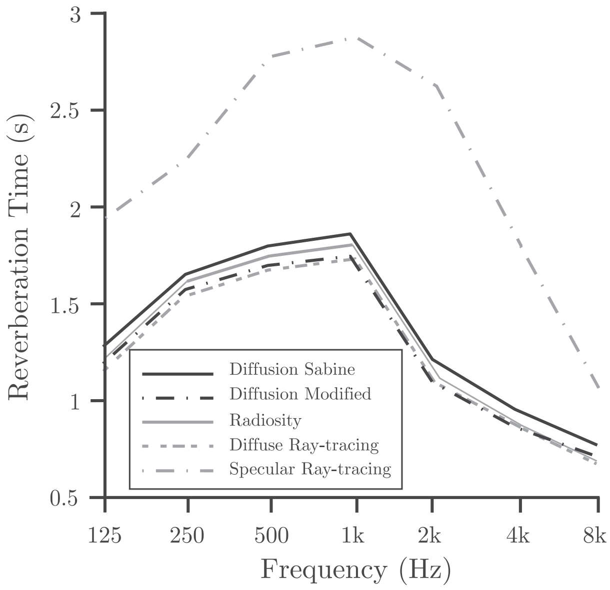

Figure 2. RT values as a function of frequency in the room under analysis.

The room is evaluated in octave-bands between 125 Hz and 8 kHz with two different absorption models. The one using the Sabine absorption factor (A_S) is denominated *diffusion Sabine* (Equation (8)) and the other adopting the modified absorption factor (A_M), *diffusion modified* (Equation (10)). Figure 2 graphs the variation with frequency of estimated RT values, calculated as T_{30}, with the diffusion equation model along with acoustical radiosity (Nosal, Hodgson, and Ashdown 2004), diffuse ray-tracing and specular ray-tracing models extracted from Nosal (2003). The diffusion equation, radiosity and diffuse ray-tracing models generally give very similar predictions showing good agreement between them, but the ones from specular ray-tracing are found to be very different as expected.

This example shows that reliable predictions of RTs can be obtained even for a configuration presenting a non-diffuse sound field.

As regards to the SPL, another example consists on an elongated room. Figure 3 represents a 20 m × 200 m × 5 m room where all walls are considered to have a coefficient of absorption of 0.2 and the air absorption exponent is $1.202 \times 10^{-3}\,\mathrm{m}^{-1}$ at 1000 Hz octave band. An impulsive source is situated at 5 m × 5 m × 2.5 m. This example has been selected to highlight how disproportionated rooms are correctly simulated using a diffusion equation model in comparison to other geometrical methods, where the accuracy of the method is proportional to the number of used rays, and therefore, to its computational time (Jing, Larsen, and Xiang 2010; Jing and Xiang 2010). The figure shows, as expected, an inhomogeneous distribution of the energy that decreases when moving away from the source.

4.2. Coupled rooms by an aperture

An interesting application of the diffusion equation model is its use on simulating coupled-volume systems. This situation occurs when given a volume where several rooms are interconnected to the main room through narrow apertures. When the natural RTs of the adjacent rooms are considerably greater than the one at the main room, it is evidenced that logarithmic energy decay is no longer lineal and at different times, the decay changes its profile. The use of coupled-rooms has been the subject of study during the last decades, as the obtaining of multi-sloped energy decay profiles able to create more adequate acoustics has been found to be challenging. These models have a practical significance in concert hall acoustics as many recent concert halls have implemented reverberation chambers coupled to the main room to produce multiple-slope energy decays (Jaffe 1995; Johnson, Kahle, and Essert 1997) and the use of theater stage shells to couple with reverberant stage houses (Jaffe 2005). However, the architectural acoustic community have yet to find a proper model to design this kind of enclosures.

In the last few years, the diffusion equation model has been proposed as an efficient alternative to simulate these models. It is assumed that the aperture size is small enough to consider it as a weak coupling, and therefore, diffusion

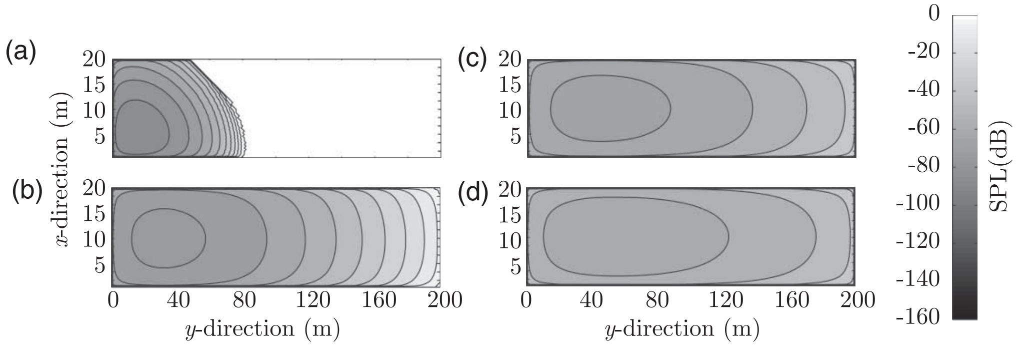

Figure 3. Empty room, where it has been represented the SPL at 1000 Hz octave band and at (a) $t = 0.2$ s, (b) $t = 1$ s, (c) $t = 2$ s and (d) $t = 5$ s.

coefficients for each sub-room are assigned based on the fact that the mean free path does not significantly change their free path distributions (Summers, Torres, and Shimizu 2004). Then, the general model for two coupled rooms, with volumes V_1 and V_2, should be described as

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D_i\nabla^2 w(\mathbf{r},t) + mcw(\mathbf{r},t) = q_i(\mathbf{r},t) \text{ in } V_i, \quad i = 1,2, \tag{13}$$

where $q_1(\mathbf{r},t) = q(\mathbf{r},t)$ and $q_2(\mathbf{r},t) = 0$.

Billon et al. (2006) were the first to propose the use of the diffusion equation model to coupled rooms. They compared the results obtained using the diffusion equation model to acoustic measurements; however, they found some limitations regarding the use of boundary conditions with absorptions larger than 0.2. After that, Xiang, Jing, and Bockman (2009) extended the effort on the same direction by using the modified absorption factor. The results of this investigation concluded that certain differences may be found in terms of time differences; however, those results were very limited since the comparisons were limited to a few cases. Very recently, Xiang et al. (2013) advanced in the effort of validating the diffusion equation model to coupled-rooms simulations by performing an extensive campaign of measurements in a 1:8 scale model and the results were compared with the corresponding computer model based on the diffusion equation model. The object of the study was to understand how the aperture size and the relative distance between source and listeners would affect the decay profile. It was concluded that, in terms of simulation, the diffusion equation model gives accurate solutions as regards to the decay parameters; however, an inherent delay always occurs, due to the fact that the diffusion equation model mainly simulates the late part of the energy propagation when the sound field becomes diffuse. This delay is evidenced by a singular point called *turning point*, defined as the time when the energy profile changes its decay value. Despite the fact that this difference may be considered as a limitation of the model, it is evidenced that the difference remains constant in all parameter combinations. Moreover, a more accurate model including energy continuity through the aperture $\mathbf{r}_b$ is demonstrated in this research, modelled as

$$\hat{\mathbf{n}} \cdot (D_1\nabla w(\mathbf{r}_b,t) - D_2\nabla w(\mathbf{r}_b,t)) = 0, \tag{14}$$

where $\hat{\mathbf{n}}$ is the normalized perpendicular direction to the aperture surface.

In order to illustrate how the diffusion equation model performs the simulation of this kind of enclosures, an example is presented. The system consists of two rooms where the primary room has dimensions of 4.88 m × 6.32 m × 7.20 m, which is equivalent to 222.06 m^3. All surfaces, except the ceiling and the aperture door, are treated with absorbing (and diffuse) material with absorption 0.41. The natural reverberation, e.g. the RT associated to each one of the individual rooms without coupling, of this primary room is calculated to be 0.47 s. As for the secondary room, its dimensions are 7.60 m × 9.76 m × 7.20 m and therefore, it has a volume of 534.07 m^3. Again, all surfaces are considered absorbing, with the same exceptions than in the primary room with an absorption coefficient of 0.22. The resulting natural RT in this case is 1.17 s. The source is situated in the middle position of the primary room. Again, the simulation is performed using the same finite-difference algorithm previously described in this article. However, this case requires us to have some considerations regarding the spatial sampling frequency, i.e. the minimum size of each discretized cell. In the previous example, there was no indication of what should be this value; however, here the aperture is taken as a reference of what the spatial sampling frequency should be, and then the temporal sampling frequency is automatically determined. In this case, the aperture size is 10 cm and therefore, the spatial sampling frequency in the direction containing the aperture is selected to be $\Delta y = 5$ cm. According to this, the sampling frequency becomes $\Delta t = 5\,\mu$s. As regards to the other directions, they can be selected at convenience ($\Delta x = 0.2$ m and $\Delta x = 0.265$ m).

Figure 4 shows the results obtained in this simulation. Figure 4(a) shows the resulting energy decay after conveniently applying the Schroeder backward integration. It is evidenced that the profile is far from the usual linear decay expected from a single room, and two clear zones can be differentiated. Although it is far beyond the scope of this paper, it is worthy to mention that in order to obtain the numerical parameters that model this complex decay (number of linear decays, decay times, level differences, etc.), a Bayesian inference model results are highly appropriate to perform this analysis (Jasa and Xiang 2012). At different time stages of the energy decay profile, the SPL in both rooms is shown. Figure 4(b) shows the SPL distribution at $t = 25$ ms; at this time the energy decay is ruled by the properties of the primary room. In this case, the average SPL is higher in the primary room compared with the secondary one. At approximately $t = 60$ ms the energy decay profile starts to modify; and the turning point is reached. In terms of SPL, both rooms have reached an energy equilibrium and the energy flow near the aperture becomes zero (Xiang, Jing, and Bockman 2009). From this point, the SPL is decreasing with a different rate, i.e. the energy decay rate becomes slower. Figure 4(d)–(f) shows how the average energy level is higher in the secondary room compared with the primary one. In some way, it could be seen as a sound source is situated at the secondary room, controlling the reverberation in the primary room to be similar to the expected from the natural RT of the secondary room (Xiang et al. 2013). To summarize, the use of a diffusion equation model for the analysis of coupled volumes, despite of certain limitations, has demonstrated to be an accurate tool.

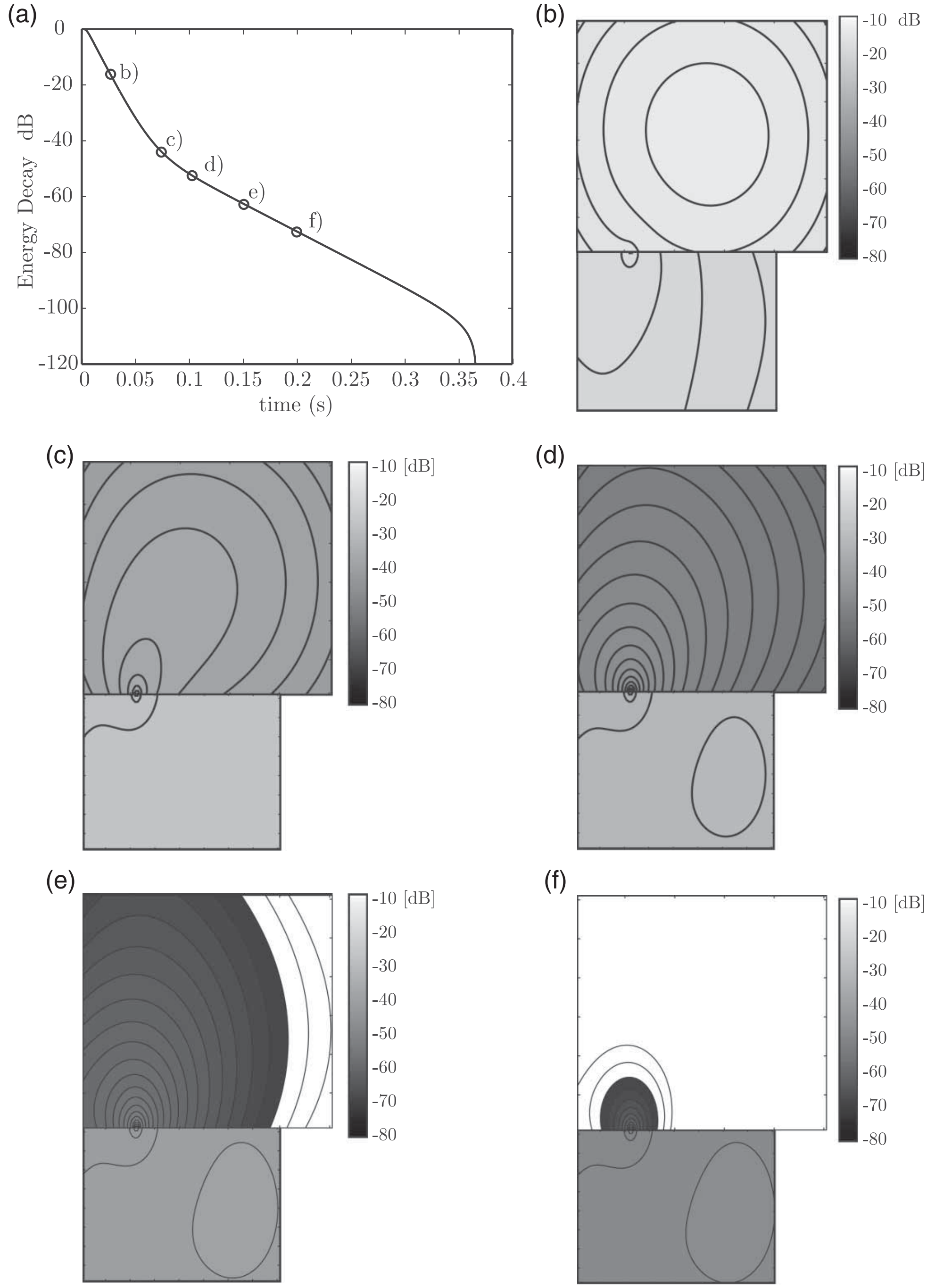

Figure 4. Application of a diffusion equation model to the simulation of coupled-room systems. (a) The characteristic double energy decay profile. (b)–(f) The resulting SPL in the system at 1000 Hz octave band and at (b) $t = 20$ ms, (c) $t = 60$ ms, (d) $t = 90$ ms, (e) $t = 150$ ms and (f) $t = 200$ ms.

4.3. Fitted rooms with objects

Another interesting case is fitted rooms. In many cases, acoustic spaces are filled with multiple objects and obstacles (furniture, machinery, benches, etc.) that scatter and absorb the sound energy. These spaces, when fitted, have a very different acoustic behaviour as when the same space is completely empty. Several approaches based on acoustic geometrical methods have been presented; however, the applications of these different models remain limited due to their assumptions concerning the obstacles or the enclosing room.

Two approaches may be used to fitted rooms when simulating using the diffusion equation model: (a) including details of the obstacles itself in the simulation by means of defining boundary conditions and (b) by statistically treating certain areas where it is assumed there is an important

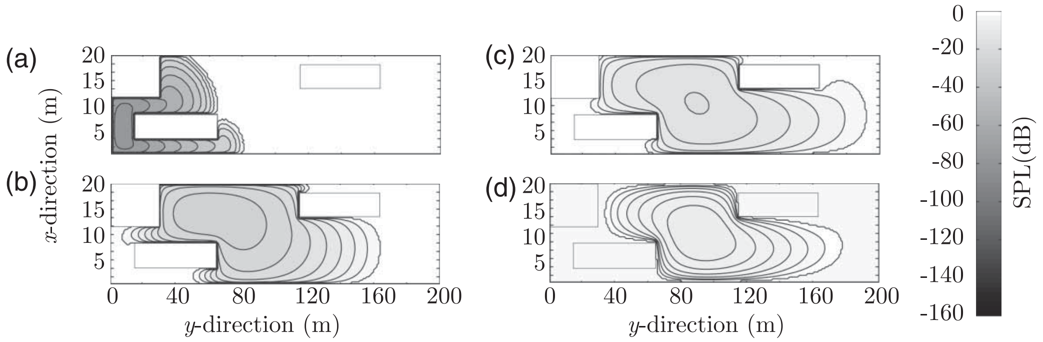

Figure 5. Room with obstacles. The figures represent the SPL at 1000 Hz octave band and at (a) $t = 0.2$ s, (b) $t = 1$ s, (c) $t = 2$ s and (d) $t = 5$ s.

number of obstacles that can produce a relevant number of scattering reflections.

The first approach can be used when the size of the obstacle is considerably big, as it may be a column or part of a wall surface. Since the diffusion equation model does not inherently consider frequency information, except for the selected absorption coefficient, and therefore, the ratio between the size of the object and the wavelength is an unknown parameter, the concept of *bigness* has to be assumed by the user. However, we have to consider that those surface properties are always completely diffuse. Using the same example as in Section 4.1, Figure 5 shows different snapshots of SPL distribution at different time steps. This example inherently shows how additional effects relative to disproportionated rooms are evidenced, where the sound field is observed to be non-uniform, which cannot be properly modelled by classical statistical theory, e.g. the Sabine equation (Kuttruff 2000). Moreover, shadow effects due to the corner diffractions, which in some methods such as ray-tracing can be difficult to properly model without any additional algorithm (Torres, Svensson, and Kleiner 2001), are evidenced.

The second approach consists on statistically treating those areas where there is an important amount of scatter objects. Considering the presence of those scatter objects in a medium, one should modify the mean-free path and the propagation of particles to be evaluated as in Valeau, Hodgson, and Picaut (2007). According to this model, two kinds of particles should be considered: one representing the diffusely reflecting surfaces of a room, and one representing the scattering obstacles located within the room. In terms of the diffusion equation model, this is equivalent to considering a spatial-dependent mean-free path λ_{eq}. In the fitting object area, the mean-free path is denoted by λ_f, whereas for the empty room, its mean-free path is indicated by λ_r. The equivalent mean-free path in the fitted room is calculated as

$$\lambda_{\text{eq}} = \frac{\lambda_r \lambda_f}{\lambda_r + \lambda_f}. \tag{15}$$

Depending on the areas where particles are propagated, λ_{eq} modifies its value: if the room is empty, λ_f is infinite and $\lambda_{\text{eq}} \simeq \lambda_r$. Conversely, if the room contains a very high number of scatterers, then $\lambda_f \ll \lambda_r$ and $\lambda_{\text{eq}} \simeq \lambda_f$. This model was properly evaluated in the original publication (Valeau, Hodgson, and Picaut 2007) by comparison with ray-tracing simulations, and good agreement between results were found. In order to evidence the effect of this approach to a fitted room, the same previous scenario has been used. The obstacles of the previous example have been substituted with fitting objects areas. To provide a visually acceptable effect, $\lambda_r/\lambda_f = 50$ has been used. Figure 6 shows the SPL distribution at the same time slots than the previous example. It is evidenced how when the energy is propagated through the fitting areas, the energy suffer a certain *delay*: the considerable amount of object scatters impedes the energy propagation, visualized as a delay regarding to the empty areas. Unlike the previous example, there is no shadow areas, since the energy is propagated through the whole area; however, the overall distribution is far from homogeneous.

The adequate model should be chosen depending on the number of objects within the room to simulate: when the number of objects is small and they can be easily modelled, first approach will provide more accurate results. However, when the number of objects is large enough, the second statistical approach may be more convenient and then, the exhausting process of modelling each one of those individual objects is considerably reduced.

4.4. *Coupled rooms by a transmitting surface*

Environmental and indoors noise are factors that have a major relevance in building comfort and quality. In building acoustics simulation, the transfer of acoustical energy from one room to another though partition walls is an important feature that the modelling technique has to incorporate. To calculate this transfer of energy, international standards are based on the statistical theory, such as UNE-EN ISO 140 (ISO 1997) and UNE-EN ISO 12354 (ISO 2000). Therefore, their predictions are reliable if a perfect diffuse field exists, which is not always applicable in real rooms (Hodgson 1996). The diffusion equation model can model this energy

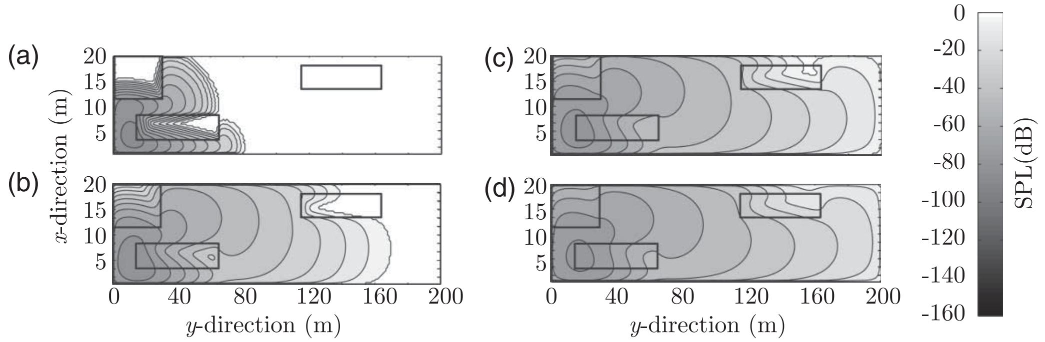

Figure 6. Fitted room with areas where the mean-free path has been modified according to Equation (15). The figures represent the SPL at 1000 Hz octave band and at (a) $t = 0.2$ s, (b) $t = 1$ s, (c) $t = 2$ s and (d) $t = 5$ s.

transfer allowing the estimation of the amount of emitted SPL that is received in adjacent rooms. The diffusion model was extended to take into account the sound transmission between rooms and allowing non-diffuse fields on both sides of the partition wall (Billon et al. 2008a). As is coupled by an aperture room case, see Section 4.2, several diffusion equations are formulated, one for each volume (Equation (13)). Moreover, the boundary conditions must express the energy exchanges between the rooms as well as its absorption at its surface. Therefore, in Equation (7) a term that stands for the energy transferred from one room to the other is included

$$D_i \frac{\partial w(\mathbf{r},t)}{\partial \hat{\mathbf{n}}_{\mathbf{i}}} + cA_{x_i} w(\mathbf{r},t) = \frac{\tau(\mathbf{r})c}{4} w(\mathbf{r},t) \text{ on } \partial V_i, \quad i = 1, 2, \tag{16}$$

where D_i is the diffusion coefficient that corresponds to the primary room, n_i the normal vector to the primary room and A_{x_i} the absorption factor of the surface in the direction towards the primary room. τ is the transmission coefficient, related with the transmission loss $TL = \log_{10}(1/\tau)$, between the adjacent room and the primary room.

Next, an example is presented to illustrate the sound energy transmission modelled by the diffusion model. In this case, the system consists of two rooms where the source room has dimensions of 40 m × 25 m × 20 m and the receiver room has 40 m × 15 m × 20 m. Both rooms have a homogeneous distribution of the absorption, it being 0.4 and 0.2 for source and receiver room, respectively. The transmission coefficient has been defined by $\tau = 0.01$. In this case, coefficients are assumed to have the same value for all frequencies. Figure 7 shows four different time steps: in Figure 7(a) the sound source is initialized and the sound is propagated only at the primary room. Once the sound energy has reached the transmission wall it is observed how part of the energy is transmitted (Figure 7(b)). From the point of view of the secondary room, the transmission wall is behaving as a energy source (Figure 7(c)). According to Billon et al. (2008b), a very good agreement is found with the statistical theory, with a maximal discrepancy inferior to 2.3 dB. Moreover, while the statistical theory is limited to configurations where the diffuse field assumptions hold, the diffusion model can predict the transmission in configurations where the reverberant sound field is not diffuse. When this method is compared with commercial ray-tracing software, similar results are found; however, ray-tracing software has shown certain limitations and it should not be applied for very low sound insulation (0–10 dB) (Rindel and Christensen 2008).

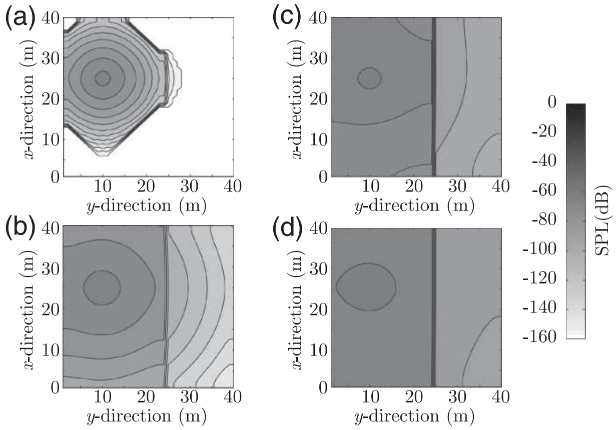

Figure 7. Coupled rooms by a partition wall where energy is transferred from the source room to the receiver room. The figures represent the SPL at 1000 Hz octave band and at (a) $t = 0.4$ ms, (b) $t = 2$ ms, (c) $t = 4$ ms and (d) $t = 10$ ms.

5. Conclusions

Despite the existence of a firmly established acoustic theory in room acoustics, it is not possible to find analytical expressions able to describe the whole acoustic phenomena occurring in a room as a function of position and time. For that reason, some simplifications should be applied to the room acoustics theory in order for performing computer simulations, having a considerable interest for engineers and architects.

Among the existing proposals to perform these acoustic simulations, the diffusion equation model has arisen as an efficient and accurate model to simulate the acoustic energy propagation, with particular interest in some scenarios where classical methods, e.g. ray-tracing, have been proved to be unsuccessful. The diffusion equation model is based on the propagation of sound particles, called *phonons*, with the same constant energy, propagating along straight lines and striking walls or scattering objects. That diffusion model consists of assuming that the sound propagation in rooms with diffusely reflecting boundaries can be compared with the movement of a single particle moving in a gas and hitting scattering objects.

This article contributes by presenting the current state-of-the-art of using the diffusion equation model with applications to several examples of potential acoustics scenarios where the diffusion equation model can be used as an appealing alternative method. Particular scenarios as disproportionated rooms or coupled-room systems can be analysed using the diffusion model, a paradigm that is able to provide accurate results with low computational resources, whereas other methods can be inaccurate, e.g. ray-tracing, or prohibitive, e.g. finite-differences time-domain.

This method has some limitations due to the assumptions of the model. The fact that completely diffusive boundary conditions need to be considered may reduce the number of practical applications; however, the recent work of Navarro et al. (2013) showed that this inherent limitation can be relaxed assuming a certain error in some acoustic parameters. Nevertheless, this error is small enough to be psicoacoustically disregarded. Another important limitation lies in the incapacity of this model to include first reflections. However, recent advances have shown (Xiang et al. 2013) that this time error can be estimated in some applications. Despite the inherent limitations of this model, the method is gaining popularity in the technical literature during the last few years, and new applications are being developed. The purpose of this manuscript is to extend the basis of the knowledge of this method to a wider public providing examples of interest in topics related to building simulation. Future efforts should be put on extending the number of applications, and not limited only to indoor building simulations, but also outdoor (Picaut, Simon, and Hardy 1999). Direct/indirect sound transmission applications and including specular reflecting surfaces into the simulations are among the future research lines which should be considered in the development of this promising alternative acoustic method.

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