

Investigation on effect of aperture sizes and receiver positions in coupled-volume systems ^{1, 2}

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Jan. 18th 2013

The author list is complete. No names of those, who made substantial intellectual contributions to the work documented in this paper, are omitted.

No part of the submitted article has been published or is under consideration for publication in any other journals.

PACS numbers: 43.55.Ka, 43.55. Br, 43.55.Mc

Running title: Aperture size effect in coupled rooms

This manuscript contains **38** pages with **10** figures and **2** tables (resulting in no more than 12 printed pages in the journal)

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¹Aspects of this work have been presented at the 21st International Congress on Acoustics, Montreal, Canada, June 2013, and the 20th International Congress on Sound and Vibration, Bangkok, Thailand, July 2013

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ABSTRACT

Some recent concert hall designs have incorporated coupled reverberation chambers to the main hall which have stimulated a range of research activities in architectural acoustics. The coupling apertures between two or more coupled-volume systems are of central importance for sound propagation and sound energy decays throughout the coupled-volume systems. In addition, positions of sound sources and receivers relative to the aperture also have a profound influence on the sound energy distributions and decays. This work investigates the effect of aperture size on the behavior of coupled-volume systems using both acoustic scale-models and diffusion equation models. In these physical and numerical models, the sound source and receiver positions relative to the aperture are also investigated. Through systematic comparisons between results achieved from both physical scale models and numerical models, this work reveals valid ranges and limitations of the diffusion equation model for room-acoustic modeling.

PACS numbers: 43.55.Ka, 43.60.Ek, 43.58. Gn, 43.55.Mc

I INTRODUCTION

This paper presents investigations on the effect of coupling-aperture sizes in acoustically coupled-volume systems. Understanding the effect of changing aperture size has practical significance in concert hall acoustics, as many recent concert halls have implemented reverberation chambers coupled to the main floor to produce multiple-slope energy decays [1, 2]. Another application is the design and adaptation of theater stage shells to couple with reverberant stage houses [3]. This paper will discuss the experimental investigations using Bayesian decay analysis based on both acoustical scale-models and the diffusion equation model.

The application of coupled reverberation chambers and stage shells in performance venues has recently prompted active research including the extension of statistical theory for coupled-volume systems [4],[5], validation and improvement of geometrical acoustics modeling[6], application of a diffusion equation model [7, 9], and a study on sound energy density distribution and sound intensity vector field within modal frequency ranges [11]. Other recent research related to coupled-volume systems can also be found in Refs. [12]-[20].

Ermann [12] studied the relationship between aperture size and sound energy decay using statistical models and a geometrical-acoustic model (CATT acousticTM). He used a decay time ratio between T_{15} and T_{60} to quantify the changes. As noted by Bradley and Wang [13], this quantifier, defined within overlapped level ranges which can obfuscate the non-single-exponential nature of the energy decays, often misrepresent the multiple-slope energy decays since a linear fit does not represent the decay function within the level range for decay quantity T_{60} . Furthermore, that work ran the geometrical acoustical simulations which are strongly limited in the late decay processes [6]. The published results [12] have indicated strong discrepancies between statistically modeled and geometrically modeled results by the quantifier used. Furthermore, no experimental validations on systematic changes of aperture sizes were reported.

A more recent work [13] has reported on investigations of varied aperture sizes in a 'virtual concert hall' using geometrical-acoustics modeling without experimental validations of decay quantifiers involved. The quantifier heavily used throughout that study is of similar nature

as those using two straight-line models. In particular, a late decay time (LDT) and the decay time T_{10} defined within a small decay portion of level ranges of 10 dB. Those quantifiers, whether LDT/T_{10} , T_{60}/T_{15} , or other ratios based on linear-fits using two straight-line models within preselected, fixed level ranges, prior to the data analysis, cannot characterize multiple-sloped decay processes with the accuracy necessary to provide deep insight into energy decay characteristics in coupled-volume systems [14, 15]. Therefore, the previous studies have not yet been able to articulate the physical changes of sound energy decay when changing aperture size.

Billion *et.al.* [7] applied a diffusion equation model to study coupled-volume systems. Their primary focus was on the feasibility and applicability of the diffusion equation model rather than the aperture size. Xiang *et.al.* [9], Jing and Xiang [17] also applied the diffusion equation model to investigate coupled-volume systems to reveal the sound energy flows across the coupling aperture. More recently, Xiang *et.al.* [15] have discussed a rigorous approach to the characterization of multiple-sloped energy decays using a parametric decay model based on the nature of Schroeder integration [21] and statistical room-acoustics [22]. The primary focus of this recent work is to introduce the approach which exploits two levels of Bayesian inference, rather than systematic investigations on aperture sizes and energy decay characteristics over large audience (receiver) areas.

This paper advances the effort by previous work [9], [12] in significant ways. This work relies on experimentally measured results achieved in acoustical scale models of two coupled rooms when changing the aperture sizes and the sound source/receiver positions relative to the coupling aperture. The results can further validate the diffusion equation models with respect to varying aperture sizes and other aspects. Using the experimentally validated diffusion equation model, this work further investigates widely varied configurations of diffusion equation models in terms of purposely changing natural reverberation times and the source/receiver positions. Bayesian energy decay analysis described in the recent publications [15, 23] is applied to both acoustical scale model and diffusion-equation model results to provide deeper insight in the energy decay characteristics and their dependence on the aperture sizes, the sound source/receiver positions. The findings discussed in this work are

also compared with those recently published [12, 13]. The comparison indicates dramatic degrees of data misrepresentations by the quantifiers used in the previous work [13]. It further motivates use of advanced decay analysis methods, such as Bayesian decay analysis employing two levels of probabilistic inference.

This paper is organized as follows: Section II describes the experimental models recently created for systematic studies on coupled volume systems in the authors' laboratory, which have not yet been documented. Section II further discusses basic results from acoustical measurements in two coupled-rooms using Bayesian analysis. Section II also briefly introduces the diffusion equation model and the relationship between aperture dimension and the meshing/time resolutions. Section III discusses experimental results achieved from the experimentally measured and numerically modeled results. Section VI compares the results recently published with those achieved within the current study. Section IV discusses some related issues. Section V concludes the paper.

II INVESTIGATION MODELS

Some recent studies [9, 15] have discussed experimental investigations using eighth scale models, including two and three coupled-rooms. The current investigations have employed further developed, improved scale models. Therefore, the following section describes the experimental models in details. The discussions in this paper focus mainly on double-slope decay characteristics achieved in two coupled-rooms.

II.1 Eighth scale models with automatic scanning system

Figure 1 (a) illustrates an eighth scale model of two adjacent rooms of different volumes, separated by a movable wall for the purpose of adjusting the coupling aperture size. Figure 1 (b) renders a view into the larger room when the movable wall entirely separates it from the smaller room. Table I lists the dimension of two rooms given in original (scaled-up) sizes. The current work investigates a narrow coupling aperture as shown in Fig.1 (a), it opens from the top to the bottom of the rooms with a narrow opening slit, the width of which ranges from 3.75 cm to 20 cm. On top of the two rooms a step-motor-driven scanning

mechanism outside the rooms moves the microphone holder inside via magnetic coupling to ensure minimum changes inside the system under test. The scanning mechanism is defined for perpendicular two-axes (XY) with a repositioning uncertainty on order of 1.0 mm.

The diffusely reflecting surfaces are designed to achieve as much diffuse reflections as possible for a wide frequency range. Figure 2 (a) shows a photograph of one complete random pattern of the surfaces, the tessellation of its four sides are identical, so that the orientation of the diffuse reflecting elements can be again randomly arranged to avoid periodically repeating patterns. Figure 2 (b) illustrates experimentally measured diffusion coefficients for two different incident angles (0 and 45 degrees) using an acoustic goniometer [24, 25]. Diffusion coefficients higher than 0.5 in 1 kHz (oct.) are considered as highly diffusely reflecting [25].

When the movable wall is completely closed, the two rooms are acoustically separated. The reverberation times in the separate rooms respectively are termed natural reverberation times. The natural reverberation times are estimated from five room impulse responses measured in each room at spatially significantly separated receiver positions. Figure 3 illustrates the experimentally measured results of natural reverberation times (given for the full-sized rooms). The variation bars represent spreading of estimated natural reverberation times over five different measurements. For the octave bands between 250 Hz and 2 kHz the averaged natural reverberation times are well separated in values, and their spatial variation is small, which is the consequence of the highly diffuse reflection surfaces of the rooms, also indicating that the sound field in each room is highly diffuse. In the octave frequency band of 1 kHz (of the full-scale dimension) all the measured data from scale-modeled rooms are analyzed and discussed in what follows.

II.2 Diffusion equation model

The acoustic diffusion equation may be considered as an analytical approximation of the acoustic radiative transfer equation, also termed transport equation [26, 27] in the acoustics literature, where sound radiation is treated in a similar way to electromagnetic radiation and where the propagation of radiation through a medium is mainly affected by absorption, emission and scattering processes. The acoustic diffusion equation model is obtained by

expanding the sound radiation in first-order spherical harmonics [29]. The resulting model assumes that the variations in energy density and energy flow remain small over one mean-free path [28]; therefore, reflected energy must dominate over absorption and the reflection events must be predominantly diffuse [29, 30].

The diffusion equation model for the sound energy density $w(\mathbf{r}, t)$ at position $\mathbf{r}$, and time t , defined on a domain V , with a sound source term $P(t)$ located at position $\mathbf{r}_s$, can be expressed by a partial differential equation with mixed boundary conditions [8, 31],

$$\begin{aligned} \frac{\partial w(\mathbf{r}, t)}{\partial t} &= D \nabla^2 w(\mathbf{r}, t) + c m w(\mathbf{r}, t) \\ &= P(t) \delta(\mathbf{r} - \mathbf{r}_s) \quad \text{in } V, \end{aligned} \quad (1)$$

$$-D \frac{\partial w(\mathbf{r}, t)}{\partial \mathbf{n}} = A(\mathbf{r}, \alpha) c w(\mathbf{r}, t) \quad \text{on } \partial V. \quad (2)$$

Equation (1) represents an inhomogeneous parabolic partial differential equation, where ∇^2 is the Laplace operator and $\mathbf{n}$ represents the unity normal outwards vector to the boundary surface. In proportionated enclosures $D = \lambda c / 3$ is the so-called diffusion coefficient with c being the speed of sound.

This diffusion coefficient takes into account the room geometry through its *mean free path* $\lambda = 4V/S$, with volume V and total interior area S , whereas the term $c m w(\mathbf{r}, t)$ accounts for atmospheric attenuation within the room, with m being the absorption coefficient of air. Finally, Eq. (2) represents a mixed boundary condition that models the local effects on the sound field by absorption on surfaces. This absorption factor takes different values according to different existing approaches [7, 8]. In general, many authors stated that the diffusion equation properly models the sound field in rooms with overall low absorption [8, 16, 31, 32]. In this paper, the modified absorption factor is adopted to perform the simulations since it has been shown the widest range of applicability in terms of overall absorption in the room under investigation [8, 29],

$$A_{\text{Modified}}(\mathbf{r}, \alpha) = \frac{\alpha(\mathbf{r})}{2[2 - \alpha(\mathbf{r})]}, \quad (3)$$

with $\alpha(\mathbf{r})$ being the absorption coefficient of the surface. Another boundary condition is the

continuous boundary of the coupling aperture, it has to fulfill the following condition (see Appendix for details):

$$\hat{\mathbf{n}} \cdot [D_1 \nabla w(\mathbf{r}, t) - D_2 \nabla w(\mathbf{r}, t)] = 0, \quad (4)$$

that represents a continuity boundary condition on interior boundaries at the aperture, where D_1 is the diffusion coefficient in the primary room and D_2 is the diffusion coefficient for the secondary room. When the two rooms are of proportionated rooms, so that $D_i = \lambda_i c/3$ with λ_i being the mean-free path of each individual rooms.

II.2.1 Finite difference implementation

In order to implement the acoustic diffusion equation model, a numerical finite difference approach is used. This approach considerably reduces implementation cost compared to other approaches such as finite element methods and it could be easily implemented. Moreover, in this particular case, the simplicity of the room model, i.e. no rounded corners or non-planar surfaces, makes worthy to use finite differences.

Recently, a Dufort-Frankel finite difference scheme has been proposed to be used in a diffusion equation model [33]. In this numerical technique, both time and space are discretized. This scheme is considered unconditionally stable, meaning any discretization value might be chosen. However, in order to ensure that the predictions converge to a fixed value with low errors, a certain relation between the space and time sampling must be established, e.g. $(\Delta t)^2/(\Delta \nu)^2 \leq 10^{-8}$ [33], where Δt is the temporal sampling difference and $\Delta \nu$ is the smallest spatial sampling difference used in any direction, defined along Cartesian co-ordinates x , y or z .

The current work investigates a narrow coupling aperture, it opens from the top to the bottom of the rooms with a narrow slit, the width of which ranges from 5 cm to 160 cm (in real size). In order to select the right set of difference values for the finite difference scheme, an appropriate way would be to choose first, the smallest spatial discretion value, forcing to get the temporal difference Δt . For other dimensions the spatial sampling difference may be selected on convenience. In the current study of coupled-volumes, the aperture widths

are narrow, spatial attention has to be given to the narrowest dimension (along Y-axis). It is straightforward to see how the smallest sample size has to be chosen to be related to the narrowest aperture size. The narrower the aperture, the smaller $\Delta\nu$ has to be. For practical purposes, the direction containing the aperture (y -axis) determines $\Delta\nu$ used in all simulations, which in this particular case, for convenience, is $\Delta y=5$ cm, since as indicated in Sec. III.1, the narrowest aperture width under consideration is 30 cm, meaning that at least six discrete points are used to model the aperture. Once this value has been fixed, it determines the maximum sampling period to be used, which in this particular case is $\Delta t = 5 \mu s$. Clearly, in case a higher resolution is needed, for instance, to properly determine turning points, highest resolution may be used, increasing the computation time. For other dimensions the spatial sampling differences can be selected at convenience ($\Delta x=0.2$ m and $\Delta z=0.265$ m in these experiments).

II.3 Model adjustment

Using the dimensions and acoustical properties of the scale model of the two coupled-rooms shown in Fig. 1 (a), the diffusion equation model of the two coupled-rooms is also established. The model adopts the same geometries of the scale model rooms. When considering each room separately the boundary conditions inside the diffusion equation model take absorption coefficients in Eq.(3) via Eq.(2) in order to achieve the same natural reverberation times as those measured in the separate scale-model rooms, taken from 1 kHz octave frequency band. According to the surface treatment of the acoustical scale models, the surfaces of the diffusion equation models are assigned to feature absorption coefficients as listed in Table I, apart from the top wall and the separating wall of the two rooms which are assigned low absorption coefficient to be 0.01. Through this adjustment of absorption properties exact averaged natural reverberation times at 1 kHz octave band measured in each scale-model room can be achieved in the diffusion equation models. The surface boundary conditions assigned according to Eq.(3) [8] are more suitable for this high absorption as listed in Table I.

After the adjustment described above, the two modeled rooms are coupled together by assigning the aperture boundary condition expressed in Eq.(9) with the same aperture width

and the height as in the acoustical scale models, the aperture opens for the entire height from the top to the bottom of the rooms. The following section compares the results from the scale model and the diffusion equation model and discusses the effect of varying aperture width and source-receiver positions.

III EXPERIMENTAL VALIDATIONS

The automatic scanning system [see Fig. 1 (a)] developed for the scale models are used throughout this experimental work. Figure 4 illustrates the top views of two configurations of the scale model coupled rooms on which this current work is focusing. The aperture size is changed systematically from wider to narrower widths. The sound source in the primary room as shown in Fig. 1 (b) is a miniature dodecahedron of with approximately omnidirectional characteristics within frequency ranges of interest, particularly between 6 kHz and 11 kHz at eighth scale model frequencies. When given for the full-size, it corresponds well to 1 kHz octave frequency band.

Two sound source positions are of interest as shown in Fig. 4. One is at the lower-left corner of the primary room [Fig. 4 (a)], while a grid for the sound receiver of three rows and ten columns is defined towards the upper left corner of the primary room. In another configuration the sound source finds itself next to the aperture in the primary room [Fig. 4 (b)]. For the receiver positions a grid of ten rows and three columns is defined towards the lower-left corner of the primary room. For every aperture size (from 20 cm wide down to 4 cm in the eighth scale models) at each sound source position, thirty room impulse responses are measured. The sound receiver (microphone) is moved to the center point of each grid as shown in Fig. 4 by the automatic scanning mechanism. After sufficient time for the mechanical system to settle down, it automatically triggers a periodic log-sweep-sine excitation followed by cross-correlation algorithm [34]. For every systematic change, the diffusion equation model also yields thirty energy impulse responses at the same receiver positions. The following section compares results of experimentally measured sound energy decay functions with those predicated by the diffusion equation model.

III.1 Comparison of energy decay characteristics

Figure 5 illustrates comparisons of double-slope decay characteristics between acoustically measured decay functions in scale models and those from the diffusion equation models. Varied aperture widths are arranged from 30, 40, 60, 80, 120 to 160 cm (given in full-size). The sound energy decay characteristics are quantified using Bayesian decay analysis [15]. Figure 5 (a) compares the results of two decay times (T_1, T_2) of the double-slope energy decays evaluated for each aperture width, respectively. The variation bars represent the estimation spreading analyzed over 30 receiver positions. The averaged decay times estimated from both experimentally measured and the diffusion-equation modeled sound energy decay functions demonstrate good agreement across all the aperture size. With increasing aperture sizes/widths the two decay times (T_1, T_2) decrease slightly. The natural reverberation times measured in each separate rooms are also plotted for ease of comparison.

Figure 5 (b) illustrates comparison results of the level differences. Figure 5 (c) compares results of the turning points from the initial decay (the first slope) to the late decay (the second slope) of the double-slope decay functions. Figure 6 conceptually illustrates the definition of the level difference ΔL and the turning point (τ_t, τ_L). The level differences and the turning point are defined in Ref. [9], the turning point is quantified by the turning point time, τ_t , and the turning point level τ_L . As illustrated in Fig. 6, the two decay times and one turning point, or the level difference can adequately describe the double-slope decay characteristics observable in two coupled rooms, when the decay times of the secondary room are clearly longer than those in the primary room. Highly similar results have also been achieved for the source-receiver configuration as shown in Fig. 10 (b). Given the good agreements of the two decay times between the diffusion equation models and the acoustical scale models [Fig. 5(a)], the consistent discrepancies of the level differences and the turning point in their time-instance (termed *turning point time* in the following) hint at the inherent nature/limitation of the diffusion equation models.

The measured turning point times in the acoustical scale models, averaged over 30 receiver positions, are clearly larger than those estimated from the diffusion equation models. The turning point time is defined from the starting of the direct-sound. The overall averaged

time difference amounts to 85.6 ms which is 7.36 times of the mean-free time (11.6 ms) in the primary room. The mean-free time is the time for the sound to travel one mean-free-path length. In other words, the diffusion equation model seems to be valid to provide accurate reverberation process predictions after a certain number of mean-free times. Figure 6 conceptually illustrates this diffusion-equation-model agreement at a later time instance with the experimentally measured decay function.

Recent works have also reported on this phenomena [9, 19]. Xiang et.al. in their previous work [9] reported that the diffusion equation model can be considered as valid after at least 2 or 3 mean-free path times. In their work, they estimated this value based on a few measurement points in different scale models of two-coupled rooms, their dimension and acoustical properties are also listed in Table I. A close comparison between two different experimental models as listed in Table I reveals intrinsic differences. In the previous investigations [9], the overall surface absorption is significantly lower than those used in the scale models recently developed for the current investigations, although the dimensions (surface areas S and volumes V) of the scale model rooms are also quite different, which can be summarized by mean-free-path lengths ($\lambda = 4V/S$).

The measured turning-point times when compared with those predicated by the diffusion equation reveal that the accurate prediction of the diffusion equation model can also be considered valid at a later time instance. Figure 6 conceptually illustrates the observed agreement between experimentally measured energy decay function and that predicated by the diffusion equation model. The comparison between the experimentally measured results and the diffusion equation modeled results in Fig. 6 suggests that the diffusion equation model is valid in a late time instance, since the experimentally measured results include the direct sound, early reflections, and the reverberation tails while the diffusion equation model only includes diffuse sound field in the room which develops after the early reflections. Table I also suggests that the time instance for the diffusion equation model to be valid depends on the overall absorption inside the room under investigation. The lower the overall absorption in the room, the earlier the diffusion equation models become valid, because rigid walls more efficiently facilitate the mixing process of the multiple reflections bouncing back-

and-forth inside the enclosure. Conversely, enclosures with higher overall surface absorption will less efficiently promote the mixing process. This phenomena was already seen for the homogeneous room case [9, 19], where the number of mean-free times to be disregarded from the measured impulse responses increases so as to compare with the diffusion-equation modeled results when the overall absorption increases. In fact, for absorption higher than 0.5 more than 3 mean-free times had to be disregarded from the measured impulse responses.

From Fig. 5 and 6 a number of observations are worth discussing:

- a) Decay times in the coupled rooms are often on the order of the natural reverberation times. When the coupling weakens, as the aperture becomes smaller, the first decay time (T_1) becomes closer to the natural reverberation time (T'_1) of the primary room. When the coupling becomes stronger, as the aperture becomes larger, the first decay time (T_1) can be shorter than the natural reverberation time (T'_1) of the primary room [see Fig. 5 (a)]. The second decay time (T_2) will generally be shorter than the natural reverberation time (T'_2) in the secondary room. With increased coupling, the second decay time (T_2) will also become shorter [see Fig. 5 (a)].
- b) The diffusion equation model can predict the reverberation process in the coupled-volume system accurately, but not without limitations. When the system is featured with diffusely reflecting surfaces and low absorption, the prediction of the reverberation process can be valid after a few mean-free times (at least 2 or 3 mean free times [9]). With increasing overall absorption, the diffusion equation model requires more time before the prediction is valid as this work demonstrates.

Over large receiver areas and across a wide range of aperture sizes, the agreement of the two decay times and consistent difference in the turning point times, and the level differences as illustrated in Fig. 6 validate the accurate prediction of reverberation process in coupled-volume systems using the diffusion equation model, yet the diffusion equation model accurately predicts the reverberation process in the enclosures only from a later time instance onwards as conceptually illustrated in Fig. 6, 6-8 times of the mean-free times in the primary room. Being aware of delayed time instance of the validity, following investigations (Sec.III.3) will change the diffusion equation models in systematic way, yet go beyond the

configurations implementable in the experimental scale models as set in Sec. II.3.

III.2 Receiver distance dependence

The configuration as illustrated in Fig. 4 (a) is of practical significance, when considering the scenario in concert halls where the coupled reverberation chambers are located behind the side walls of the main floor. The double-slope or multiple-slope decay profiles should be better understood over the audience area with an increasing distance to the side wall. Figure 7 illustrates averaged values of decay times and the level differences over three rows as a function of receiver positions evaluated from the experimental results of the scale models. Receiver position 1 corresponds to the very right column, close to the aperture, while receiver position 10 corresponds to the far-left column, far away from the aperture [see Fig. 4 (a)]. For clarity, only three aperture widths (30, 60, and 120 cm) are shown. With the receiver position going away from the aperture, the decay times (T_1, T_2) change slightly. A slight increase of the level difference (2 - 3 dB) with an increasing distance to the side wall is observed as illustrated in Fig. 7 (b). Overall, with the narrow slit-shaped aperture the double-slope profiles of the reverberant energy decays change only slightly with regards to the distance from the aperture.

III.3 Sound source near the aperture

Figure 4 (b) shows another group of experimental measurements where the sound source is placed next to the aperture, while the receiver area is covered by a grid of 10 rows by 3 columns towards the lower-left corner of the primary room. This configuration is motivated by the practical application as the stage shell can be designed in partially separating the stage from the reverberant stage house. The overall distance from the grid to the sound source is comparable to the grid as defined in Fig. 4 (a). Highly similar results and a similar agreement with those of the diffusion equation models in terms of decay times have also been observed as discussed in previous section. Figure 8 illustrates the comparison between the level differences of systematically varied aperture widths for the two different configurations as shown in Fig. 4 (b). It suggests that the level differences for the narrow aperture widths,

when the acoustical coupling is weak, are significantly lower than those of the configuration in Fig. 4 (a). Put differently, the second decay slope will be much shallower relative to the initial decay slope if the sound source is placed near the aperture for the cases when the coupling aperture width is smaller.

IV APERTURE SIZE EFFECT

After validating the diffusion equation model of coupled-volume systems using the experimental investigations in acoustical scale models, this section discusses further investigations using the diffusion equation model with widely varied parameters.

In order to achieve distinct double-slope decay profiles, the natural reverberation times T'_1 of the primary room is assigned to be 0.56 s while T'_2 of the secondary room to be 4.1 s by adjusting the surface absorptions. Figure 9 (a) shows the changes of the energy decay functions when the aperture width decreases from 8 cm down to 1.3 cm. With decreasing aperture width the two decay slopes change slightly. Significant changes are observed in the level differences or the starting level of the second slope decay. For clarity, Fig. 8 (b) magnifies three decay curves taken from Fig. 8 (a), for aperture width of 1.3 cm, 1.8 cm and 3 cm. Table II lists relevant decay parameters analyzed using Bayesian probabilistic inference [15]. With decreased aperture width, the decay time T_1 changes from 0.56 s, 0.55 s, to 0.54 s, while the decay time T_2 changes from 3.77 s, 3.65 s, to 3.34 s. The slight changes in decay time values of the second slope decay are illustrated in the decay curves by three nearly parallel decay slopes, but the second-slope decay starts at three different levels, reflected by the level difference of 13.9 dB, 20.4 dB and 26.6 dB.

When using the late decay time as defined by a recent work [13], the late decay time becomes 0.67 s, 2.45s and 3.76 s, respectively, clearly misrepresenting the underlying physics of the decay profile changes when the aperture width decreases. The results discussed here suggest that those quantifiers, whether LDT/T_{10} , T_{60}/T_{15} , or other ratios based on linear-fits using two straight-line models within preselected, fixed level ranges, prior to the data analysis, cannot characterize multiple-sloped decay processes with the accuracy necessary to provide deep insight into energy decay characteristics in coupled-volume systems. Therefore,

the previous studies have not yet been able to articulate the physical changes of the sound energy decays when changing the aperture size.

V CONCLUDING REMARKS

This paper has discussed systematic investigations on aperture size effect, and the source-receiver positions using an experimental model consisting of two coupled rooms, where each room can be considered as a proportionated space. An eighth scale model of the two rooms facilitates systematic measurements of room impulse responses when changing the aperture size and the source, receiver positions. The diffusion equation model is also established for the experimental model. The comparison of results obtained from the acoustical scale models and from the diffusion equation model has a number of implications.

The diffusion equation model is implemented within the finite-difference framework. In order to model the enclosures with narrow (slit) apertures, the meshing conditions, following the finite-difference scheme governed by the Courant conditions [], have to precede the meshing conditions governed by the mean-free path lengths as recent publications have pointed out [31, 7]. In this work one-sixth of the narrowest coupling width is assigned which is experimentally proven to be effective to predict the changes in the energy decays in the reverberation tails.

The diffusion equation validly models room-acoustic reverberation processes when a diffuse sound field is established. Two coupled-rooms generating double-slope energy decay profiles can effectively support the 'time-delay' comparisons in terms of turning point estimates, otherwise the sound energy decays will dominantly demonstrate single-slope decays in highly diffuse sound fields within proportionated single-space rooms. Bayesian decay analysis has been proven to be effective to characterize multiple-slope decay profiles. The comparison between the experimentally measured turning points of the double-slope decays with those from the diffusion equation model reveals that the diffusion equation modeled results are accurately comparable with those from the experimentally measured results after certain number of the mean-free times starting from the direct sound of the measured room impulse responses, which indicates that the diffusion equation model is only valid to pre-

dict the room-acoustic reverberation processes in the later time instance after the diffusion sound field is established. When the system is featured with diffusely reflecting surfaces and low absorption, the diffusion-equation-based prediction of the reverberation process can be valid after a few mean-free times (at least 2 or 3 mean free times). With increasing overall absorption, the model is only valid for late reverberation as this work demonstrates. This work leads to better understanding of the diffusion equation applied to modeling room-acoustics reverberant fields, which after sufficiently many diffuse reflections that take place on the room surfaces render a sound field whose reverberation processes can be described as diffusion processes using the same equation governing the propagation of particles inside a scattering medium.

This work has primarily focused on source-receiver locations relative to the aperture, and on systematic changes in aperture size (widths) for the given overall absorption coefficients on the enclosure surfaces. Systematic investigations on different degrees of absorptions are beyond the scope of the current study, they may be potential research topics for future effort. The discussion about the valid ranges of the diffusion equation models should also be generalized in single-space simulations in the near future.

VI ACKNOWLEDGMENTS

The authors are especially grateful to Craig Schaeffer, who has built the scale-model rooms with automatic high-resolution scanning system in the scope of his MS. degree thesis. Thanks are also due to Angela Ostrowski, Zichun Zhou, who contributed to the collection of experimental data from scale models of coupled rooms. We are also grateful to Dr. Jason Summers for many insightful discussions, to John Paul Nichols for careful proofreading and editing of English-writing.

References

- [1] J. C. Jaffe, Selective reflection and acoustic coupling in concert hall design, *Proceedings of the Music and Concert Hall Acoustics*, MCHA 1995, Ed. by Y. Ando and D. Nosen Academic Press, New York, 1995, pp. 85-94.
- [2] R. Johnson, E. Kahle, and R. Essert, Variable coupled cubage for music performance, in *Proc. Music & Concert Hall Acoustics*, eds. Y. Ando & D. Nosen, 1997, Academic Press, New York, pp. 373 - 385.
- [3] J. C. Jaffe, Innovative approaches to the design of symphony halls, *Acoust. Sci. & Tech.* bf 26 (2005), pp.240-243.
- [4] J. E. Summers, R. R. Torres, and Y. Shimizu, Statistical-acoustics models of energy decay in systems of coupled rooms and their relation to geometrical acoustics, *J. Acoust. Soc. Am.* 116 (2004), pp. 958-969.
- [5] J. E. Summers, Accounting for delay of energy transfer between coupled rooms in statistical-acoustics models of reverberant-energy decay, *J. Acoust. Soc. Am. Express Letters*, 132 (2012), pp. EL129-EL134.
- [6] J. Summers, R. Torres, Y. Simishu, and B. L. Dalenbaeck, Adapting a randomized beam-axis-tracing algorithm to modeling of coupled rooms via late-part ray tracing, *J. Acoust. Soc. Am.* 118 (2005) 1491-1502.
- [7] A. Billon, V. Valeau, A. Sakout, and J. Picaut, On the use of a diffusion model for acoustically coupled rooms, *J. Acoust. Soc. Am.* **120**, 2043 - 2054 (2006).
- [8] Y. Jing, and X. Xiang, On boundary conditions for the diffusion equation in room-acoustic prediction: Theory, simulations, and experiments, *J. Acoust. Soc. Am.*, **123**, pp. 145-153 (2008).
- [9] N. Xiang, Y. Jing and A. Bockman, Investigation of acoustically coupled enclosures using a diffusion equation model, *J. Acoust. Soc. Am.*, **126** (2009) 1187-1198.

- [10] S. M. Pasareanu, M. C. Remillieux, and R. A. Burdisso, Energy-based method for near-real time modeling of sound field in complex urban environments, *J. Acoust. Soc. Am.*, **132** (2012) 4647-3658.
- [11] M. Meissner, Acoustic energy density distribution and sound intensity vector field inside coupled spaces, *J. Acoust. Soc. Am.*, **132** (2012), 228 - 238.
- [12] M. Ermann, Coupled volumes: Aperture size and the double-sloped decay of concert halls, *Build. Acoust.* **12**, 114 (2005).
- [13] D. Bradley and L. M. Wang, Optimum absorption and aperture parameters for realistic coupled volume spaces determined from computational analysis and subjective testing results", *J. Acoust. Soc. Am.* **127**, 223-232 (2010).
- [14] N. Xiang, P. Robinson, and J. Botts, Comments on "Optimum absorption and aperture parameters for realistic coupled volume spaces determined from computational analysis and subjective testing results" [in *J. Acoust. Soc. Am.* 127, 223-232 (2010)] (L), *J. Acoust. Soc. Am.*, **128** (2010), pp. 2539-2542.
- [15] N. Xiang, P. Goggans, T. Jasa, and P. Robinson, Bayesian characterization of multiple-slope sound energy decays in coupled-volume systems, *J. Acoust. Soc. Am.*, **129**, 741-752 (2011).
- [16] Y. Jing, and N. Xiang, A modified diffusion equation for room-acoustic predication (L), *J. Acoust. Soc. Am.*, **121**, 3284 - 3287 (2007).
- [17] Y. Jing, and N. Xiang, Visualization of sound energy across coupled rooms using a diffusion equation model, *J. Acoust. Soc. Am. Express Letters*, **124**, pp. EL360-EL365 (2008).
- [18] F. Marttelotta, Identifying acoustical coupling by measurements and prediction-models for St. Peter's Basilica in Rome, *J. Acoust. Soc. Am.* **126** (2009), pp. 1175 - 1186.
- [19] J. Escolano, J. M. Navarro and J. J. López, On the limitation of a diffusion equation model for acoustic predictions of rooms with homogeneous dimensions. *J Acoust Soc Am.* **128**. 1586-1589 (2010).

- [20] H.J. Pu, X.J. Qiu and J.Q. Wang, Different sound decay patterns and energy feedback in coupled volumes, *J. Acoust. Soc. Am.* **129**, 1972-1980 (2011).
- [21] M. R. Schroeder, New method of measuring reverberation time, *J. Acoust. Soc. Am.*, **37** (1965), 409-412.
- [22] L. Cremer, H. A. Mueller, and T. Schultz, *Principles and Applications of Room Acoustics*, Chapter II.3.3 - II.3.4, Applied Science Publishers, New York, 1982, Vol. 1
- [23] T. Jasa and N. Xiang, Nested sampling applied in Bayesian room-acoustics decay analysis, *J. Acoust. Soc. Am.*, **132** (2012), 3251-3262.
- [24] P. Robinson, and N. Xiang, On the subtraction method for in-situ reflection and diffusion coefficient measurements, *J. Acoust. Soc. Am., Express Letters*, **127** (2010), pp.EL99 - EL104.
- [25] T. Cox and P. D Antonio, *Acoustic Absorbers and Diffusers: Theory, Design and Application*, Chapter 4, 2nd Ed. (Taylor & Francis, London, 2009).
- [26] Y. Jing, E. Larsen, and N. Xiang, One-dimensional transport equation models for sound energy propagation in long spaces: Theory, *J. Acoust. Soc. Am.*, **127** (2010), pp. 2312-2322.
- [27] Y. Jing, and N. Xiang, One-dimensional transport equation models for sound energy propagation in long spaces: Simulations and experimental results, *J. Acoust. Soc. Am.*, **127** (2010), pp.2323-2331.
- [28] J. Picaut, L. Simon and J. D. Polack, A mathematical model of diffuse sound field based on a diffusion equation, *Acta Acustica united with Acustica*, **83**, 614 - 621, (1997).
- [29] J. M. Navarro, F. Jacobsen, J. Escolano, J. Lopez, A Theoretical Approach to Room Acoustic Simulations based on a Radiative Transfer Model, *acta acust. w. ACUSTICA*, **96** (2010), pp. 1078 - 1089.
- [30] C. Visentin, N. Prodi, V. Valeau, and J. Picaut, A numerical investigation of the Fick's law of diffusion in room acoustics, *J. Acoust. Soc. Am.*, **132** (2012), pp.3180-3189.
- [31] V. Valeau, J. Picaut and M. Hodgson, On the use of a diffusion equation for room-acoustic prediction, *J. Acoust. Soc. Am.* **119**, 1504 - 1513, (2006).

- [32] A. Billon, J. Picaut and A. Sakout, Prediction of the reverberation time in high absorbent room using a modified-diffusion model, *Appl. Acoust.*, **69**, 68–74, (2008).
- [33] J. M. Navarro, J. Escolano, and J. J. López, Implementation and evaluation of a diffusion equation model based on finite difference schemes for sound field prediction in rooms, *Appl. Acoust.*, **73**, 659–665, (2012).
- [34] S. Müller, and P. Massarani, Transfer-Function Measurements with Sweeps, *J. Audio Eng. Soc.* **49**, 443-471 (2001).

Appendix: Aperture modeling

A proper model of a coupled-volume system consisting of several different spaces, requires redefinition of diffusion coefficients. According to Billon et al. [7], a coupled-volume system consists of the application of Eq. 1 over each different space having each one a different diffusion coefficient. However, the previous literature has not accumulated the modeling mechanism of the aperture or interface between each two volumes. This aperture acts as a boundary condition causing neither absorption nor scattering over the particles traveling between both volumes. Therefore, in order to implement the model it is necessary to provide a mathematical description of the phenomena occurring in the interface between two volumes. For this purpose, consider a weak coupling so that their mean-free path distributions in individual rooms do not significantly change when connecting them together via the coupling aperture [4]. In the primary room the diffusion coefficient is denoted by D_1 , with volume V_1 , whereas D_2 is the diffusion coefficient for the secondary room with volume V_2 .

The aperture coupling mechanism in a diffusion equation model is derived using an *acoustic radiative transfer equation* [26, 29]. This model expands classical geometrical room acoustic modeling algorithms incorporating a propagation medium and objects within the room that can absorb and scatter sound particles.

For the acoustic radiative transfer equation, the boundary conditions are expressed in terms of the sound radiance $L(\mathbf{r}, \hat{\mathbf{s}}, t)$ which is defined as the energy flow at position $\mathbf{r}$ per unit normal area per unit solid angle per unit time t , where the normal area is perpendicular to the flow direction $\hat{\mathbf{s}}$. In absence of sources at boundaries, the reflected sound radiance in a direction $\hat{\mathbf{s}}$ as a consequence of a sound particle traveling from direction $\hat{\mathbf{s}}'$ and striking over a surface at position $\mathbf{r}_b$ is expressed as

$$L(\mathbf{r}_b, \hat{\mathbf{s}}, t) = \int_{\Omega^-} R_F(\mathbf{r}_b; \hat{\mathbf{s}}', \hat{\mathbf{s}}) L(\mathbf{r}_b, \hat{\mathbf{s}}', t) (\hat{\mathbf{s}}' \cdot -\hat{\mathbf{n}}) d\Omega', \quad (5)$$

where R_F is the surface scattering or reflecting function with units of sr^{-1} , defined as the probability that a particle at $\mathbf{r}_b$ moving in the $\hat{\mathbf{s}}'$ direction will be reflected into a new direction $\hat{\mathbf{s}}$. The sound radiance leaving the surface is determined by solving the incoming sound radiation integral over the negative hemisphere Ω^- [see Fig. 10 (a)].

In the particular case of the aperture modeling between both volumes, the interface does not produce any scattering or absorption over particles passing from one volume to another. Therefore, $R_F(\mathbf{r}_b; \hat{\mathbf{s}}', \hat{\mathbf{s}}) = 1$ only if $\hat{\mathbf{s}}' = \hat{\mathbf{s}}$; in any other case, the reflecting function should take value zero. To consider the boundary condition it should be taken into account that the direction-integrated sound radiation function at the boundary has to be the same at both parts of the interface:

$$\int_{\Omega^+} L_1(\mathbf{r}_b, \hat{\mathbf{s}}, t) (\hat{\mathbf{s}} \cdot \hat{\mathbf{n}}) d\Omega = \int_{\Omega^-} L_2(\mathbf{r}_b, \hat{\mathbf{s}}', t) (\hat{\mathbf{s}}' \cdot \hat{\mathbf{n}}) d\Omega', \quad (6)$$

with L_1 and L_2 being the sound radiance integrated at the hemisphere $\Omega^+ \in V_1$ and $\Omega^- \in V_2$, respectively [see Fig. 10 (b) for details].

After integration the diffusion approximation renders an expression for the sound radiation function (see [29] for details) and the boundary condition becomes

$$\frac{w_1(\mathbf{r}_b, t)}{4} + \frac{1}{2c} \mathbf{J}_1(\mathbf{r}_b, t) \cdot \hat{\mathbf{n}} = \frac{w_2(\mathbf{r}_b, t)}{4} + \frac{1}{2c} \mathbf{J}_2(\mathbf{r}_b, t) \cdot \hat{\mathbf{n}}, \quad (7)$$

where $w_1, \mathbf{J}_1 \in \Omega^+$ represent the sound energy density and energy flow at the aperture boundary in the primary room while $w_2, \mathbf{J}_2 \in \Omega^-$ in the secondary room, respectively.

If the integration area is considered small enough, it is straightforward to assume any air absorption phenomenon in the energy density at both parts of the interface occurs; then $w_1(\mathbf{r}_b, t) = w_2(\mathbf{r}_b, t) = w(\mathbf{r}_b, t)$. Therefore, Eq. (7) reduces to

$$\hat{\mathbf{n}} \cdot \mathbf{J}_1(\mathbf{r}_b, t) = \hat{\mathbf{n}} \cdot \mathbf{J}_2(\mathbf{r}_b, t) \quad (8)$$

Applying Fick's law, the continuity equation for the aperture between two coupled volumes results

$$\hat{\mathbf{n}} \cdot (D_1 \nabla w(\mathbf{r}_b, t) - D_2 \nabla w(\mathbf{r}_b, t)) = 0, \quad (9)$$

that represents a continuity boundary condition on interior boundaries at the aperture.

Table 1: Dimensions, absorption and natural reverberation times of two different experimental models.

		New Scale models	Scale models in Ref. [9]
Primary Room	Dimension (l, w, h)	4.88 m, 6.32 m, 7.20 m	4.8 m, 6.4 m, 7.0 m
	Volume ₁	222.06 m ³	215.04 m ³
	Absorption ₁	0.412	0.28
	Mean Free Path ₁	3.98 m	4.60 m
	Natural RT ₁	0.469 s	0.46 s
Secondary Room	Dimension (l, w, h)	7.60 m, 9.76 m, 7.20 m	6.8 m, 7.2 m, 7.0 m
	Volume ₂	534.07 m ³	342.72 m ³
	Mean Free Path ₂	5.36 m	4.66 m
	Absorption ₂	0.218	0.16
	Natural RT ₂	1.17 s	1.17 s

Table 2: Decay parameters analyzed using Bayesian decay analysis from diffusion-equation model predicted decay functions shown in Fig.7 (b).

Aperture width (cm)	1.3	1.8	3.0
A_1 (dB)	-5.08	-5.13	-5.21
T_1 (s)	0.564	0.554	0.54
A_2 (dB)	-31.69	-25.77	-19.15
T_2 (s)	3.77	3.65	3.34
ΔL (dB)	26.61	20.64	13.94
Turning point time (ms)	372	299	224
Turning point level (dB)	-35.1	-28.06	-22.55

Captions

Figure 1. (Color online) Eighth scale model of two coupled rooms. (a) Photo of the model rooms with a movable wall between the two rooms for adjusting the coupling aperture sizes. A step-motor-driven scanning mechanism is installed on the top of, and outside the models. It moves a microphone holder inside via magnetic coupling. (b) Top view into the larger model room to show the diffusely reflecting surfaces of interior walls. Two omni-directional model sound sources covering a frequency range between 300 Hz - and 45 kHz find themselves in one room corner.

Figure 2. (Color online) Diffusely reflecting surfaces and experimentally measured diffusion coefficients. (a) photograph of one complete random pattern of the surfaces, the tessellation of its four sides are identical. (b) Measure diffusion coefficients for two different incident angles.

Figure 3. (Color online) Natural reverberation times measured in two separate scale-model rooms (converted into full scale). The variation range represents the estimation spreading from five spatially significantly separated receiver positions.

Figure 4. Top view of scale models of two coupled rooms, showing the sound source position, the receiver grid definition, and the aperture location. The aperture size(width) is subject to change systematically. (a) Sound source finds itself in the lower-left corner of the primary room, the far most away from the coupling aperture. For the receiver positions, a grid of three rows and ten columns is defined towards the upper left corner of the primary room. (b) Sound source finds itself next to the aperture in the primary room. For the receiver

positions a grid of ten rows and three columns is defined towards the lower-left corner of the primary room.

Figure 5. (Color online) Comparisons of double-slope decay characteristics between acoustically measured decay functions in scale models and ones from the diffusion equation models. the aperture widths change from 30, 40, 60, 80, 120 to 160 cm. (a) Two decay times (T_1, T_2) of the double-slope decays. A top view of the models showing the source location, the aperture location, and the receiver grid is also included. The natural reverberation times are also plotted for ease of comparison. (b) Level differences (c) The time instance of the turning points.

Figure 6. (Color online) Conceptual double-slope decay functions from experimentally measured Schroeder decay function and the diffusion equation model predicted one, indicating the valid range of the diffusion equation model.

Figure 7. (Color online) Receiver distance dependence from the aperture for three aperture widths of 30 cm, 60 cm, and 120 cm. The receiver position is numbered from 1 for the closest distance to the aperture, to 10 for the farthest distance. (a) Decay times (T_1, T_2). (b) Level differences.

Figure 8. (Color online) Comparison of the level differences between the two configurations as shown in Fig. 10 (a), the sound source is far from the aperture, and (b) the sound source is close to the aperture.

Figure 9. (Color online) Trend of energy decay profiles predicated by the diffusion equation model when decreasing the aperture sizes. The natural reverberation times are adjusted to be ($T'_1 = 0.46s, T'_2 = 4.1s$). (a) Energy decay functions when the aperture width changing from 10 cm down to 1.3 cm. (b) Energy decay functions at aperture width of 1.3 cm, 1.8 cm and 3 cm, taken from (a) in magnified presentation. The decay parameters are listed in Table II.

Figure 10. Concept of phase-space and radiative particle transfer (a) xxx xxx xxx (b) xxx xxx xxx.

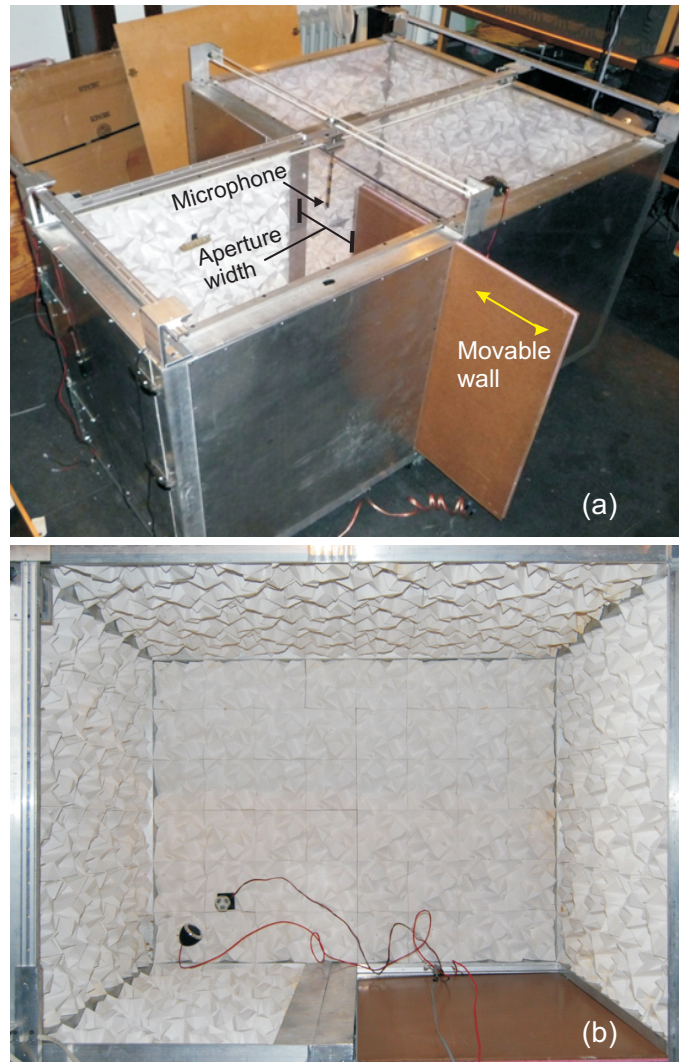


Fig. 01

Figure 1: (Color online) Eighth scale model of two coupled rooms. (a) Photo of the model rooms with a movable wall between the two rooms for adjusting the coupling aperture sizes. A step-motor-driven scanning mechanism is installed on the top of, and outside the models. It moves a microphone holder inside via magnetic coupling. (b) Top view into the larger model room to show the diffusely reflecting surfaces of interior walls. Two omni-directional model sound sources covering a frequency range between 300 Hz - and 45 kHz find themselves in one room corner.

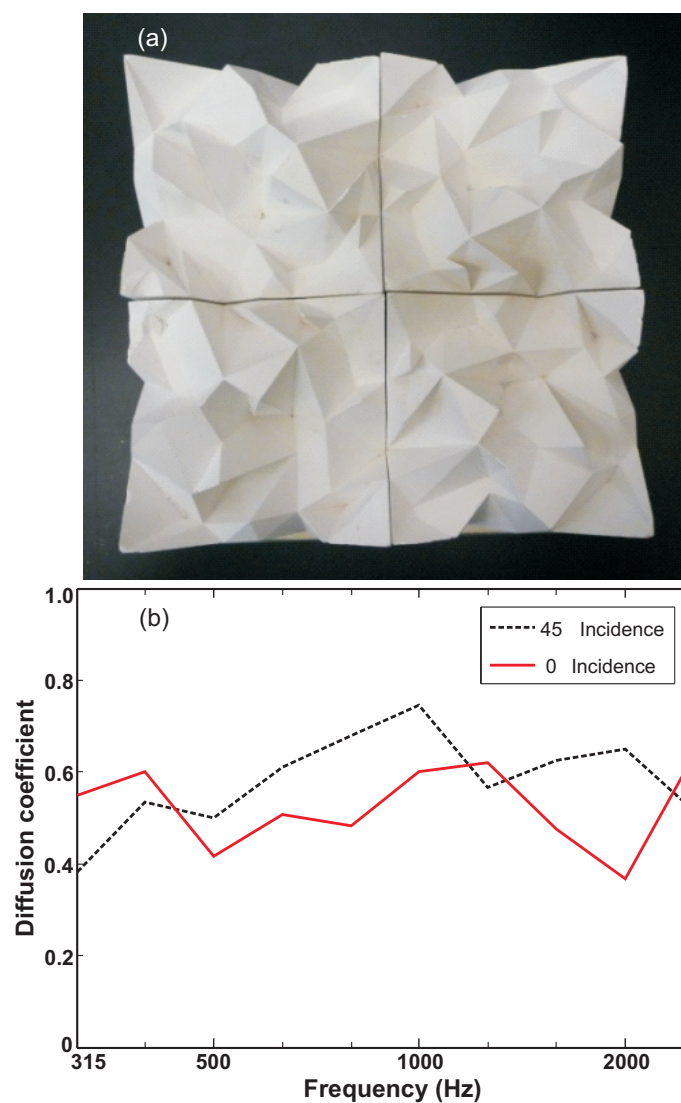


Fig. 02

Figure 2: (Color online) Diffusely reflecting surfaces and experimentally measured diffusion coefficients. (a) photograph of one complete random pattern of the surfaces, the tessellation of its four sides are identical. (b) Measure diffusion coefficients for two different incident angles.

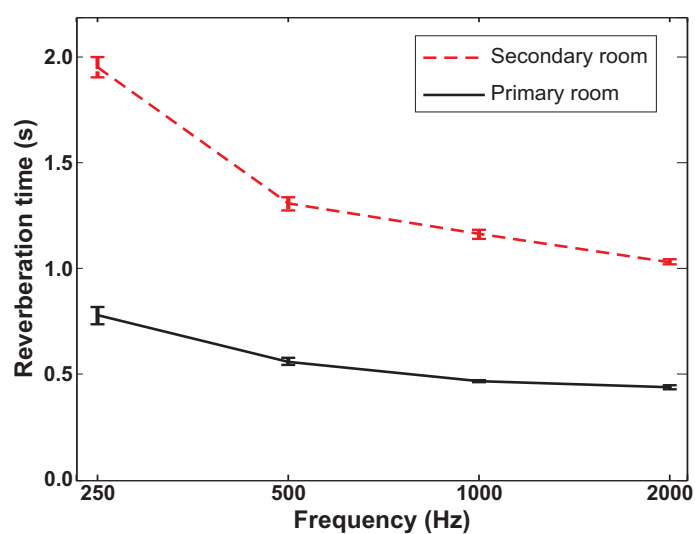


Fig. 03

Figure 3: (Color online) Natural reverberation times measured in two separate scale-model rooms (converted into full scale). The variation range represents the estimation spreading from five spatially significantly separated receiver positions.

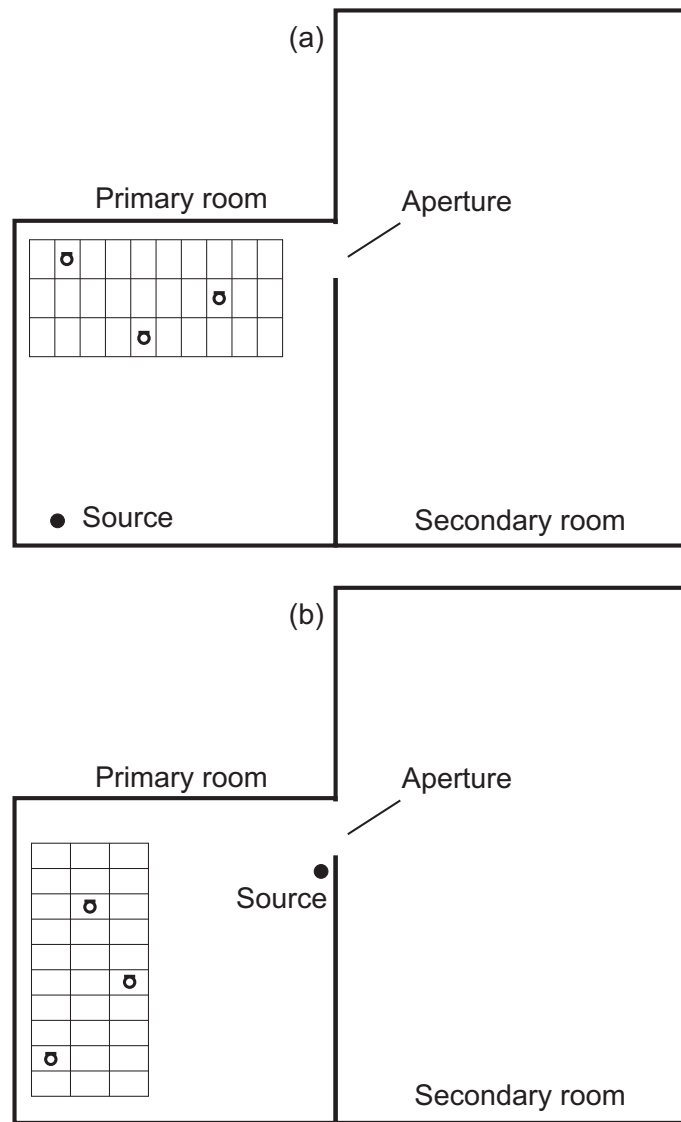


Fig. 04

Figure 4: Top view of scale models of two coupled rooms, showing the sound source position, the receiver grid definition, and the aperture location. The aperture size(width) is subject to change systematically. (a) Sound source finds itself in the lower-left corner of the primary room, the far most away from the coupling aperture. For the receiver positions, a grid of three rows and ten columns is defined towards the upper left corner of the primary room. (b) Sound source finds itself next to the aperture in the primary room. For the receiver positions a grid of ten rows and three columns is defined towards the lower-left corner of the primary room.

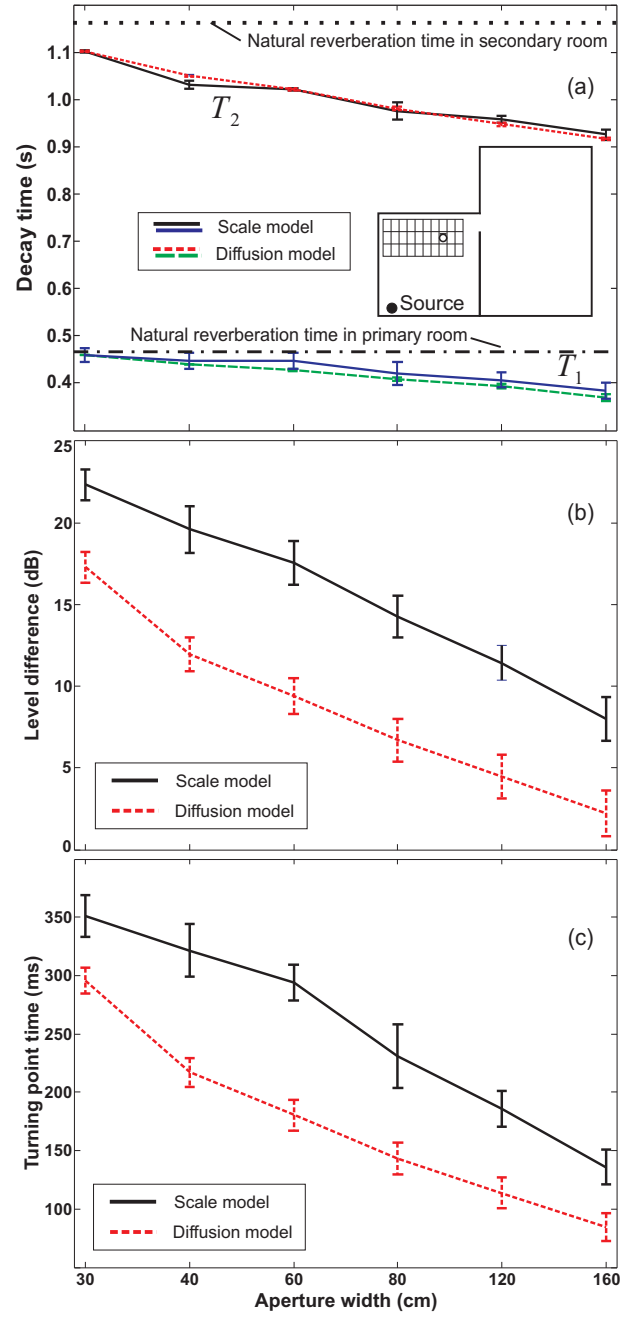


Fig. 05

Figure 5: (Color online) Comparisons of double-slope decay characteristics between acoustically measured decay functions in scale models and ones from the diffusion equation models. the aperture widths change from 30, 40, 60, 80, 120 to 160 cm. (a) Two decay times (T_1, T_2) of the double-slope decays. A top view of the models showing the source location, the aperture location, and the receiver grid is also included. The natural reverberation times are also plotted for ease of comparison. (b) Level differences (c) The time instance of the turning points.

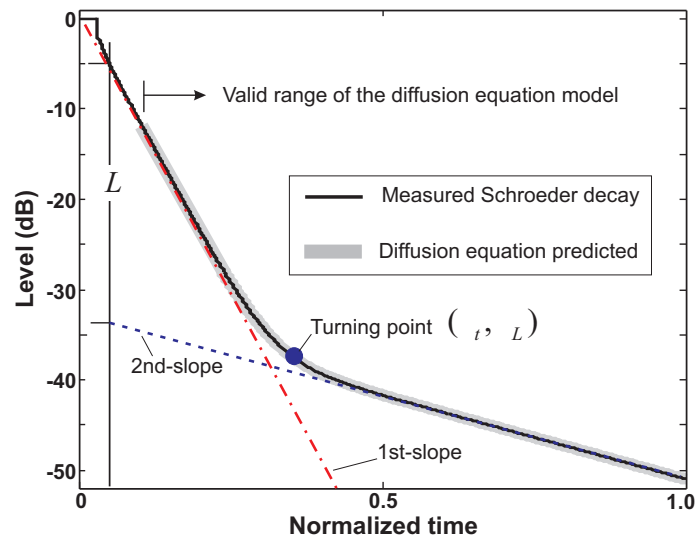


Fig. 06

Figure 6: (Color online) Conceptual double-slope decay functions from experimentally measured Schroeder decay function and the diffusion equation model predicted one, indicating the valid range of the diffusion equation model.

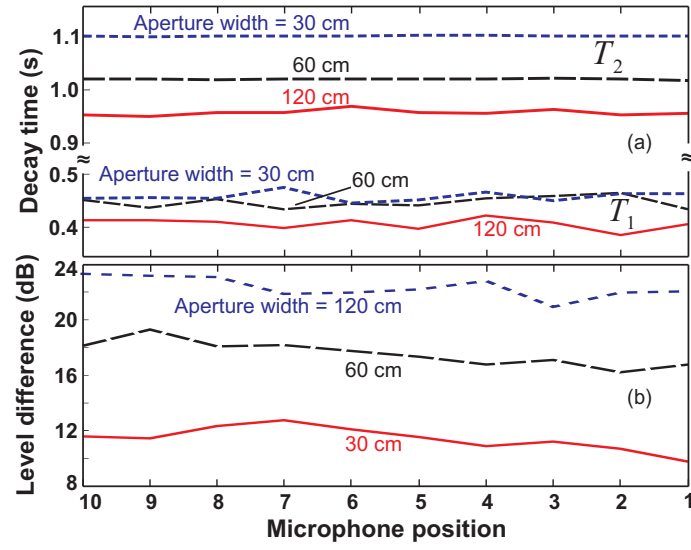


Fig. 07

Figure 7: (Color online) Receiver distance dependence from the aperture for three aperture widths of 30 cm, 60 cm, and 120 cm. The receiver position is numbered from 1 for the closest distance to the aperture, to 10 for the farthest distance. (a) Decay times (T_1, T_2). (b) Level differences.

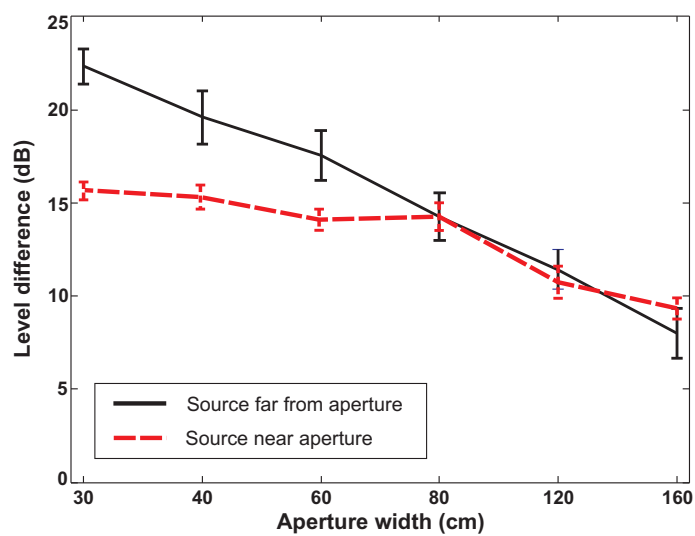


Fig. 08

Figure 8: Comparison of the level differences between the two configurations as shown in Fig. 4 (a), the sound source is far from the aperture, and (b) the sound source is close to the aperture.

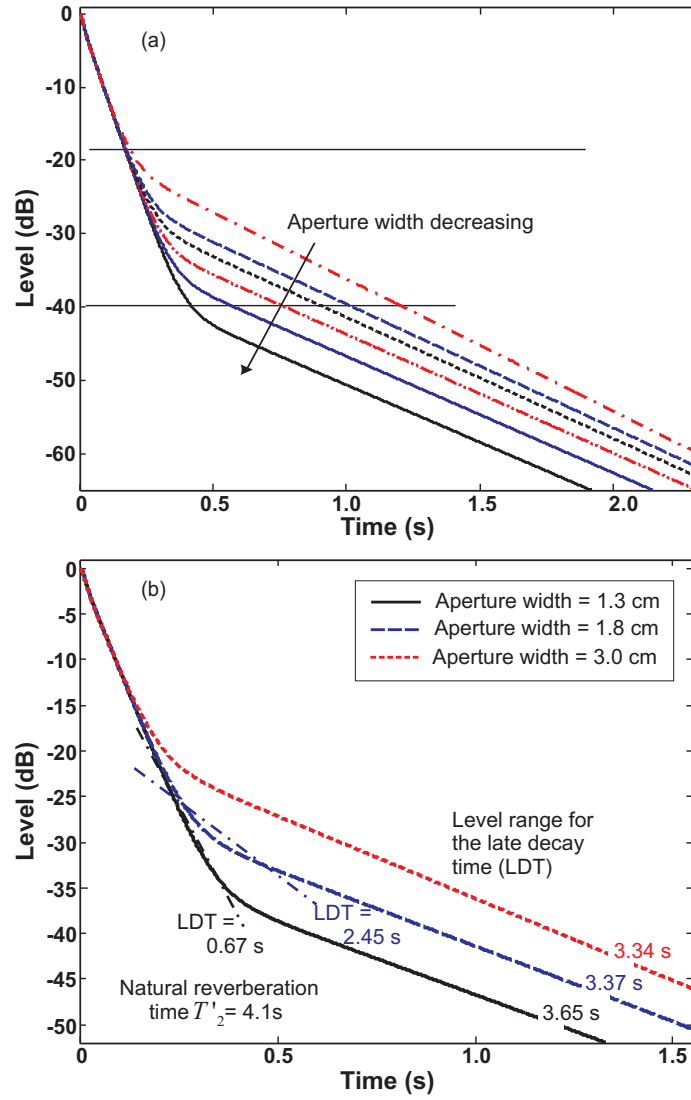


Fig. 9

Figure 9: (Color online) Trend of energy decay profiles predicated by the diffusion equation model when decreasing the aperture sizes. The natural reverberation times are adjusted to be ($T'_1 = 0.46$ s, $T'_2 = 4.1$ s). (a) Energy decay functions when the aperture width changing from 10 cm down to 1.3 cm. (b) Energy decay functions at aperture width of 1.3 cm, 1.8 cm and 3 cm, taken from (a) in magnified presentation. The decay parameters are listed in Table II.

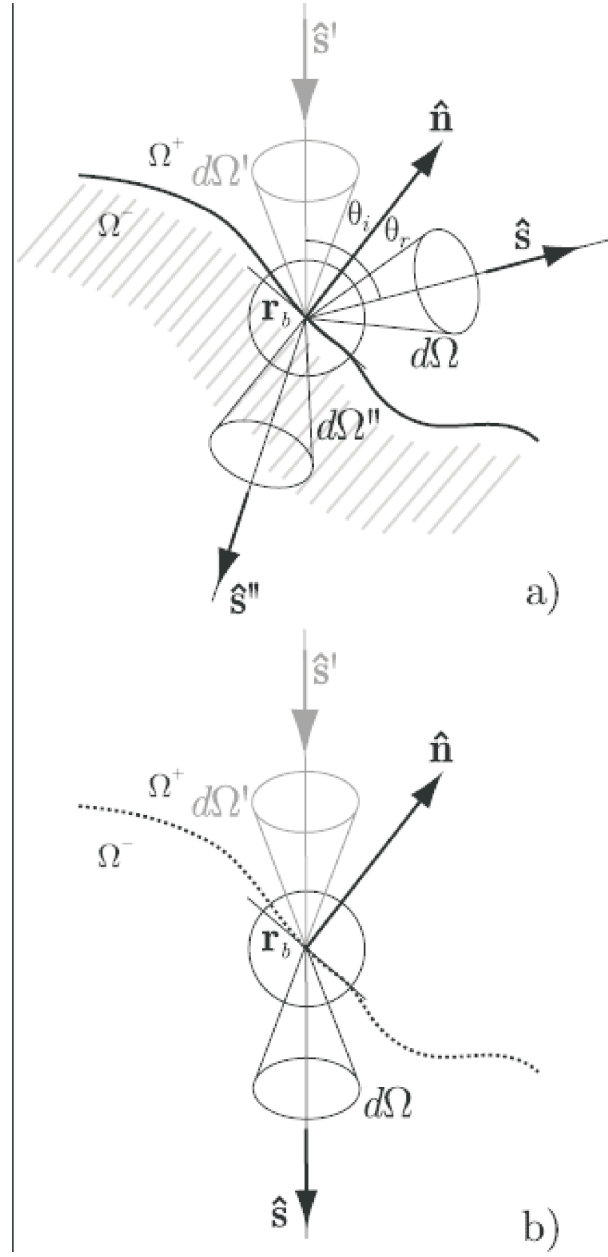


Figure 10: Concept of phase-space and radiative particle transfer (a) xxx xxx xxx (b) xxx xxx xxx.