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Investor Sentiment and Statistical Moments of the Return Distribution in the German Stock Market: A Three-Stage Empirical Analysis

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Murcia, 10 de Septiembre de 2022

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Frankfurt am Main, September 10, 2022

E. Hövel

Emile D. Hövel, M.Sc.

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“How much easier it is to be critical than to be correct.“
- Benjamin Disraeli (1804–1881) -

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Resumen

Esta tesis contribuye al avance de la literatura existente en esta área a través de una perspectiva holística en la investigación y el desarrollo de modelos que relacionan el sentimiento de los inversores y el rendimiento del mercado de valores alemán, utilizando como índice de referencia el CDAX.

Los estudios empíricos realizados hasta la fecha se han basado principalmente en datos del mercado de EE. UU., existiendo escasa literatura sobre el mercado alemán. Los hallazgos obtenidos para otros países no pueden aplicarse de forma generalizada al comportamiento del mercado alemán, especialmente porque Alemania parece estar fuertemente influenciada por las tendencias globales y los sentimientos de los inversores como consecuencia de su alta dependencia del comercio exterior.

En consecuencia, la evidencia empírica para Alemania en este campo de investigación, que incluye perspectivas transversales y longitudinales, es escasa. Dado que existen varios enfoques para medir y evaluar los vínculos entre el sentimiento de los inversores y los movimientos del mercado de capitales, en esta tesis se propone un sistema de categorización del sentimiento de los inversores propio. La muestra, abarca un periodo de 20 años, concretamente, desde 2001 hasta 2021.

Con respecto a la estructura de la tesis, se realiza en primer lugar una revisión completa de la literatura actual sobre la eficiencia del mercado y el sentimiento de los inversores. A continuación, se expone la metodología y para, posteriormente, analizar los resultados obtenidos. Cabe destacar el análisis empírico en tres etapas realizado en esta tesis.

En primer lugar, se realiza un análisis de componentes principales basado en el sentimiento de riesgo del inversor con el fin de mejorar el rendimiento del modelo.

En segundo lugar, se aplica un modelo de redes neuronales para abordar cómo varía la prima de riesgo de la confianza de los inversores. Se establece un modelo de redes neuronales que evalúa el rendimiento del mercado alemán a partir de 73 indicadores de sentimiento del inversor sin reducir dimensiones y realizando pruebas fuera de la muestra.

En tercer lugar, se realiza un análisis exploratorio en Twitter sobre el sentimiento de los inversores en el mercado de valores alemán en tiempos de Covid-19.

El estudio investiga el impacto de incorporar datos no estructurados en el análisis del sentimiento de los inversores para mejorar el poder discriminatorio y la precisión predictiva y emplea técnicas de procesamiento elaboradas. El estudio exploratorio se basa en un conjunto de datos únicos seleccionados a mano de casi dos millones de tweets en el mercado de valores alemán, recopilados exclusivamente para este estudio. En este contexto, se investiga y destaca la importancia del sentimiento de los inversores en las redes sociales para evaluar la volatilidad en el mercado bursátil alemán.

Como resultado, los tres estudios empíricos abordan numerosas cuestiones relevantes, y se plantean y discuten nuevos desafíos dignos de investigación en la parte final de la tesis.

Palabras clave: Distribución de la rentabilidad, Sentimiento de los inversores, Modelos multifactoriales, Análisis de componentes principales, Redes de gran memoria de corto plazo, Red neuronal recurrente artificial, Procesamiento de lenguajes naturales, Mercado de valores alemán

Abstract

This dissertation contributes to an increasing body of literature through a holistic perspective in research and model development to investigate the relationships between investor sentiment and the lower- and higher-order statistics of the return distribution in the German stock market utilizing the market-wide CDAX stock index as an exemplary sample.

Since empirical studies on investor sentiment are conducted mainly with US-market-based data, comparatively few academic contributions are made to the German investor sentiment literature. Moreover, previous findings for other countries cannot necessarily be generalized to Germany, especially as Germany appears to be mainly influenced by global trends and investor sentiments owing to its high dependence on foreign trade.

Consequently, the empirical evidence for Germany in this research domain, which includes both cross-sectional and longitudinal perspectives, is sparse. As various approaches exist to measure and assess the links between investor sentiment and capital market movements, a proprietarily defined investor sentiment categorization system is established, in which each investor sentiment indicator is assigned. This dissertation's underlying investor sentiment sample consists of all three categories of the dedicated categorization system for investor sentiment indicators and covers up to 20 years to 2021.

With regard to the thesis structure, a comprehensive overview of the literature and current research on market efficiency and investor sentiment is initially elaborated before the applied methodology and the evaluation results are analyzed. Of particular note is the three-stage empirical analysis conducted in this thesis:

First, a principal component analysis-based investor sentiment risk factor is established to improve model performance in traditional cross-sectional multi-factor models as measured by the corrected coefficient of determination and additional metrics.

Second, the application of Long Short-Term Memory (LSTM) artificial recurrent neural network architecture models to account for time-varying investor sentiment risk premia explaining and predicting the return distribution's lower- and higher-order statistics leads to notable findings.

A performant model for the German stock market results from fitting a deep neural network fed with 73 sentiment indicators without dimension reduction and performing out-of-sample tests.

Third, an insightful exploratory Twitter study of social investor sentiment in the German stock market in times of COVID-19-induced market turmoil is elaborated. The study investigates the impact of incorporating unstructured data into investor sentiment analysis to improve discriminatory power and predictive accuracy and employs elaborate processing techniques. The exploratory study is based on a unique hand-curated dataset of almost two million tweets on the German stock market, exclusively collected for this study. In this context, the importance of investor sentiment in social media for volatility in the German stock market is investigated and highlighted.

As a result, all three empirical studies address many vital matters, although new challenges worthy of investigation are as well raised and discussed in the final part of the thesis.

Keywords: Return Distribution, Investor Sentiment, Multi-factor Models, Principal Component Analysis, Long Short-Term Memory, Artificial Recurrent Neural Network, Deep Learning, Natural Language Processing, German Stock Market

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LIST OF ABBREVIATIONS

AAII	American Association of Individual Investors
AMSI	Acertus Market Sentiment Indicator
APT	Arbitrage Pricing Theory
ARCH	Autoregressive Conditional Heteroscedasticity
CAPM	Capital Asset Pricing Model
CBOE	Chicago Board Options Exchange
CDAX	Composite DAX
COVID-19	Coronavirus Disease 2019
DAX	Deutscher Aktienindex
DERI	Dynamic Equity Risk Indicator
DJIA	Dow Jones Industrial Average
ECB	European Central Bank
EMH	Efficient-Market Hypothesis
EMT	Efficient Markets Theory
EUREX	European Exchange
EURIBOR	Euro Interbank Offered Rate
FEARS	Financial and Economic Attitudes Revealed by Search
FIRST	Integrated Financial Market Information System
FY	Fiscal Year
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GDP	Gross Domestic Product
GRS test	F-Test proposed by Gibbons et al. (1989)
HML	Risk-Factor High-Minus-Low

IBOR	Interbank Offered Rates
Ifo	Institut für Wirtschaftsforschung
LSTM	Long Short-Term Memory
MSE	Mean Squared Error
MV	Market Value
NLP	Natural-Language Processing
OLS	Ordinary Least Squares
P	Price Index (Refinitiv)
PCA	Principal Component Analysis
PTBV	Price-to-Book Value (Refinitiv)
RESET	Regression Equation Specification Error Test
RI	Total Return Index (Refinitiv)
RMRF	Market Risk-Premium
RNN	Recurrent Neural Network
SD	Standard Deviation
SENT	Risk-Factor Investor Sentiment
SMB	Risk-Factor Small-Minus-Big
TED	Treasury-EuroDollar rate
US	United States (of America)
USA	United States of America
VDAX	DAX Volatility Index
VIF	Variance Inflation Factor
VIX	Volatility Index
VPS	Virtual Private Server
WML	Risk-Factor Winners-Minus-Losers

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LIST OF SYMBOLS

α	Constant (Ordinate section)
β	Slope Parameter
$E[x]$	Expected Value of x
ε	Error Term
F_i	Market Factor i
g^c	Candidate Value Gate
g^f	Forget-gate Layer
g^o	Sigmoid Gate
g^u	Input-Update-gate Layer
h	Slope Parameter of the Value Factor
h_t	Filtered Cell State in LSTM Cell
H_0	Null Hypothesis
H_1	Alternative Hypothesis
HML	Risk Factor High-Minus-Low
HS	High-Sentiment
$\boldsymbol{I^c}$	Projection Matrix in LSTM Cell
$\boldsymbol{I^f}$	Projection Matrix in LSTM Cell
$\boldsymbol{I^o}$	Projection Matrix in LSTM Cell
$\boldsymbol{I^u}$	Projection Matrix in LSTM Cell
λ_i	Risk Premium i
LS	Low-Sentiment
m	Market Portfolio
m_t	Memory Vector in LSTM Cell

NS	Neutral-Sentiment
p	P-Value
ψ	Slope Parameter of the Sentiment Factor
R^2	Coefficient of Determination
$\bar{R}^2$	Corrected Coefficient of Determination
ρ	Pearson's Correlation Coefficient
r_i	Return of the i-th Security or Portfolio
R_f or r_f	Risk-free Interest Rate
R_m or r_m	Return of the Market Portfolio
$RMRF$	Market Risk Premium
$SENT$	Investor Sentiment Risk Factor
$SENT_{PCA}$	PCA-based Investor Sentiment Risk Factor
s	Slope Parameter of the Size Factor
SSE	Error Sum of Squares
SSR	Regression Sum of Squares
SST	Total Sum of Squares
$\sigma_{i,m}$	Covariance of the i-th Investment and the Market Portfolio
σ_m^2	Variance of the Market Portfolio
σ	Standard Deviation
SMB	Risk Factor Small-Minus-Big
t or T	Formula Symbol of Time
$\boldsymbol{W^c}$	Recurrent Weight Matrix in LSTM Cell
$\boldsymbol{W^f}$	Recurrent Weight Matrix in LSTM Cell
$\boldsymbol{W^o}$	Recurrent Weight Matrix in LSTM Cell
$\boldsymbol{W^u}$	Recurrent Weight Matrix in LSTM Cell
WML	Risk Factor Winners-Minus-Losers

1 INTRODUCTION

"Now, the question is no longer, as it was a few decades ago, whether investor sentiment affects stock prices, but rather how to measure investor sentiment and quantify its effects" (Baker and Wurgler 2007, p. 130).

1.1 PROBLEM DEFINITION

Research in the field of capital markets frequently concentrates on the investment decisions of individual investors, who, following their personal risk tolerance, establish a proportion between high-risk assets and a risk-free investment r_f for the allocation of their portfolio.

Additionally, a considerable number of research models in the field of capital markets posit that security returns are normally distributed and that investors, as a whole, exhibit rational behavior. According to the proposed framework, the determination of pre-investment decisions is solely dependent on the projected return $E[r_i]$, the potential risk associated with investment chances,[1] and the investor's risk-aversion level.

Reflections on this concept culminate in the Capital Asset Pricing Model (CAPM).

Empirical research for several decades has critiqued this idealized framework, providing evidence to the contrary. For instance, Fischer Black et al. (1972) noted that there is a discrepancy between the returns earned and the level of risk assumed, even in diversified portfolios.

[1] According to the μ-σ theory prevailing in the Capital Asset Pricing Model, this is the portfolio variance or standard deviation. Nowadays, not conclusive, Value-at-Risk, and Expected Shortfall, as well as higher-order statistical moments, such as Skewness and Kurtosis, are also applied in portfolio risk management.

This insight underpins the argument that returns from diversified portfolios are not solely determined by a single risk factor and that other factors beyond the market risk "beta factor" β_i of the CAPM must be taken into account.

Subsequently, Ross (1976) developed the Arbitrage Pricing Theory (APT) as a response to the empirical limitations of CAPM. The APT posits that stock market returns can be explained by both stock-specific and macroeconomic risk factors. In contrast to the CAPM, the APT permits the incorporation of several risk factors as potential determinants of security yields.

This opening from the CAPM beta coefficient to other factors laid the foundation for developing factor models with higher empirical explanatory power, leading to empirically recognizing factor-specific risk premiums in global stock markets.

The divergence between classical portfolio theory and empirical observation is due to the concept of a hypothetical market portfolio, which is assumed to be held by all investors. According to the CAPM, only one stock-specific characteristic (beta factor) determines returns on the stock market, representing the standardized, systematic, or non-diversifiable risk of investment i versus the return of the market portfolio r_m.

The observation of deviations from the expected returns predicted by the CAPM (referred to as "return anomalies") has generated significant scholarly discussion, which has led to the understanding that the selection of factors in the APT model can be challenging. This results from the reality that risk factors in the Arbitrage Pricing Theory are frequently specific to the country, not always identifiable, and not constant over time.

It has yet to be determined through research which specific risk factors are the most effective in APT.

Nevertheless, the flexibility of the APT model enables the utilization of investor sentiment risk factors in addition to the traditional factors established by Fama and French (1993) and Carhart (1997). Why this seems to make logical sense with regard to the efficient-market hypothesis (EMH) will be shown in detail throughout this dissertation.

The research field of investor sentiment (sometimes referred to as "market sentiment") ascribed to Behavioral Finance is not new per se.

The fundamental work in this field includes Russell and Thaler (1985) and Lakonishok et al. (1994). They strengthen the assumption that (irrational) investor behavior significantly influences stock market returns. Based on findings of the prospect theory by Kahneman and Tversky (1979), it is assumed that, contrary to the EMH of Fama (1970), investor behavior is based on imprecise heuristics and follows specific patterns.

The idea is as follows: If these specific patterns are systematic and, therefore, may be predictable, investor sentiment could contribute to explaining and predicting stock market returns.

Meanwhile, there is empirical evidence to support this theory. The most recent findings include that investor sentiment can also be captured in (almost) real-time from unstructured data of social networks. In the last decade, the European Commission has financed the development of an "Integrated Financial Market Information System" as part of the FIRST project, in which the Stuttgart Stock Exchange in Germany was one of the participants, which automatically evaluates the investor sentiment picture of unstructured data from social networks and condenses it into an investor sentiment indicator. Other projects followed in various areas, including outside the financial sector.

Since Hwang (2011) shows that investor sentiment is country-specific and, after reviewing relevant journals, no comparative analysis for the German stock market seems to be available so far, an honest empirical reflection of the topic is considered a valuable research contribution to the German stock market and investor sentiment research.

The German stock market is of particular interest in the context of this research. Unlike the US, for instance, for which some empirical evidence on investor sentiment research is already available, Germany is highly dependent on exports and, thus, on global welfare in terms of the structure of its economy.

Specifically, Germany regularly has the world's largest trade surplus after China (FY 2020), and German export-oriented companies are most likely susceptible to international developments and investor sentiment, a trend that could become even more important in an increasingly globalized world.

The present thesis aims to illuminate the abovementioned issue by comprehensively comparing investor sentiment indicators derived from various sub-areas of previous research on investor sentiment. This examination endeavors to empirically assess the potential explanatory power of these indicators in relation to returns in the German stock market, as well as essential risk metrics commonly employed in portfolio management, such as skewness and kurtosis.

The present thesis results are highly contemporary and relevant and provide valuable contributions to modern portfolio and risk management (Hövel and Gehrke 2022a, 2022b).

1.2 SCOPE AND STRUCTURE OF RESEARCH

This thesis analyzes investor sentiment and lower- and higher-order statistics of return distributions on the German stock market cross-sectionally with a rather traditional approach and longitudinally with a state-of-the-art methodology. The studies aim to determine the appropriateness of incorporating a sentiment risk factor, based on principal component analysis (PCA), into both conventional asset pricing theory models and current machine learning techniques utilizing recurrent neural networks (RNNs).

In the latter, out-of-sample tests are also conducted to explore the predictability of trends based on investor sentiment. Furthermore, the objective is to evaluate the extent to which investor sentiment is generated in the social media sphere, where the involved actors come from, and which tweet characteristics are of particular relevance.

In order to structure the research into a readable format, this dissertation is divided into nine chapters, which is also beneficial in structurally responding to the research hypotheses.

In the first chapter, the problem definition and an elaboration of the present dissertation's objectivities and structure take place.

In addition, Chapter 1 presents the research objectives, hypotheses, and their implied relevance to this dissertation and the academic community.

In general, Chapter 2 first considers the stock market itself and shows why it is of great interest as a research object when it comes to the hypotheses of this dissertation.

It is to be understood as a fundamental introduction to the research object "stock market" and why this is particularly relevant in the social sciences, although numbers dominate it. It clarifies which purpose the stock market fulfills and which investment strategies can be pursued in principle. Depending on the assumption about the efficiency of the stock market, the question arises whether to invest actively or passively or to pursue specific strategies to monetize the stock market's risk most efficiently from the investor's perspective. In addition, a brief digression is made on risk and return and why a differentiated view of risk is particularly important for this thesis.

From the general to the specific, the German stock market is considered, and its particularities are briefly outlined. The historical developments of factor models are also described subsequently. These can provide information on various risk factors in structure-testing cross-sectional studies, which generate explanatory contributions for returns.

In the third chapter, a distinguished categorization of different investor sentiment sources is carried out based on various current and historical empirical research results. Several contemporary concepts of investor sentiment by renowned researchers are introduced to gain a deeper understanding of the methodology applied in the later stages of the dissertation. The chapter summarizes the history and existing concepts of investor sentiment research, highlighting its importance to academic research and its contributions to risk- and portfolio management. A specific categorization for investor sentiment indicators is presented, and the respective essential literature in the context of this dissertation are visualized in tables in an overview.

The fourth chapter of this thesis is the first of three empirical study sections. After an introduction and outline of the development of research hypotheses, the classical or traditional cross-sectional approach is described to test risk factors structurally with regard to their ability to generate explanatory contributions for returns and to what extent. The chapter concludes with regression diagnostics and robustness tests before interim conclusions are presented.

The fifth chapter refers to the strengths and weaknesses of the standard traditional approach with respect to investigating explanatory contributions from the traditional investigation of potential risks in the German stock market.

In particular, the concept of time-varying risk premia is presented, which is especially relevant for investigating investor sentiment, since cross-sectional multi-factor models cannot account for it. The chapter also highlights the relevance of higher statistical moments, such as skewness and kurtosis, as risk measures in risk- and portfolio management, which may be (partially) explained by investor sentiment.

In addition, a different method is proposed compared to the first part of the empirical study in Chapter 4, which is based on an artificial RNN and Long Short-Term Memory (LSTM) neuron architecture. The theoretical foundations of artificial RNNs and LSTM neurons are presented in a differentiated manner in this context.

The utilized approach shows that good trend results with high model quality can be achieved in the explanation and trend prediction.

After presenting further interim results, in which the previous findings are once again summarized in the light of the research hypotheses, the transition to the next chapter finally follows.

Chapter 6 represents the third and final stage of the empirical analysis. In the empirical explorative analysis, both applied methods, such as natural language processing and text mining, are presented, and the study's essential findings are examined in more detail. This final empirical analysis section illustrates the importance of the highly relevant and contemporary social investor sentiment using an extensive collection of tweets on the German stock market explicitly collected for this thesis.

Chapter 7 summarizes the results of the empirical studies, leading to a final conclusion about the research hypotheses. Thereby, the empirical study results are thoroughly reexamined, regarding every research hypothesis defined in the first chapter.

In Chapter 8, the relevant empirical findings of the dissertation are taken up and discussed in the context of larger theoretical perspectives. Also, the findings obtained in this work are placed in the context of previous empirical research findings.

Finally, the last chapter presents an outlook and more interesting open questions that emerged during the research and represent room for further empirical analysis.

The limitations of the empirical research results are also explicitly addressed in this section. In the following Figure 1, an overview of the structure of the dissertation is presented for better clarification. The arrows indicate how the sections intertwine to investigate the subject of the dissertation.

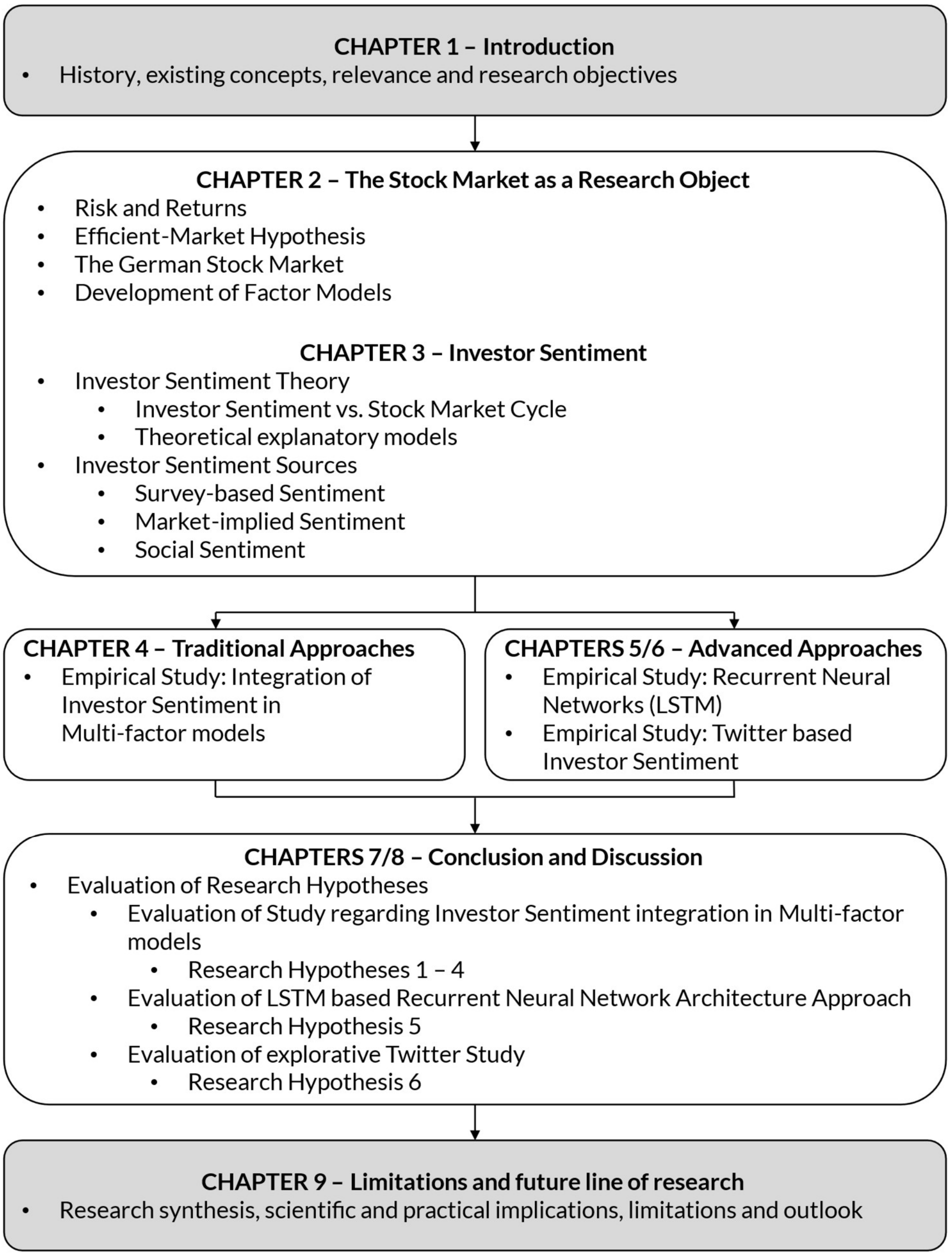

Figure 1. Thesis Structure.

[Source: Author's representation]

1.3 OBJECTIVES

This dissertation makes a valuable contribution to the rapidly expanding corpus of literature on investor sentiment by focusing on the analysis of return distributions. Specifically, it offers a comprehensive analysis of trends in the German stock market through the examination of three sub-studies.

In order to determine the research hypotheses to be investigated in this thesis, the following steps are considered necessary in the coherent structure of this dissertation:

Initially, the extant body of literature in the field of research on investor sentiment illustrates that sources of investor sentiment can be categorized into three distinct groups: survey-based sentiment, market-implied sentiment, and social sentiment. The existing literature also shows a variety of approaches to examine the explanatory and predictive power of investor sentiment for stock market returns. Most of the research seems to be based on US market data. This dissertation summarizes the sparse research on the German stock market to date and visualizes the essential literature work in the context of this dissertation for each identified investor sentiment category.

In this context, it is also emphasized that both lower- and higher-order statistics of return distributions are of heightened importance in portfolio risk management. Therefore, considering a possible link with investor sentiment seems meaningful, following initial approaches to explaining risk through investor sentiment in the US market.

Second, research results from other countries cannot per se be generalized to countries like Germany. This is partly because the structure of individual countries and economies may vary; therefore, a different degree of market sensitivity to changes in investor sentiment can be assumed. The German economy is strongly export-oriented and appears particularly interesting in light of investor sentiment research, as an exceptional sensitivity to investor sentiment may be assumed.

Hence, in the first empirical part of this dissertation, an investor sentiment risk factor determined by PCA is integrated into models that take into account multiple factors, such as those proposed by Fama and French (1993) and Carhart (1997). For the factor construction, 73 hand-curated investor sentiment indicators are considered.

The aim is to examine whether and to what extent established risk factors based on a contemporary sample provide explanatory contributions to the German stock market in a cross-sectional study.

Compared with older empirical studies, this analysis reveals changes in the explanatory contributions of risk factors over time. Furthermore, it is examined whether integrating the investor sentiment risk factor into the already established models incorporating multiple factors as proposed by Fama and French (1993) and Carhart (1997) provides further advantages in empirical model quality.

Third, there is a glaring methodological weakness in the traditional multi-factor model-based approach when measuring the explanatory contributions of risk factors over time. The challenge is that the existing analyses are often cross-sectional and usually cover a relatively large period of time. Risk factors, however, tend not to make constant explanatory contributions over time. Instead, the explanatory contributions vary over time and may add a zero-sum in the cross-section. Accounting for this problem of so-called time-varying risk premia in terms of a rolling analysis is extremely difficult and hardly of any practical relevance.

Conversely, the rolling window has to be defined ex-ante, but the window width is also not stable over time. On the other hand, the influencing factors to be investigated would also have to be rolled over in shorter or longer time windows, although there is no established procedure. In the case of the PCA-based risk factor, one would have to redo the factor construction for each rolling window.

Briefly summarized, Investor sentiment is not time stable, and the explanatory contribution consequently fluctuates over time (time-varying risk premia). In the cross-section, positive and negative premia might cancel each other out so that the actual relationship between investor sentiment and market development might not be visible to the full extent in the cross-section.

Therefore, a different methodology is applied in the second part of the empirical analysis. Based on a deep artificial RNN containing LSTM neurons, time-varying risk premia are considered. If patterns exist, for instance, that certain investor sentiment indicators become dominant at specific points in time, this can likely be represented by the applied methodology.

However, unlike in the area of PCA-based risk factor construction from the cross-sectional study, no dimension reduction may occur.

The obvious disadvantage is the increased computational effort to train the model weights. In addition to examining any explanatory contributions, tests are also performed for predictive power in out-of-sample tests, with the finding of surprisingly good predictive power in trend projection. More precisely, using the market turbulence triggered by COVID-19 in the German stock market in 2020 as an example, this innovative model is tested empirically for predictive power in an extensive out-of-sample test. The investigation also includes higher-order statistics of the return distribution in the German stock market, such as skewness and kurtosis, as these also make portfolio risks measurable in addition to variance, especially if tail risks are to be taken into account in the return distribution, which corresponds to a more contemporary approach to portfolio risk management.

Fourth, in conclusion to the empirical investigations of this dissertation, the most recent investor sentiment category, social sentiment, is observed. The first two empirical studies merely focused on investor sentiment indicators in the first two categories: market-implied investor sentiment and survey-based investor sentiment. Unlike the analysis focusing on more extended time periods, such as month-based time series, social sentiment analysis is suitable for shorter periods up to the intraday range.

If one assumes that the stock market is becoming increasingly efficient due to ever-better global networking, then the third and last category of investor sentiment (social investor sentiment) could play an increasingly important role. This is because, in an incrementally more efficient overall market, so-called anomalies (i.e., market inefficiencies) will decline or, at least, level out more quickly than in the past, as was the case with well-known calendar anomalies and arbitrage limits.

In addition, the third part of the empirical study addresses whether investor sentiment is upstream if it is based mainly on changes in market psychology and expectations rather than actual fundamentals, as Jong et al. (2017) have empirically tested with Dow Jones returns. In their Twitter investor sentiment-based study, 40% of the stock returns studied influenced tweets, but 73% of the tweets studied influenced daily stock returns.

Exclusively for this dissertation, almost two million tweets from the news service Twitter, including metadata on the German stock market, were compiled.

The explorative analysis of the data shows which actors and topics are currently relevant for social investor sentiment.

Finally, because there is essentially no unified scientific consensus in the literature on investor sentiment and especially on the German stock market, about the explanatory and predictive power, as well as the determinants of investor sentiment, this dissertation aims to answer the following research hypotheses in the context of the three empirical analyses:

$\boldsymbol{RH_1}$: *Investor sentiment contributes to explaining return variances in the German stock market.*

$\boldsymbol{RH_2}$: *Investor sentiment is a contra-indicator for stock market developments.*

$\boldsymbol{RH_3}$: *The integration of an investor sentiment risk factor into multi-factor models leads to a higher model quality compared to the Fama-French and Carhart target portfolio regression models, expressed by the adjusted coefficient of determination* $\bar{R}^2$.

$\boldsymbol{RH_4}$: *Incorporating an investor sentiment risk factor into multi-factor models leads to a lower alpha range in the Fama-French and Carhart target portfolio regression models.*

$\boldsymbol{RH_5}$: *By taking the time-varying characteristics of investor sentiment into account, the explanatory power of investor sentiment increases perceptibly compared to traditional cross-sectional analyses.*

$\boldsymbol{RH_6}$: *Investor sentiment-based events in social networks have an impact on market developments in the German stock market.*

2 THE STOCK MARKET AS A RESEARCH OBJECT

"History has shown that the price of shares and other assets is an important part of the dynamics of economic activity, and can influence or be an indicator of social mood. An economy where the stock market is on the rise is considered to be an up and coming economy. In fact, the stock market is often considered the primary indicator of a country's economic strength and development" (Singh 2011, p. 242).

2.1 RISK, RETURNS, AND INVESTMENT STRATEGIES

The stock market is a part of the capital market, and the capital market is a social system (Kommer 2018) and is, therefore, attractive as an object of study for phenomena that fundamentally fall within the scope of social sciences research.

As far as investment strategies in the capital market are concerned, the various approaches can be summarized as follows:

The art or challenge of the investor in the capital market, especially in the stock market, has always been to beat the market or a benchmark derived from it. If one does not want to generate structural and persisting excess returns (Jensen's alpha),[2] it is convenient to buy the market portfolio or an approximation of it and, depending on the degree of risk aversion, mix in so-called risk-free investments, usually short-term government bonds of high rating.

Conversely, the rarely questioned basis of trading is often no longer, to a large extent, trivial when it comes to active investing. Especially when it comes to outperforming the market, the first fundamental question is whether this challenge facing the typical investor can be solved systematically.

[2] Structural or autonomous excess returns are suspected in the alpha constant (Jensen 1968).

Meanwhile, academic research has always been concerned with explaining returns and, at best, with the development of forecasting models, coming to different conclusions that often have to do with the efficiency of markets.

At this juncture, it must be recalled that it is often easier to falsify a hypothesis than to confirm it scientifically. According to Karl Popper, a statement is falsifiable precisely when there is an observational proposition with which the statement can be challenged, which disproves it if it is true (Miller 2007). Since, at any point in time, there are likely to be investors who beat the market as a benchmark in terms of active investing with a more favorable Sharpe ratio, the hypothesis that active management is fundamentally inferior to passive investing can be considered falsified.

Unfortunately, two other questions arise at this point, the hypotheses of which cannot be falsified without difficulty. Indeed, it becomes interesting when the question is whether investors can sustainably beat a benchmark over the long term. This is followed by how exactly the long-term should be defined. To complicate matters, if a window of, for example, ten years is defined, during which an investor succeeds in beating the market in terms of its risk-return profile after costs and, thus, outperforms the market, the question follows whether this performance is due to the genius of the investor or merely coincidence. The empirical studies quickly showed that only a few investors in shares and equity funds could beat their benchmark index for more than three years. This is particularly true when risk, costs, and taxes are considered.

The most critical insights from a financial market research perspective on these complex issues are presented in the following. In general, it can be assumed that underperformance of an active investor increases with the length of the period under consideration (Odean 1998; Cici and Gibson 2012; Meyer et al. 2012).

Among the first important considerations in this context are those of Jensen (1968), who, in his late 1960s publication on the performance of 115 (actively managed) mutual funds in the period from 1945 to 1964, stated:

> "The evidence on mutual fund performance […] indicates not only that these 115 mutual funds were on average not able to predict security prices well enough to outperform a buy-the-market-and-hold policy, but also that there is very little evidence that any individual fund was able to do significantly better than that which we expected from mere random chance. It is also important to note that these conclusions hold even when we measure the fund returns gross of management expenses (that is assume their bookkeeping, research, and other expenses except brokerage commissions were obtained free). Thus on average the funds apparently were not quite successful enough in their trading activities to recoup even their brokerage expenses. […] The evidence does indicate, however, a pressing need on the part of the funds themselves to evaluate much more closely both the costs and the benefits of their research and trading activities in order to provide investors with maximum possible returns for the level of risk undertaken" (Jensen 1968, p. 415).

Around 30 years later, in 1997, financial economist Mark Carhart presented his findings on 1,892 actively managed equity funds over a 35-year period from 1962 to 1993 and came to similarly sobering conclusions. At that time, this was the largest and most complete database of actively managed mutual funds with no survivorship bias. As a result, he found that most funds failed to beat their benchmarks. He comes to the following conclusion in his empirical study:

> "The results do not support the existence of skilled or informed mutual fund portfolio managers" (Carhart 1997, p. 57).

Malkiel (1995) achieved similar results. Eugene Fama, who was later awarded the Nobel Prize, also found in another study with Kenneth French that only 3% of US equity mutual fund managers outperformed their benchmark between 1984 and 2006. To make matters worse, the authors note that luck cannot be ruled out as a cause of this outperformance (Fama and French 2010).

Recent studies agree with the tenor and even suggest that the underperformance of actively managed funds declined until the recent past and, therefore, no improvement is in sight (Dyakov et al. 2017).

Therefore, the fundamental question remains, even against the backdrop of ever-better availability of information and increasing market efficiency: Why do investors make operational decisions that are almost certainly worse (in the long run) than the development of the market as a whole? If a further distinction is made between retail and institutional investors, Kommer (2018) has found that the returns achieved by retail investors are far below those published by mutual funds.

One reason for this is the procyclical behavior of investors (performance chasing). The typical investor only enters certain funds or asset classes when they achieve above-average returns over several periods.[3] As a result, investors miss out on a large part of the returns shown in the fund prospectus. However, this is not intended to be a plea for a particular investment philosophy. Instead, it is a matter of scientifically examining how returns are generated. A standard theory is that returns always compensate for a particular type of risk (Kommer 2018).

However, this does not mean that every risk taken by the investor is compensated.[4] To shed light on this, it is necessary to identify the risks that generate returns. Some risks are already known and have been empirically confirmed in various studies and countries. Other return-generating risks are still unknown or cannot be systematically assumed to cause returns, as these risks are often not constant over time.[5]

In this dissertation, the investigation focuses on a particular risk: investor sentiment. The various statistical moments, for instance, that are examined in the second part of the empirical study in this dissertation (Chapter 5) are well-known risk metrics in portfolio management to which a relationship is presumed.

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[3] This observed behavior can already be taken as an indicator of an investor sentiment cycle, which is discussed in more detail in Chapter 3.1.

[4] For more information, see "idiosyncratic/unsystematic risk" in Chapter 2.2.

[5] For more information, see “time-varying risk premia” in Chapter 5.2.

2.2 EFFICIENT-MARKET HYPOTHESIS

To comprehend why investigating return-generating risk factors is crucial in exploring a possible link between investor sentiment and stock market returns, it is necessary to digress and look at the EMH. The EMH is a popular but controversial hypothesis with strengths and weaknesses, as well as some empirical evidence and some evidence that has been rejected. Even among financial scientists, there is no consensus on whether the EMH holds (Sewell 2012). In a broad chronological meta-study that begins in the 16th century, Sewell (2011) examines the state of empirical research on the EMH to that point and finds that only about half of the papers examined conclude that the EMH holds. The ambivalence is also evident in his summary, in which the issue is whether the EMH is valid.

Sewell concludes:

> "…Economics is a social science, and a hypothesis that is asymptotically true puts the EMH in contention for one of the strongest hypothesis in the whole of the social sciences. Strictly speaking the EMH is false, but in spirit is profoundly true. Besides, science concerns seeking the best hypothesis, and until a flawed hypothesis is replaced by a better hypothesis, criticism is of limited value" (Sewell 2011, p. 7).

The mixed evidence to date on the EMH suggests that it is probably, at least, not permanently valid, which is an essential prerequisite for analyzing the possible link between investor sentiment and stock market returns. However, since this dissertation mainly deals with investor sentiment, only the most critical positions are highlighted in this section.

Fama (1970) established a theory that the prices of assets traded in the capital market reflect all available information (see Figure 2). He assumes a so-called semi-strong information efficiency. Without new information, Fama argues, asset prices follow a random walk. Only when new information becomes available may asset prices change in a non-random direction. Consequently, no market participant can beat the market in the long run except by using non-public information or luck.

Fama's work was also based on, among others, fundamental work by Regnault (1863), Bachelier (1900), and Samuelson (1965) and provides a valuable basis for research on market anomalies (typically, deviations from the CAPM), representing a theoretical foundation of modern portfolio theory (Elton et al. 2007).

Fama also makes further assumptions for the fully efficient market that are not very realistic, such as no transaction costs, full availability of all information to everyone free of charge, and homogeneous expectations about the development of each security in the market among market participants.

A market that meets all these requirements is described as a frictionless market. Fama goes on to declare that a market can be efficient even if the strict requirements of the frictionless market are not fully met, as long as a sufficient number of market participants have access to, for instance, all publicly available information. Fama implicitly assumes that this sufficient number of market participants acts rationally.

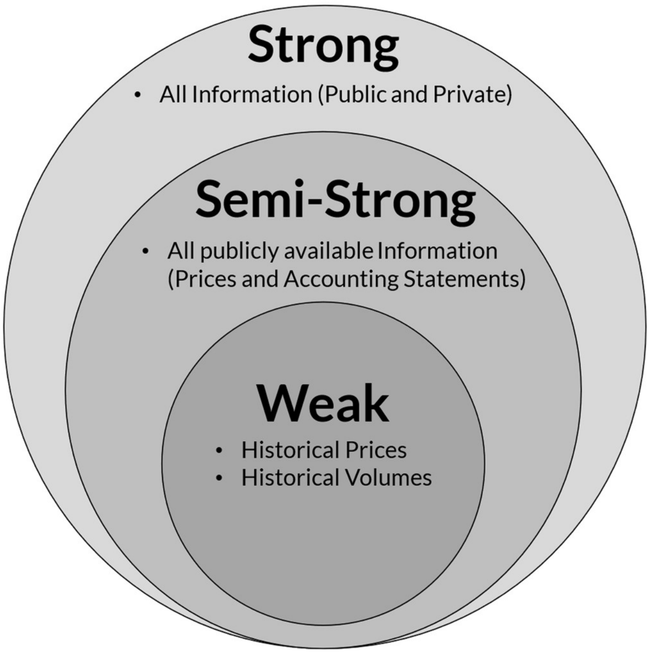

Figure 2. Degrees of Informational Efficiency.

[Source: Author's representation based on (Fama 1970)]

Empirical tests of the EMH have been conducted quasi-ever since its tremendous popularity was associated with publication (Fama 1970) and led to mixed results, as described above. Robert J. Shiller, Paul Krugman, Daniel Kahneman, Amos Tversky, and Richard Thaler advocate other explanations of price determination and information processing in markets.

However, after the 1970s, the efficient markets theory (EMT) was the prevailing opinion. Belief in the theory was shaken by a series of discoveries of anomalies, mainly in the 1980s, and evidence of excessive volatility in returns. In his contributions of 1989 and 2003, Malkiel summarizes the essential features of the EMT, which are still valid today (Malkiel 1989, 2003).

In his more recent work, he confirms the assumptions of EMT with minor limitations based on his empirical investigations:

> "As long as stock markets exist, the collective judgment of investors will sometimes make mistakes. Undoubtedly, some market participants are demonstrably less than rational. As a result, pricing irregularities and even predictable patterns in stock returns can appear over time and even persist for short periods. Moreover, the market cannot be perfectly efficient, or there would be no incentive for professionals to uncover the information that gets so quickly reflected in market prices, a point stressed by Grossman and Stiglitz (1980)" (Malkiel 2003, p. 80).

Eventually, empirical contributions in behavioral finance occurred in the 1990s (Shiller 2003). For the German stock market, Fahling et al. (2019) investigated a sample of 194 (actively managed) mutual funds and concluded that:

> "…Active funds can and do create value in terms of abnormal returns, but these are mostly offset by expenses. Regression results prevent a rejection of the null hypothesis, indicating that active funds in general do not create significant value in form of alpha" (Fahling et al. 2019, p. 73).

Currently, due to the availability of higher computing power, extensive empirical testing of the often more complicated models from the field of behavioral finance can take place, which, related to stock market pricing, often culminates in the field of investor sentiment research, which is further described in Chapter 3.

2.3 THE GERMAN STOCK MARKET

In contrast to other work in the investor sentiment research space, which mainly focuses on the US market, this thesis looks at the German stock market. This is because empirical research results on investor sentiment from other markets are not necessarily transferable to the German stock market. For instance, Corredor et al. (2013) examine the German stock market and other European countries regarding the relevance of investor sentiment in influencing stock market returns. It has been established through previous research that stock characteristics alone are not the sole determinant of the effect of investor sentiment and that country-specific factors also play a significant role in the influence of investor sentiment on the market.

Another reason for this investigation's relevance is that the German stock market structure makes it particularly interesting for investor sentiment research. The German economy is characterized by a high degree of foreign trade intensity. This implies that global economic processes strongly influence the German economy and, thus, the stock market, which hosts many companies that participate directly or indirectly in the export business. The German economy, for its part, co-determines international economic processes. The exchange of production goods dominates the commodity structure of German foreign trade. Germany's most important trading partners are the US, China, France, and the Netherlands (Forner 2022).

Due to the high degree of globalization-related integration of German capital market companies and the intense focus on exports, the German economy is susceptible to international developments and investor sentiment. Thus, the German stock market is particularly worthy of investigation, as investor sentiment could have a unique link to stock market yields compared to other markets.

The Composite DAX (CDAX) stock market index, which is the subject of investigation in the empirical studies of this dissertation, was introduced by Deutsche Börse AG on September 17, 1993, as a complementary index to the renowned DAX German stock index. While the DAX, with only 40 (until 2021: 30) blue chips, comprises a small but substantial number of stocks, the CDAX contains all German stocks listed on the Frankfurt Stock Exchange in the General Standard and Prime Standard. Foreign shares are not included in the CDAX.

Focusing on the CDAX rather than the DAX, which only contains blue chips, is vital for this research because previous results show that shares of smaller companies are especially sensitive to investor sentiment (Kumar and Lee 2006; Barber and Odean 2008).

It is worth highlighting that there has been a decrease in the quantity of stocks listed on the CDAX in recent years. The German stock market has undergone a shift, with more companies opting for voluntary delistings and moving to the unofficial regulated market (open market). This can provide certain advantages, such as the elimination of certain reporting requirements, for these companies. As a result of this trend and other factors, there has been a noticeable level of consolidation over time, which can be traced in Table 1.

Table 1. Mean Quantification of Securities in the Monthly CDAX Data Set.

Year	**2001**	**2002**	**2003**	**2004**	**2005**
Shares	787	771	727	703	679
Year	**2006**	**2007**	**2008**	**2009**	**2010**
Shares	672	682	676	645	611
Year	**2011**	**2012**	**2013**	**2014**	**2015**
Shares	582	554	509	481	441
Year	**2016**	**2017**	**2018**	**2019**	**2020**
Shares	424	420	422	423	411

Note. This table illustrates the data for the years 2001 to 2020, displaying the average number of shares that were included in the CDAX for each year within that time frame.

[Source: Author's representation]

2.4 DEVELOPMENT OF MULTI-FACTOR MODELS

The CAPM is often the starting point for many empirical studies on capital market theory. Since tests of market efficiency are often tests of the classical CAPM extended by additional risk factors, the development and assumptions of this model are relatively extensively presented in this subsection.

The following explanations serve to better comprehend, in particular, the first part of the empirical study in this dissertation (see Chapter 4).

The CAPM is a reference equilibrium model with restrictive assumptions of modern finance theory and describes fundamental relationships between (expected) return and risk of a security or a portfolio of securities.

The model is based on the work on portfolio theory by Markowitz (1952), who postulates a linear relationship between return and risk. The first notable mention of this theory is in a paper by Tobin (1958), in which he extends Markowitz's model to include the possibility of a risk-free investment r_f as part of his separation theorem.

Various further developments followed independently in the 1960s (Ziemer 2018). Of note in the context of the work on the CAPM is the contribution of Sharpe (1964), who defines the risk of an investment exclusively as its standard deviation. In the context of work on the CAPM, Markowitz and Sharpe were awarded the Nobel Memorial Prize in Economic Sciences in 1990.

The work of Lintner (1965) on the valuation of risky financial products and the Treynor ratio (Treynor 1965) also formed essential foundations of the CAPM, which was finally completed by Mossin (1966) with his assumptions on market equilibrium (Equation 1).

$$E[r_i] - r_f = \beta_{CAPM,i} \cdot \left(E[R_m] - r_f\right) \tag{1}$$

In the CAPM, $E[r_i]$ stands for the expected return of a security i, while $E[r_m]$ represents the expected return of the market portfolio.[6]

[6] While theoretical derivations of the CAPM refer to securities, which in principle also include bonds, empirical studies usually refer exclusively to the stock market or stock market portfolios.

The CAPM "beta factor" (Equation 2) of a security i versus an efficient market portfolio m is defined as the quotient of the covariance of the expected return of the security i with the expected return of the market portfolio m to the variance of the market portfolio m.

$$\beta_{CAPM,i} = \frac{\sigma_{i,m}}{\sigma_m^2} \tag{2}$$

Thus, the beta factor assumes a linear and positive relationship between the expected return of a risky investment i and the expected return of the market portfolio m. Since the market portfolio concept is idealized, it is substituted in practice by a diversified benchmark portfolio, typically a market-wide index. The beta coefficient is then determined using the ordinary least squares (OLS) regression from historical data.

The theoretical expected return model CAPM empirically represents a one-factor model. The basis for the consideration is the market model of Sharpe (1963) (see Equation 3) with $i = 1, \dots, I;\ t = 1, \dots, T$.

$$r_{i,t} = \alpha_{CAPM,i} + \beta_{CAPM,i} \cdot r_{m,t} + \varepsilon_{i,t} \tag{3}$$

Following Sharpe (1963), the market model is combined with the CAPM and results in the empirical single-factor model (Equation 4).

$$r_{i,t} - r_{f,t} = \alpha_{CAPM,i} + \beta_{CAPM,i} \cdot \left(r_{m,t} - r_{f,t}\right) + \varepsilon_{i,t} \tag{4}$$

The alpha factor indicates the systematic difference in returns compared with the benchmark portfolio (in empirical research: Jensen's alpha)[7] or the market portfolio in an idealized world. At this point, the empirical evidence contradicts the CAPM theory, which denies the existence of a systematic differential return.

The search for positive alpha, a structural excess return over the approximated market portfolio, is, thus, a concept that assumes that market risk cannot be the only determinant of return, even in diversified security portfolios.

The difference $r_{i,t} - r_{f,t}$ represents the return premium of the considered portfolio in the period t. $\varepsilon_{i,t}$ is an unobservable (error) term that represents the securities-specific risk. This corporation-specific, so-called idiosyncratic or unsystematic risk can be considered to be eliminated through sufficient diversification.

The beta coefficient of the CAPM only represents the systematic (market) risk since it is assumed that a market-wide (index) portfolio is sufficiently diversified to eliminate unsystematic risk. According to Sharpe (1964), security returns are influenced by the issuer (unsystematic risk or idiosyncratic risk), as well as by the development of the overall market (systematic risk or market risk).

Systematic risks are inherent in the market and cannot be eliminated, even by diversification.[8]

Conversely, according to theory, unsystematic risks are issuer-specific and can be eliminated by sufficiently large diversification(Fischer 2014).

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[7] The alpha of Jensen (1968) is employed to ascertain the abnormal or excess return of a security or portfolio of securities when compared to the predicted expected return. It is calculated by utilizing a theoretical performance benchmark and not a market index.

[8] Since the theoretical unsystematic risk is negligible in a sufficiently diversified portfolio, only the systematic risk (beta coefficient) is commonly referred to as portfolio risk in the CAPM.

Consequently, total portfolio risk (see Figure 3) is measured as the standard deviation of the portfolio returns and consists of systematic risk (non-diversifiable) and unsystematic risk (diversifiable).

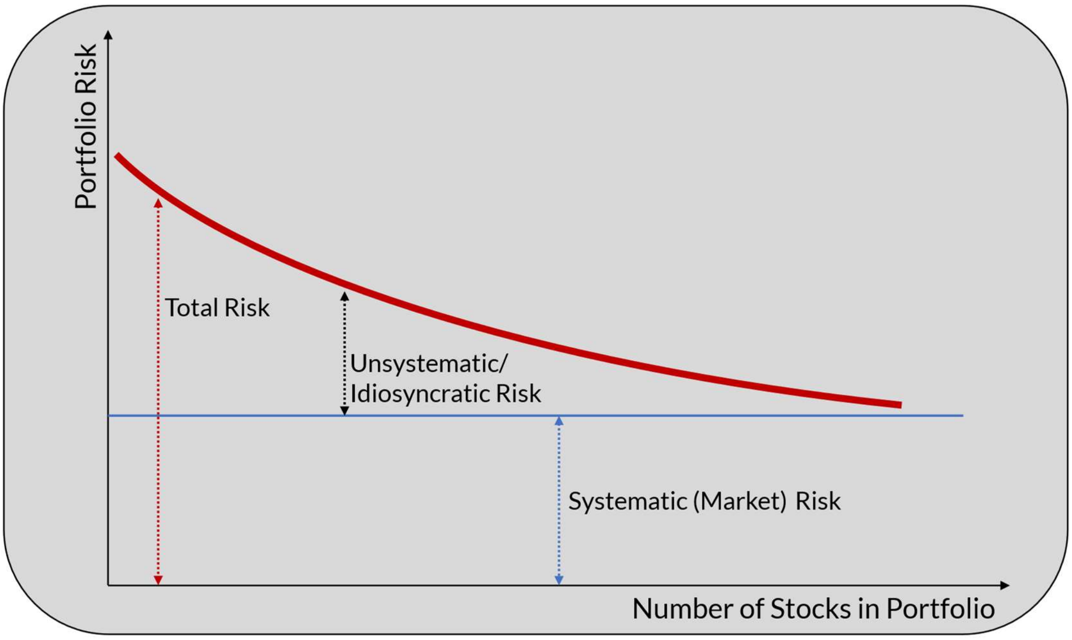

Figure 3. Composition of portfolio risk.

[Source: Author's representation based on Sharpe (1964).]

It must be noted at this point, however, that the considerations that after sufficient diversification, only the CAPM beta coefficient remains as a risk factor quasi as a residual are purely theoretical.

As a result, the hypothesis regarding the CAPM beta factor as the only relevant risk factor in diversified portfolios proved counterfactual in empirical studies in the early 1970s. Merton (1973) notes that the expected returns of risky securities may differ from those of risk-free investments, even in the absence of systematic risk. Merton's work, therefore, implies that the existence of other factors determining securities returns is likely. Since the late 1970s, the CAPM beta concept and the EMH have been increasingly challenged (see Chapter 2.2). Empirical work shows other return anomalies that contradict the EMH. The observation of arbitrage limits further refutes the market equilibrium assumption.

In order to address the empirical shortcomings of the CAPM, Ross (1976) developed the APT, which is based on the assumption that n market factors F_1, $F_2, \dots, F_n$ determine stock returns (see Equation 5).

$$r_i = E(r_i) + \beta_{1,i}F_1 + \beta_{2,i}F_2 + \cdots + \beta_{n,i}F_n + \varepsilon_{it} \quad (5)$$

In Equation 5, $\beta_{n,i}$ represents security-specific coefficients that measure the sensitivity of the return to the n-th factor. In contrast to the CAPM, the APT does not require market equilibrium but rather only an arbitrage-free capital market.[9]

These assumptions also may not have a lot in common with empirical reality. However, the assumptions are somewhat less restrictive compared with the CAPM. Moreover, the rather unrealistic assumptions of the CAPM include, among other ideas, that all investors can lend and borrow unlimited amounts of cash at a risk-free interest rate and trade without transaction costs and taxes.

Furthermore, it is assumed that all investors are able to trade in securities divided into arbitrarily small packages. In this idealized concept, all investors also have homogeneous expectations and assume that all information is available to all investors simultaneously (Arnold 2008).

[9] A market equilibrium exists when supply and demand for a good or service (here: security) are in balance, resulting in a stable price for that good or service. An arbitrage-free capital market, on the other hand, refers to a market in which there are no opportunities to make risk-free profits by buying and selling financial instruments (arbitrage). In a market without arbitrage, the prices of financial instruments accurately reflect all available information, and market participants cannot make a risk-free profit by buying and selling the same or similar financial instruments. Although both concepts have to do with market stability and equilibrium, they refer to different aspects of the market and different types of equilibrium.

Consequently, the APT is an arbitrage model based on fewer preconditions than the CAPM, which is an equilibrium model (Ziemer 2018). Ross shows that in an APT model, each factor F_i must be assigned a risk premium λ_i (see Equation 6).

$$E(r_i) = r_f + \beta_{1,i}\,\lambda_1 + \beta_{2,i}\,\lambda_2 + \cdots + \beta_{n,i}\,\lambda_n \tag{6}$$

Studies based on these assumptions by Chen et al. (1986), Cochrane (1992), Holmström and Tirole (2001), and Liu (2006) provide empirical evidence for the general suitability of the APT model. Others, such as Banz (1981), Basu (1983), and Rosenberg et al. (1985), find that company size and ratios, such as price-earnings ratio or book-to-market value ratio, contribute to explaining stock returns.

An interim result of these findings is the three-factor model (see Equation 7) developed by Fama and French (1993, 1996), which empirically demonstrates a higher empirical model quality (= higher model goodness of fit) than the CAPM.

$$E[r_i] - r_f = \beta_{FF,i} \cdot E[RMRF] + s_i \cdot E[SMB] + h_i \cdot E[HML] \tag{7}$$

$RMRF$ (Market Return Minus Risk-Free Return) is an alternative notation for the market risk premium $r_m - r_f$. This indicates that market risk is a risk factor like SMB and HML (which will be explained in the following sentences) and that market risk, as assumed in CAPM, is not the only substantial risk factor that plays a role in generating explanatory contributions for excess returns in the stock market. SMB as a size risk factor (Small Minus Big) represents the portfolio return difference between companies with small and large market capitalization. HML (High Minus Low) represents a value risk factor: The portfolio return difference between companies with high and low book-to-market value ratios. Jegadeesh and Titman (1993) observe the momentum effect, which is taken up by Carhart (1997) to extend the Fama-French three-factor model (see Equation 8).

$$\begin{aligned} E[r_i] - r_f = \beta_{Carhart,i} \cdot E[RMRF] + s_i \cdot E[SMB] + h_i \cdot E[HML] \\ + w_i \cdot E[WML] \end{aligned} \tag{8}$$

The momentum factor WML (Winners Minus Losers) represents the portfolio return difference between companies with good and bad performance in the previous year. According to Carhart (1997) and Fama and French (1993, 1996), the three- and four-factor models based on the CAPM are still among the most renowned multi-factor models. However, they are not carved in stone but are subject to constant evolutionary development and adaptation (Fama and French 2015, 2016, 2017).

3 INVESTOR SENTIMENT

“The intelligent investor is a realist who sells to optimists and buys from pessimists“ (Graham 1949, p. 31).

3.1 INVESTOR SENTIMENT THEORY

Classical capital market theory starts from the ideal-typical state of flawless capital markets, with the corresponding assumptions described in the preceding chapters. A long-term cyclical trend follows a growth path in which the stock market continually appears to anticipate real economic growth (Figure 4).

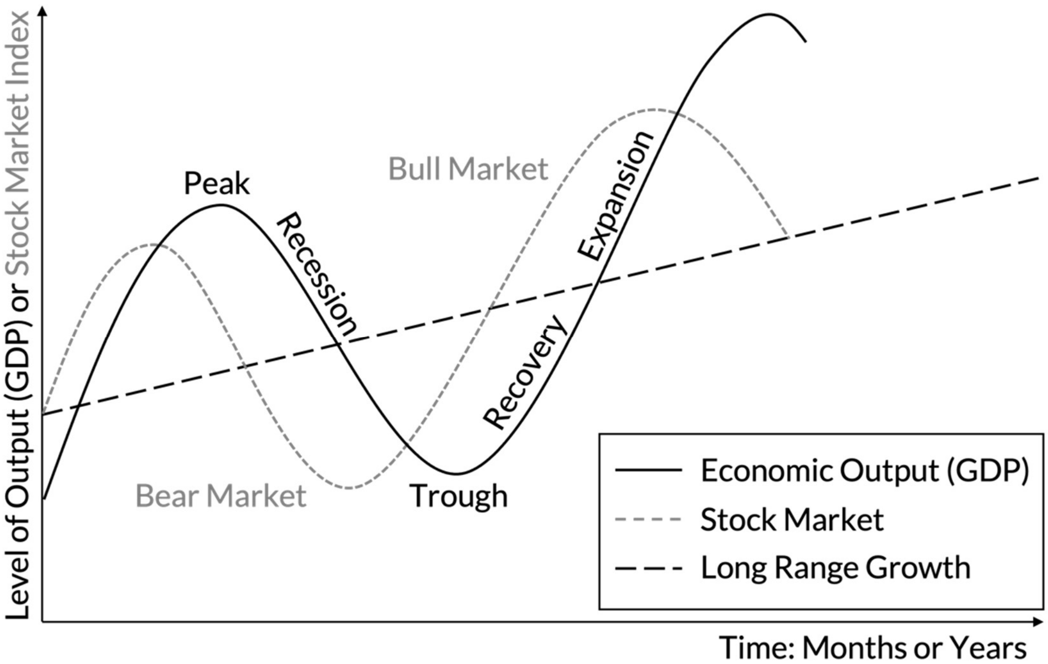

Figure 4: Stock Market Cycle leads Business Cycle.

[Source: Author‘s representation based on (Schmidt 2021)]

The relationship described in Figure 4, if one considers the stock market on its own, can also assume significantly shorter cycles, which nevertheless follow a major overarching cycle.

Also, the cycle shown in Figure 4 is an idealistic representation, which is not always as evident in empirical reality. The relevance of the stock market as a sub-segment of the capital market becomes clear in connection with overall economic development. This embedding of the research topic in larger, overarching concepts also clarifies that areas of economic development that cannot be intuitively linked to the topic under investigation may also be affected in the context of second-round effects.

Since, according to theory, the expectations of market participants are homogeneous, stock returns are analyzed in market equilibrium. An actual or equilibrium fair price is obtained for each stock. Price fluctuations are unpredictable, with random deviations from this value (random walk). The relationship between risk and return of shares is determined only by idiosyncratic or unsystematic and systematic risk.

However, capital market imperfections are due to transaction costs, information asymmetries, boundedly rational behavior, liquidation costs, and non-risk diversified portfolios and arbitrage limits.[10]

As described in more detail in the previous chapters, multi-factor (APT) models are to be understood as further developing the classical capital market theory, which finds its expression in the CAPM.

[10] Arbitrage limits are imposed to mitigate the potential for excessive profits to be derived from price differentials across various markets. These limits are often established by regulatory bodies or exchanges as a means of preventing market manipulation and ensuring market efficiency. The specifics of these limitations can vary by market, and may take the form of restrictions on the quantity of a particular financial instrument that can be transacted within a specified time frame, or limits on the degree of exposure to underlying asset price movements. These limitations are dynamic and subject to change depending on market conditions.

With the integration of investor sentiment-specific risk factors, new approaches increasingly influence the models based on the original CAPM.

In contrast to the previous chapters, the assumed rationality of the investor, in particular, is examined in more detail here. The "Theory of Games and Economic Behavior" developed by Morgenstern and Neumann (1944) is the starting point. This theory, in which rationally acting agents maximize the expected value of their risk utility function, represents the basis of rational action in decisions under risk.

Furthermore, it postulates that if a decision maker's preference for risky alternative actions satisfies the axioms of independence, continuity, and completeness, then a utility function exists, for which expected utility reflects the decision maker's preference.

The expected utility-based approach is also taken up by Kahneman and Tversky (1979) in the context of their prospect theory. The central statement of this theory is that decisions are based on heuristics, which are subject to various cognitive biases.

The utilization of investor sentiment in stock valuation is based on the assumption that heuristics due to biases lead to irrational errors, resulting in misleading investment decision-making, which follows predictable patterns (Kumar and Lee 2006).

In addition to the disposition effect, the over-confidentiality bias plays a unique role. Other well-known phenomena, such as herd behavior (social proof), which is repeatedly observed among investors, certainly play a role as well. There are already numerous publications on topics about cognitive biases. A decent review of the research to date in this area and the individual relevant cognitive biases that appear to play a role can be found in the work of Zindel et al. (2014).

In the present thesis, however, a clear distinction must be made. It is not primarily a matter of investigating which cognitive biases people and, thus, investors display in order to show that the market is not always rational or efficient. This research aims to investigate whether and from which sources these psychological market forces are at work on the German stock market and how to quantify them.

Positive and euphoric investor sentiment is often observed in connection with subsequent negative returns and vice versa. Therefore, investor sentiment is regularly interpreted as a contra-indicator. In fact, every purchase is always matched by a sale. An important signal that a stock bull market is coming to an end is a rapid increase in share turnover, with prices rising at the same time. In such a phase, shares change from "solid" to "shaky hands" (Kostolany 2015).

Despite the fact that the linkages between behavior and economics remain unclear, Baker and Wurgler (2006) were able to demonstrate these reversion patterns through empirical evidence. The perceptible herd behavior plays a crucial role in this process. As shown in Figure 5, an example of a typical investor sentiment cycle (depicted in yellow) is demonstrated in correlation to the stock market cycle (shown in blue).

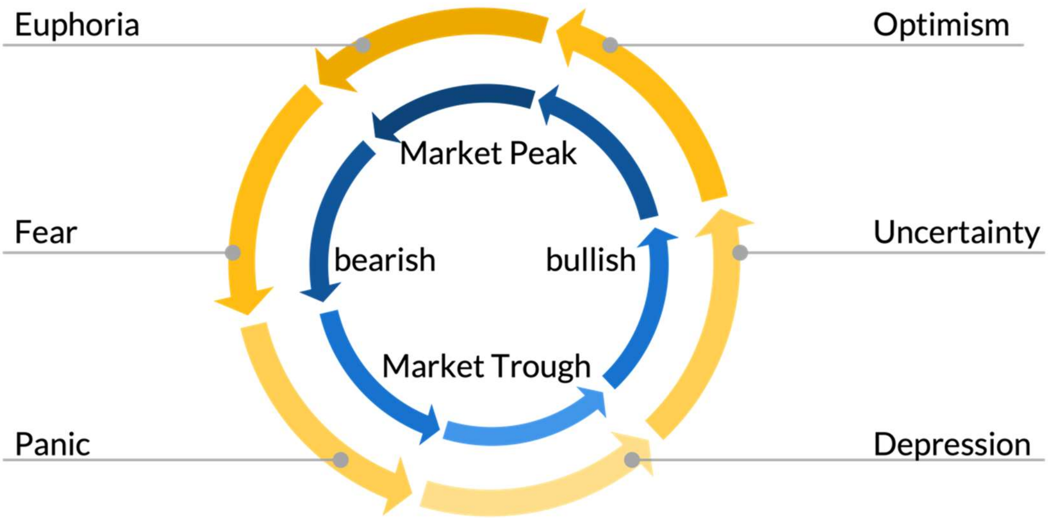

Figure 5. Investor Sentiment Cycle and Stock Market Cycle.

[Source: Hövel and Gehrke (2022a)]

Figure 5 shows the typical picture of the investor, who (expensively) buys full of optimism at the peak of a stock bull market and then irrationally sells in panic when prices are at their lowest (cheapest). The phenomenon of investor behavior, particularly the cognitive and emotional factors that drive the acquisition of assets at a premium and their subsequent disposal at a discount, is essential to investor sentiment research.

The apparent rationality of acquiring assets at a low cost and disposing of them at a high value to generate profits is often at odds with observed investor behaviors.

When defining investor sentiment correspondingly, based on empirical observations, investor sentiment can be considered as the prevailing opinion of investors about the expected price development in a market. This opinion is based on various factors, including economic reports, stock market wisdom, seasonal factors, and national and global events. According to Fama's theory, even assuming semi-strong information efficiency, the vast majority of these factors should already have been considered in price formation. Similar to technical chart analysis, some groups believe in it, and others do not. It is acknowledged that certain market events may be driven by self-fulfilling prophecies, where market participants' expectations and actions can shape the event's outcome.

The demarcation from the research object is clear: This thesis is not primarily concerned with identifying whether investor sentiment works, as this would open Pandora's box, and there is already a great deal of research in the field (see Chapter 3.2).

In essence, although it would be no less attractive, this thesis explicitly does not directly deal with psychological or behavioral anomalies but instead focuses on the capital market. Many studies show empirically that links between investor sentiment and stock market performance are recurrent, at least in the short run. Therefore, the research questions of this thesis explore whether investor sentiment on the German stock market is empirically measurable to explain stock market returns or to be utilized for prediction models.

In the example of technical analysis, it is possible that the underlying psychological component often matures into a self-fulfilling prophecy in the market and is regularly confirmed in empirical evidence (Menkhoff 1997). Alternatively, to use the words of Baker and Wurgler:

> "Now, the question is no longer, as it was a few decades ago, whether investor sentiment affects stock prices, but rather how to measure investor sentiment and quantify its effects" (Baker and Wurgler 2007, p. 130).

If investor sentiment can be measured based on the cyclical structure, it can be assumed that positive investor sentiment will be followed by the actual negative performance (negative returns or higher risk) and vice versa (contra-indicator).

This research hypothesis, together with the research hypothesis that sentiment helps explain stock market returns, is also tested in the context of this dissertation on the German stock market.

$\boldsymbol{RH_1}$: *Investor sentiment contributes to explaining return variances in the German stock market.*

$\boldsymbol{RH_2}$: *Investor sentiment is a contra-indicator for stock market developments.*

The assumption that investor sentiment serves as a contra-indicator of market developments is also based on several other conjectures. On the one hand, it is assumed that already invested market participants are optimistic about price expectations, which is reflected in a positive investor sentiment value.

On the other hand, uninvested investors tend to be pessimistic about price potential, which is reflected in a negative investor sentiment value.

Any other investor behavior, such as investing despite negative expectations or not investing despite positive expectations, would be irrational.[11]

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[11] This does not mean that such behavior does not exist. If investment guidelines stipulate that, for example, a cash holding or holding of government bonds must not exceed a relative fraction of portfolio assets defined ex ante, then the responsible manager must invest, even if he is pessimistic. The reverse is also conceivable, so equity positions may have to be sold or reallocated to other asset classes if the equity ratio in the portfolio is too high or threatens to exceed the limits of the mandator's investment guideline. However, such cases are not perceived in this thesis as influencing the overall market development.

Meanwhile, it can be assumed that in a market in which most market participants are already invested, only a few remain to generate additional demand, leading to further price increases (see Figure 6).

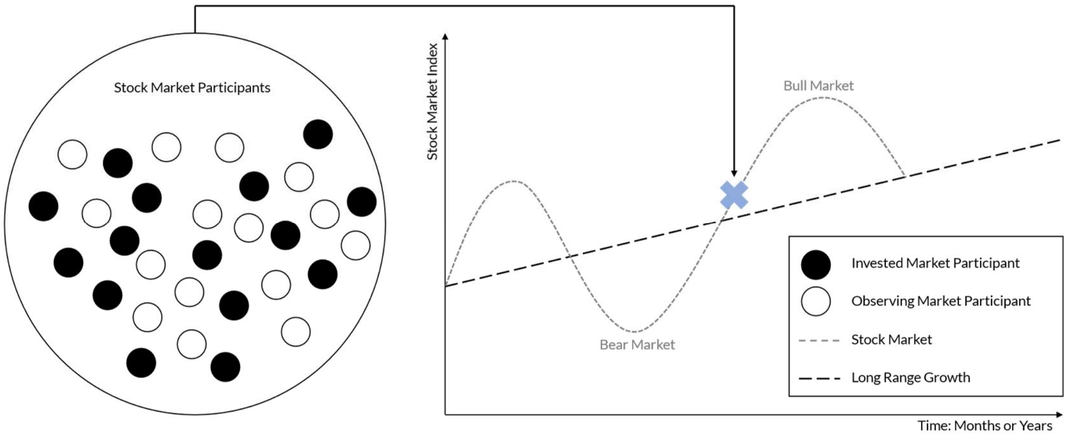

Figure 6. Stock Market Demand Model.

[Source: Author's representation]

Therefore, the additional demand potential is minimal and negatively correlated with positive investor sentiment. This phenomenon causes stock prices to rise even when economic indicators continue to deteriorate, as they did at the beginning of 2019 in Germany, as long as the majority of those market participants willing to sell have already sold their stocks, as was the case at the end of 2018 in the German stock market.

It can be summarized at this point that pricing models based on the CAPM, which consider the market-psychological investor sentiment component, thus represent a synthesis of modern portfolio theory, according to Markowitz (1952) and behavioral finance theory. At this point, the definition of investor sentiment will be discussed in more detail after the theoretical background has been examined.

The study of investor sentiment explores the impacts on the evolution of markets or specific financial instruments. This field aims to gauge the sentiment of those involved in the market. Therefore, investor sentiment can be utilized to make educated assumptions about future price developments.

It also provides better explanations for current price developments, especially when there is no cause for these developments that fundamental data could explain. The knowledge obtained from analyzing investor sentiment can serve as a foundation for both trading decisions made over a short period of time and investments made for the long haul.

According to Baker and Wurgler (2007) investor sentiment refers to the attitudes and expectations surrounding future financial returns and investment hazards that are not accounted for by fundamental information. This means that events such as airplane crashes and even soccer game outcomes can have an impact on investor sentiment (Kaplanski and Levy 2010; Edmans et al. 2007).

Recent research in capital markets on the advancement of multi-factor models suggests that there is a possibility of additional interpretative and improvement potential in unidentified risk factors. Thus, it is vital to examine if factors that cannot be easily measured, such as investor sentiment, may exert an influence on share prices in addition to the commonly recognized factors included in popular multi-factor models.

Russell and Thaler (1985) demonstrate that market participants exhibit certain irrational behaviors. Lakonishok et al. (1994) have noted that by exploiting the irrational behavior of investors, one could generate higher returns for the same level of risk. Tversky and Kahneman (1973) and Kahneman and Tversky (1979) provide a classifying explanation and understanding for this (irrational) decision-making.

The prospect theory proposed by them argues that decisions are based on the expected subjective utility, which implies that perceptions and expectations of potential gains or losses are formed through the use of heuristics. These heuristics are frequently utilized to improve the speed and effectiveness of decision-making in uncertain situations, despite the fact that they give rise to cognitive biases and predictable errors. Crucial to research in the economic context here is the word predictable. The over-confidentiality bias, which presumably plays a significant role in this context, is, for example, stressed and verified through actual observation and experimentation in the study of Daniel et al. (1998).

Besides the well-established tendency for investors to exhibit loss aversion, herd behavior influences investor sentiment significantly. Through his research, De Bondt (1998) discovered that investors tend to adhere to basic patterns in price movements and do not always exhibit rational decision-making.

An example is the aforementioned technical chart analysis, where decisions are made based on chart patterns independent of fundamental data. Among other findings, Barber and Odean (2013) show that individual investors underperform standard benchmarks, sell profitable assets while holding losing ones ("disposition effect"), have their purchase decisions strongly influenced by limited attention and past returns, and tend to hold undiversified stock portfolios.

Bradshaw (2002, 2004) states that also experienced stock analysts rely on heuristics when making decisions. Given the fact that heuristics are not entirely precise, it is reasonable to assume that stocks may be temporarily mispriced, resulting in their market value not always aligning with their intrinsic value.

If heuristics lead to predictable errors, there is a rational reason to consider that these errors can be converted into risk factors and successfully incorporated into multi-factor models. These findings are essential because they suggest that retail and, professional, institutional investors are affected.

Given the complexity and impracticality of analyzing individual investor behavior, research often centers on models that assume certain general behaviors of investors as a group.

Jackson (2003) supports the credibility of this assumption and presents evidence that the aggregate of individual trading decisions follows a discernible pattern.

Baker and Wurgler (2007) demonstrate that investor sentiment plays a significant role in explaining stock market returns by examining aggregate investor behavior. Other notable contributions to this field include Long et al. (1990), who have studied the effect of irrational investors (they refer to them as “noise traders”), and Barberis et al. (1998), who have looked into the psychological underpinnings of investor sentiment.

The results of prior studies concur with the perspective put forth by Fama and French (1993), which posits that the risk factors included in the three-factor model serve as general indicators ("mimicking returns") for a wide range of abnormal stock valuation patterns and risks that the three-factor model does not specifically address.

Since investor sentiment is not intended to represent individual investor behavior but rather general market sentiment, it is considered at the aggregate level in this dissertation.

3.2 INVESTOR SENTIMENT SOURCES

The fundamental challenge in analyzing investor sentiment is to quantify it. Various sources present at least five different categories of investor sentiment. For a structured approach, it is essential to categorize the various investor sentiment indicators, which is not always possible to do clearly and distinctly.

The use of investor surveys to directly measure sentiment is a popular method to be viewed as one category. However, it is often not carried out using proper statistical methodologies such as defining the primary population or randomizing the survey participants. This is discussed in more detail in Chapter 3.2.1.

Another approach is to infer investor sentiment from financial market data that looks ahead. This can be done by using indicators such as implied volatility and put-call ratios of options, which are often used to determine the prospects of investors. These indicators are widely used as indirect measures of investor sentiment. In this dissertation, this type of investor sentiment is referred to as "market-implied sentiment" and is discussed in Section 3.2.2.

Social investor sentiment is currently a topic of significant debate within the field of behavioral finance. Various indicators can be subsumed under this category, such as news or media sentiment, which includes media reports from various sources, not just traditional news outlets. In this concept, not only is the content of the news important, but also the amount of coverage it receives. In this third category, depending on the source, further differentiation is made between text mining and sentiment analysis techniques to extract information about investors' moods from unstructured data in social networks.

Other metadata are occasionally collected or evaluated in addition, for example, the subjectivity or frequency of search queries or postings in the section on social investor sentiment (see Chapter 6).

However, household internet search behavior is also considered a separate category in some cases. Finally, non-economic factors are occasionally considered a separate category. Some sources say that many non-economic events affect investor sentiment on a daily basis, which, in turn, influences risk aversion and trading behavior (Edmans et al. 2007). This dissertation summarizes all these issues as "social sentiment," which is further explained in Section 3.2.3.

Consequently, the investor sentiment to be aggregated in this thesis is divided into three categories, which may sometimes include sources in the social sentiment domain that are listed as separate categories in other studies.

The division chosen in this dissertation does not influence the defined research hypotheses from Chapter 1.3. The following subsections also define which areas of investor sentiment are included in the respective categories in this dissertation and how individual categories are explicitly referred to in the further course of this thesis to resolve specific research questions.

Additionally, it is important to note that all investor sentiment indicators share a common goal of providing a clear distinction between optimism and pessimism in terms of investor sentiment.

Many research studies that examine the relationship between investor sentiment and stock market returns utilize basic statistical methods such as Pearson's correlation coefficient, linear regression, and nonlinear causality tests. Often, the three-factor model proposed by Fama and French (1993, 1996) is selected as the benchmark model, and the four-factor model proposed by Carhart (1997) is applied less frequently. It is noteworthy that the majority of studies have focused on the US market. There is a lack of robust studies on the German stock market, but there are connections to existing research on the German stock market.

As an example, research conducted by Finter et al. (2012) using data from the German market showed that certain stocks are more affected by investor sentiment from survey and market-implied sources. However, Finter et al. (2012) did not find any significant correlation with future stock returns.

Additionally, Krinitz et al. (2017) utilized a Granger causality test to confirm that media-based investor sentiment affects the returns of the German stock market. Furthermore, it is crucial to acknowledge that investor sentiment should be differentiated into different time horizons, including short-term, medium-term, and long-term observations. It is widely acknowledged that investor sentiment exhibits distinct differences between institutional and private investors, despite the fact that they both fall within its purview.

When assessing investor sentiment, it is ideal to consider and account for these distinctions in order to make more accurate evaluations.

Figure 7 shows the typical procedure for moving from the investor sentiment indicator from various sources to a risk factor that can be examined in, for example, a multi-factor model. It can be seen that a dimension reduction takes place in the third step. Thus, the figure mainly represents the first of this dissertation's three empirical investigation sections (Chapter 4).

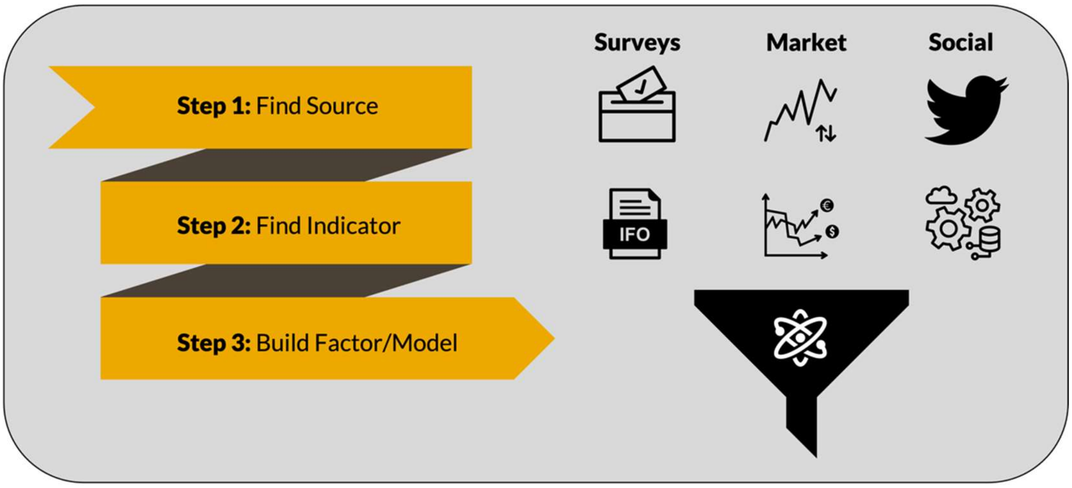

Figure 7. Utilization of Investor Sentiment.

[Source: Author's representation]

3.2.1 Survey-based Sentiment

Collecting investor sentiment through surveys has been widely used in stock return research for many years. The first individuals to utilize this method, such as Shiller et al. (1996), aimed to gather the sentiment of institutional investors by conducting semi-annual surveys on their perceptions of the US and Japanese markets.

A more sophisticated approach for determining investor sentiment is to employ survey-based sentiment indexes. Internationally, particularly in the US market, well-known examples of these indices include the UBS/Gallup Index of Investor Optimism, the Conference Board Consumer Confidence Index, and the University of Michigan Consumer Sentiment Index.

Also, the American Association of Individual Investors (AAII) Sentiment Survey has been an increasingly widely followed indicator among individual investors since 1987 and provides insight into investor sentiment. The weekly survey results are made available to the public through financial publications such as Barron's and Bloomberg and, by AAII's admission, are regarded as essential information by financial professionals. The aforementioned survey-based investor sentiment indices have been shown to possess a degree of predictive power regarding financial market indicators (Brown and Cliff 2004, 2005; Aboura 2016).

Notable analyses in the area of survey-based investor sentiment research in the US market include those (not conclusive) by Clarke and Statman (1998), Fisher and Statman (2000), Shiller (2000), Brown and Cliff (2004, 2005), and Verma and Soydemir (2006). Although the results are mixed (which is likely, as Hengelbrock et al. (2013) recognize, due to differences in sample periods, methodology, and prediction time frames), these earlier studies find long-term predictability of stock market yields.

Among other matters, Schmeling (2007, 2009) also examines the German stock market and finds a robust correlation between investor sentiment and future returns in Germany. Other work has found short-term predictability (e.g., at the weekly and monthly level, as in Brown and Cliff (2004)) and day effects (Hengelbrock et al. 2013).

The previous research on the relationship between survey-based investor sentiment and financial market indicators has yielded various findings. Qiu and Welch (2004) examined the relationship between customer satisfaction and investor sentiment. Hengelbrock et al. (2013) explored the varying timeframes in which survey-based investor sentiment affects market prices. Additionally, Hilliard et al. (2016, 2020) demonstrate that a risk factor built on weekly survey-based investor sentiment provides significant explanatory power in understanding stock market returns.

In the meantime, survey-based indicators have also established themselves on the German market, such as the Ifo index for the business climate or the various indices of Sentix GmbH, which directly capture investor sentiment for the financial sector. However, according to Da et al. (2014), applying such investor sentiment indices may have considerable limitations. For instance, most survey-based datasets are available at weekly or monthly intervals. Such indices have certain drawbacks and may be limited in some cases (i.e., when shorter time intervals are investigated).

Survey-based investor sentiment also provides significant explanatory contributions for stock market returns after controlling for variables in the Fama-French three-factor model and Carhart's four-factor model.

Tiwari et al. (2018, 2021) explore whether investor sentiment impacts different markets built on weekly surveys conducted by the German company Sentix GmbH. They posit that nonlinear causality analyses offer more comprehensive explanatory power than linear models. The following tabular overview illustrates the most crucial work/milestones in the field of survey-based investor sentiment in the context of this dissertation.

Table 2. Key Research Contributions in the Area of Survey-based Investor Sentiment Analysis.

Source	**Database**	**Applied method(s)**	**Key finding**
Shiller et al. (1996): Why Did the Nikkei Crash? Expanding the Scope of Expectations Data Collection	Biannual mail surveys from 1989 to 1994 in Japan and the US	Interpretative social science (Rabinow and Sullivan, 1979); behavioral listings (Sternberg, 1987)	Investor sentiment is country-specific; first indications for an investor sentiment cycle
Qiu, Lily; Welch, Ivo (2004): Investor Sentiment Measures	UBS/Gallup investor sentiment survey data	Pearson correlation coefficient	Investor sentiment derived from surveys is found to be associated with the additional return on small companies
Brown and Cliff (2005): Investor Sentiment and Asset Valuation	Investor's Intelligence survey-based investor sentiment data	Integration in Fama-French Three-factor Model + further risk factors	A correlation exists between survey-based investor sentiment and the excess return on small firms over the next 1-3 years
Hengelbrock et al. (2013): Market Response to Investor Sentiment	Survey-based sentiment indicators from Germany (Sentix) and the US (AAII)	Methodology proposed by Brown and Cliff (2005); Bootstrap Simulation; Event Study	Investor sentiment, as measured by various indicators, has a strong correlation with future stock market returns over a medium-term horizon, according to the findings of this study. Additionally, the study finds evidence of a positive impact on stock market returns on the day of announcements in the German market

Source	Database	Applied method(s)	Key finding
Tiwari et al. (2018, 2021): Investor Sentiment Connectedness: Evidence from Linear and Nonlinear Causality Approaches	Survey-based sentiment indicator (Sentix)	Kernel-based multivariate nonlinear causality test; Granger Causality.	Investor sentiment is country-specific, but spillover effects occur; findings suggest that linear causality models may not be suitable for analyzing the relationship between investor sentiment and stock market returns due to the presence of nonlinearity and structural breaks

Note. This table provides an overview of important research contributions in the area of survey-based investor sentiment analysis.

[Source: Author's representation]

3.2.2 Market-implied Sentiment

As described prior, market-implied investor sentiment is based on market information that comes directly or indirectly from the market itself and is usually directed toward the future. Trading volume is an essential and straightforward indicator mentioned in this context (Gervais et al. 2001; Hou et al. 2009). While a high trading volume is associated with a tendency toward price appreciation, a low trading volume is associated with price depreciation.

However, it is not easy to attribute these empirically observable relationships to investor sentiment, as the corresponding appreciation can also be viewed as the price of liquidity in the market, which increases the fungibility of the asset being traded. Also, irrational behaving investors ("noise traders") influence prices whose intensity of movement cannot be explained by fundamental data (Brown 1999; Barber and Odean 2008). Furthermore, volatility indices exist, such as the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), which are proposed, for example, by Whaley (2000) and Baker and Wurgler (2007).

The UBS DERI Risk Indicator also measures daily sentiment on the global financial markets. As a counterpart to the VIX on the German stock market, the “Volatility DAX” VDAX(-NEW)[12] exists.

Other so-called sentiment indices, which are based on market-implied data, include established market variables. For example, the Acertus Market Sentiment Indicator is a composite index aiming to gauge the market's overall sentiment by taking into account various indicators. The five variables that make up this index are the Price-to-earnings ratio (a measure of stock market valuation), price momentum (a measure of market psychology), realized volatility (a measure of recent risk), high-yield bond returns (a measure of credit risk), and the TED spread (a measure of systemic financial risk). These variables are weighted in descending order of importance in the index calculation, with the Price-to-earnings ratio being the most substantial factor.

This approach allows for a comprehensive assessment of the market sentiment by considering multiple factors that are believed to be indicative of investor sentiment. A further measure is a discount of closed-end funds (the case in which the net asset value of a mutual fund does not equal its market price) as an essential measure in this context (Zweig 1973; Lee et al. 1991; Swaminathan 1996; Neal and Wheatley 1998; Elton et al. 1998).

[12] The "VDAX-NEW" replaced the VDAX in 2016. Unlike its predecessor, the VDAX, the VDAX-NEW can be replicated with a portfolio of options on the DAX actually traded on the futures market. While the VDAX-NEW is interpolated from the next two sub-indices to 30 days, the VDAX was based on an option term of 45 days. The VDAX was not based on actually traded options, but on fictitious option prices. While a theoretical option price model is required for the calculation of the VDAX, the VDAX-NEW is determined from the variance of a special option portfolio. The calculation of the VDAX-NEW is based exclusively on at-the-money options. The VDAX-NEW reached its highest value to date with an index level of 93.3 on March 16, 2020 during the COVID-19 pandemic; on this day, it also reached its historic highest daily closing price of 86.01 points.

Fluctuations in closed-end fund discounts are highly correlated with fluctuations in investor sentiment, indicating a strong connection between the two variables. This may suggest that changes in investor sentiment have a direct impact on the pricing of closed-end funds.

A substantial body of research utilizes market-implied investor sentiment measures to assess investor sentiment. These indicators include mutual fund flows (Brown et al. 2003), put-call ratios (Dennis and Mayhew 2002), and various measures of trading activity (Kumar and Lee 2002, 2006; Barber and Odean 2008). Baker and Wurgler (2006) developed a composite investor sentiment indicator based on six underlying indicators and found it to have a significant explanatory contribution to stock market returns. Brown and Cliff (2004) analyzed market-based and survey-based sentiment measures and determined that many of these measures are correlated.

The exemplary market-implied measures mentioned above also have a significant drawback. Da et al. (2014) state:

> "Although market-based measures have the advantage of being readily available at a relatively high frequency, they have the disadvantage of being the equilibrium outcome of many economic forces other than investor sentiment."

Another concern is seen in the fact that some indicators can be procyclical and, thus, reinforce each other in second-round effects. For instance, an increase in trading activity can lead to heightened interest from market participants, which in turn can result in a further increase in trading volume, thus amplifying investor attention. This cyclical process can perpetuate itself, leading to significant fluctuations in trading activity.

Other notable studies on the relationship between investor sentiment and stock market returns include those of Lee et al. (1991), who proposed that investor sentiment influences stock returns; and Goetzmann et al. (2000), who discovered an inverse relationship between the daily inflow of funds into equity funds and the yields of those funds. Grinblatt and Keloharju (2000) demonstrated that the influx of liquidity from foreign investors could potentially have an impact on the fluctuation of share prices, and Brown et al. (2003) likewise revealed a relationship between the inflow of funds into equity funds and the yields of those funds.

Contemporary research on sentiment analysis using survey data also considers the significance of market-implied sentiment indicators in their research. They recognize the need to incorporate measures derived from financial market data to gain a more comprehensive understanding of the relationship between investor sentiment and stock market returns.

Yoshinaga and Castro Junior (2012) observe that sentiment influences Brazilian stock market returns based on a sentiment index derived from market-implied data. Kumari and Mahakud (2016) use a custom-developed investor sentiment index to show its significance in the Indian stock market. Bahloul and Bouri (2016) use generalized autoregressive conditional heteroscedasticity (GARCH) analysis to observe that investor positioning affects returns and volatility in US futures markets.

Zha (2018) illustrates the influence of market-implied data, such as turnover rate or new issues, on the Chinese stock market returns. Félix et al. (2020) demonstrate that implied volatilities have significant explanatory power on stock returns. The following tabular overview illustrates the most critical work/milestones in the field of market-implied investor sentiment in the context of this dissertation.

Table 3. Key Research Contributions in the Area of Market-implied Investor Sentiment Analysis.

Source	**Database**	**Applied method(s)**	**Key finding**
Lee et al. (1991): Investor Sentiment and the Closed-End Fund Puzzle	Wiesenberger's Investment Companies Services 68 Mutual Funds with CUSIP identifiers from 1960 until 1986; WSJ; CRSP	Pearson correlation coefficient. Linear regression	The principal outcome that can be drawn from the body of research is that fluctuations in the discounts of closed-end funds can be used as a reliable indicator of investor sentiment

Source	**Database**	**Applied method(s)**	**Key finding**
Brown et al. (2003): Investor Sentiment in Japanese and U.S. Daily Mutual Fund Flows	U.S. data from TrimTabs, which contains data from 1998 until 1999; Japanese data from QUICK Corporation, which contains data from 1998 until 2000	Investor Sentiment Factor generation from mutual fund flows using a version of the Fama-MacBeth (1973) framework; Integration in Fama-French Three-factor Model + further risk factors	In the studied time frame, the inclusion of an investor sentiment factor in the model significantly enhanced the explanation of stock returns beyond that provided by traditional return factors such as size, value, and momentum
Kumar and Lee (2006): Retail Investor Sentiment and Return Comovements	>1.85 million transactions made by >60,000 retail investors from 1991 until 1996	Integration of investor sentiment risk factor in Fama-French Three-factor Model + momentum risk factor by Carhart (1997)	The findings indicate that the systematic behavior of retail traders plays a considerable role in the return patterns of stocks that are heavily traded by retail investors, particularly when these stocks are also difficult to arbitrage
Baker and Wurgler (2006, 2007): Investor Sentiment and the Cross-Section of Stock Returns; Investor Sentiment in the Stock Market	Composite sentiment measure based on six underlying indicators from 1963 to 2001	Integration of investor sentiment risk factor in Fama-French Three-factor Model + momentum risk factor by Carhart (1997)	Empirical evidence for the investor sentiment cycle; Stocks that present challenges in terms of arbitrage or valuation are particularly susceptible to the influence of investor sentiment

Note. This table provides an overview of important research contributions in the area of market-implied investor sentiment analysis.

[Source: Author's representation]

3.2.3 Social Sentiment

The utilization of social sentiment, or news-based sentiment, as a means of capturing investor sentiment, is a relatively recent development. Differentiating between sentiment that is solely based on the amount of specific data, such as message and search frequency, and social sentiment that incorporates a qualitative aspect through text mining methods, such as evaluating the positivity or subjectivity of tweets, is crucial. One major benefit of social sentiment is its quick accessibility, which can even be near-instant or in real-time, depending on the approach used. This, unlike sentiment collected through surveys, makes it valuable for traders who make decisions on a daily basis.

In terms of promptness, the study conducted by Checkley et al. (2017) illustrates that investor sentiment obtained from social media platforms can have an instantaneous effect on the stock market.

Studies based on information quantity include Antweiler and Frank (2004), who studied financial news from Yahoo Finance. Allen et al. (2015) also extract investor sentiment from financial news. Dimpfl and Jank (2016) show that search engine queries impact stock returns.

Preis et al. (2013) show the importance of social sentiment based on Google trends on stock market returns. Solanki and Seetharam (2018) implement the FEARS index derived from Google trends into an APT model and discover a significant impact on stock market returns. Joseph et al. (2011) also consider the impact of Google searches on individual companies.

Other studies, such as Liu et al. (2007), begin with weblogs analysis. Though, social sentiment is not without controversy, Sorto et al. (2017) observe that news-based sentiment does not affect Dow Jones returns. Thus, they support Fama's EMH. Uncertain results on Twitter social sentiment are also shown in the study of Gustafsson and Granholm (2017).

Nevertheless, investor sentiment analyses based on tweets are mostly considered promising. O'Connor et al. (2010) observe that sentiment from surveys is sometimes correlated with Twitter sentiment by up to 80%. Jong et al. (2017) show that Twitter-based sentiment affects Dow Jones returns. Bhardwaj et al. (2015) shed light on the possibility of using social networks as a source of social sentiment for the Indian stock market.

Wang et al. (2011) likewise experimented with investor sentiment extraction from Twitter. Bollen et al. (2011) also show, based on daily data, that Twitter sentiment leads to significant results.

Liu (2012) demonstrates techniques for Twitter investor sentiment extraction. Sul et al. (2017) demonstrate that trading strategies based on Twitter sentiment can succeed. Pagolu et al. (2016) use Twitter-based sentiment to predict stock returns. They demonstrate a significant association between sentiments expressed on the social media platform Twitter and fluctuations in stock market returns on an intraday basis.

Additionally, their study illustrates a significant association between sentiments gleaned from tweets and the performance of the Dow Jones Industrial Average. Fang and Peress (2008) find empirical evidence that stocks receiving less coverage in the media have higher average returns than those in the media spotlight. In addition to Twitter, less widely used social networks are also examined. Rechenthin et al. (2013) show that sentiment from "stock chatter" significantly affects daily returns.

Chen and Lazer (2013) derived investment strategies from monitoring and classifying Twitter feeds. Zhang (2013a) observed a negative correlation between sentiment status and the Dow Jones. Using Pearson's correlation coefficient, Dickinson and Hu (2015) examine the correlation between investor sentiment and stock return changes. Gusev et al. (2015) show specific news-based investor sentiment model applications. Guo et al. (2017) demonstrate that social media-based investor sentiment is significant in the Chinese stock market. Yu et al. (2013) use risk, in addition to stock returns, as indicators of companies' short-term performance.

Since the field of social media analysis is one of the recent areas in which investor sentiment has been most analyzed, the most current research will be reviewed at this point before moving on to the empirical exploratory study of this dissertation in Chapter 6.

Among the most recent empirical reviews of social sentiment are Hamraoui and Boubaker (2022), which examine the Tunisian stock market based on Twitter investor sentiment and find relatively weak correlations using Pearson correlation and Granger causality tests.

Duz Tan and Tas (2021) examine the effect of social media on the components of the S&P index for US, European, and emerging markets from the perspective of international investors using firm-specific Twitter investor sentiment. They propose that Twitter investor sentiment is more influential for small and emerging firms, which aligns with the existing literature that suggests small firms are challenging to value and emerging firms have high information asymmetry. They conclude that investors can use social media investor sentiment to influence their trading strategies from a practical perspective.

Other interesting studies deal with the correlations of investor sentiment and price development in the stock market based on specific events. Swathi et al. (2022) find that positive and negative opinions are essential indicators of upcoming stock prices in the present stock market. They use LSTM-based sentiment analysis to predict stock prices using Twitter data. Also, Jin et al. (2020) could make interesting findings based on this methodology.

For example, Lete Segura (2022) analyzes whether there is a relationship between overall political sentiment on Twitter and stock market movements, more specifically, the US S&P 500, DJIA, and Nasdaq Composite indices.

To investigate this, he focused on the 2020 US presidential election and the Twitter hashtag "#Election Day". However, no firm conclusions could be drawn because the results were not statistically significantly different from zero. However, some trends could be identified.

Becker et al. (2021) conducted a study in a similar context. They analyze the impact of investor sentiment variables on stock market returns for periods before and after the election of Donald Trump in 2016. At this time, they applied a traditional approach, namely the CAPM and the Fama-French three-factor model, supplemented with investor sentiment measures.

The results suggest that the relationship between investor sentiment and abnormal returns is more negative in the post-election periods, strengthening the contra-indicator theory. The following tabular overview illustrates the most important work/milestones in the field of social investor sentiment in the context of this dissertation.

Table 4. Key Research Contributions in the Area of Social Investor Sentiment Analysis.

Source	**Database**	**Applied method**	**Key finding**
Bollen et al. (2011): Twitter mood predicts the stock market	9,853,498 tweets posted by approximately 2.7M users in 2008	Granger Causality analysis; Self-organizing Fuzzy Neural Network to predict DJIA values	The relation between public mood and stock market values is almost certainly non-linear
Rechenthin et al. (2013): Stock chatter: Using stock sentiment to predict price direction	67,849 posts (Yahoo Finance message boards) regarding eleven widely-traded stocks (large number of posts and high trading volumes) in 2011	"Boosted"decision tree; Artificial Neural Network for classification	Dimensionality reduction (PCA) is unsuitable for the case; Artificial neural networks are suitable for determining that markets are predictable
Dickinson and Hu (2015): Sentiment analysis of investor opinions on Twitter	DJIA performance and corresponding Tweets from 2014 until 2015	Pearson correlation coefficient; Random forest; n-gram and word2vec investor sentiment prediction	A correlation between sentiment and price exists; this correlation varies between the different DIJA titles considered
Jin et al. (2020): Stock closing price prediction based on sentiment analysis and LSTM	96,903 comments made by stockholders on stocktwits from 2013 until 2018; AAPL stock data	LSTM improved by empirical mode decomposition	R^2close to 1 shows a high model goodness of fit for LSTM-based models
Swathi et al. (2022): An optimal deep learning-based LSTM for stock price prediction using Twitter sentiment analysis	A Twitter dataset, which is not described in detail	Teaching and Learning-based Optimization model with LSTM; Random Forest, Logistic Regression; RNN	The applied method with LSTM is superior to the other methods

Note. This table provides an overview of important research contributions in the area of social investor sentiment analysis.

[Source: Author's representation]

4 TRADITIONAL APPROACH: RISK FACTOR INTEGRATION IN MULTI-FACTOR MODELS

This section examines empirical approaches to integrating investor sentiment risk factors, building on the previous chapter, in which the theoretical foundations and historical development of multi-factor models and investor sentiment sources were considered. The most important findings of the basic introductory chapters (see Chapters 1-3), which are relevant to this part of the empirical study, are summarized once again in a very abbreviated form.

As part of this initial empirical analysis (Hövel 2018; Hövel and Gehrke 2022a), a PCA-based sentiment risk factor specifically required for this study is developed. Supported by recent studies' results, this thesis first utilizes an investor sentiment risk factor explicitly derived for this purpose to empirically examine whether the hypotheses developed from the literature on the integrability of investor sentiment risk factors on the German stock market can be rejected or confirmed.

4.1 INTRODUCTION TO THE EMPIRICAL ANALYSIS

These days, the capital market and modern portfolio theory leave practically no room to take investor sentiment into account when valuing shares or share portfolios. Further developments of the CAPM and widely adapted multi-factor models do not consider the possible effects of investor sentiment on stock returns and do not provide sufficient explanations for the return anomalies observed in empirical studies. The focus of rational investors is instead, when following EMH theory, on the diversification of idiosyncratic risks.

However, empirical results in the literature (see Chapter 3.2) support the assumption that investor sentiment can make an explanatory contribution to stock market yields.

The German stock market presents a unique opportunity to study the impact of traditional risk factors, particularly in relation to investor sentiment.

Structural characteristics of the German capital market have revealed variations in the significance and explanatory power of classical risk factors in comparison to other markets (Ziegler et al. 2007; Hanauer et al. 2013).

Furthermore, Germany's highly export-oriented economy and large foreign trade surplus make it particularly susceptible to global developments and investor sentiment. This study aims to examine the evolution of established risk factors' explanatory power and significance in a contemporary sample in comparison to prior cross-sectional studies. It is essential to note that the significance of these factors is also contingent on the sample size being studied.

Additionally, this study will examine the degree to which an investor sentiment-based risk factor is able to deliver further explanatory power. The findings of this study indicate that an APT model incorporating Carhart's risk factors outperforms a single-factor CAPM model in explaining excess yields in the German stock market. Furthermore, the inclusion of a sentiment risk factor in the APT model yields additional advantages.

These findings are crucial for the investigation of asset allocation principles. Nevertheless, it is important to note that securities market analysis primarily centers on the investment decisions of risk-averse investors and often assumes normally distributed security returns and rational investors. The observation is that even though diversified portfolios are supposed to reduce risk, the returns generated by these portfolios were not always proportional to the amount of risk taken on. This realization prompted the understanding that the yields on a diversified portfolio are not exclusively impacted by a single market risk factor, such as the beta coefficient in the Capital Asset Pricing Model.

This reflection forms the basis for the APT by Ross (1976), which allows for multiple factors to determine stock yields as it became known that the proportionality of risk and return is sometimes not given despite sufficient diversification (Fischer Black et al. (1972)).

However, selecting the appropriate factors in the APT model can be challenging as risk factors can vary depending on the country, may not always be fully understood, and can change over time. Although Fama and French (1992, 1993) and Carhart (1997) established four factors, the optimal combination of factors for APT models remains uncertain.

In recent years, newer factors such as investor sentiment have been studied internationally (e.g., Gutierrez and Perez-Liston (2021); Hadi and Shabbir (2021); Jiang et al. (2021)).

4.2 DEVELOPMENT OF RESEARCH HYPOTHESES

Investor sentiment, as described by Baker and Wurgler (2007) as the perception of future cash flows and risks that cannot be explained by fundamental data, has been proven to have a significant impact on future price movements.

This, in turn, can serve as a basis for making informed decisions regarding short-term trading and long-term investment strategies. It is also worth noting that unexpected events such as airplane crashes or lost soccer games can also have an impact on investor sentiment, as demonstrated by Kaplanski and Levy (2010) and Edmans et al. (2007).

Recent developments in capital market research have indicated that there may be additional, yet-to-be-discovered risk factors that could potentially be incorporated into multi-factor models in order to improve their predictive power. As such, it is of utmost importance to investigate whether factors such as investor sentiment, which can be challenging to quantify, may also contribute to stock valuation.

Given the inherent difficulty in capturing the behavior of individual investors, research often concentrates on models that posit generalizations about the actions of investors as a group. Studies by Baker and Wurgler (2007) have shown that investor sentiment plays a significant role in explaining stock market returns based on aggregate investor behavior. Additionally, Long et al. (1990) and Barberis et al. (1998) have made important contributions to our today's comprehension of the impact of irrational market participants (sometimes called "noise traders") and the psychological basis of investor sentiment, respectively.

Quantifying investor sentiment is a challenging task that has been the focus of much research in the field of financial analysis. As outlined in previous chapters, one established method for determining sentiment is through the use of investor surveys. However, as is often the case, survey participants may not meet the requirements of inferential statistics or randomization, making it difficult to draw accurate conclusions from the data collected.

Furthermore, in order to gain a comprehensive understanding of investor sentiment, it would be ideal to differentiate short-term, medium-term, and prolonged monitoring.

Investor sentiment analysis encompasses not only sentiment indicators based on surveys, but also market-implied sentiment which is inferred from future-oriented market information. Indicators like implied volatility and put-call ratios from options are commonly employed to signify the outlook of investors and are typically regarded as secondary markers of investor sentiment. Another area that is gaining popularity is the analysis of news and social sentiment, which encompasses a wide range of information, including online media reports. In this field, not only the quality but also the quantity of news is taken into account. This is currently an area of much discussion in the sphere of investor sentiment research.

All sentiment indicators share the common goal of representing the duality of investor opinion, specifically, positivity and negativity. This reflects the ongoing effort to provide a comprehensive and nuanced understanding of the underlying sentiment driving financial markets.

Investigating the relationship between investor sentiment and stock market returns has been the subject of numerous studies (see Chapter 3), which have employed various analytical techniques such as nonlinear causality tests, linear regression, and Pearson's correlation coefficient. In many of these studies, the three-factor model of Fama and French is commonly utilized as the control model, with some studies also using the multi-factor model proposed by Carhart (1997).

Despite recent research on this subject from around the world (such as those by Gao and Liu 2020; Jun Xiang Huang et al. 2020; Li et al. 2020; P.H. and Uchil 2020; Steyn et al. 2020; Zaremba et al. 2020; Al-Nasseri et al. 2021; Dunham and Garcia 2021; Gutierrez and Perez-Liston 2021; Hadi and Shabbir 2021; Jiang et al. 2021), there are relatively fewer studies on the German stock market, despite its unique capital market organization and architecture.

Finter et al. (2012) conducted a study on German market data and discovered that certain groups of stocks are more responsive to investor sentiment. However, the study did not find any substantial explanatory power for subsequent share yields.

Krinitz et al. (2017) utilized a Granger causality test and discovered that sentiment derived from media sources has a significant effect on the returns of the German stock market, further emphasizing the need for additional examination of the German share market within this context.

Additionally, there has been a plethora of research that has investigated risk factors from this sentiment category. However, from a contemporary perspective, the underlying data samples used in these studies may no longer be considered current, leading the inquiry into the extent to which traditional risk factors influence the yields have undergone any changes.

In light of this, it would be valuable to also investigate whether incorporating a sentiment-risk factor into established APT models could further improve the quality of these models. This research question gives rise to four hypotheses, which will be addressed in the empirical analysis that follows.

$\boldsymbol{RH_1}$: *Investor sentiment contributes to explaining return variances in the German stock market.*

$\boldsymbol{RH_2}$: *Investor sentiment is a contra-indicator for stock market developments.*

$\boldsymbol{RH_3}$: *The integration of an investor sentiment risk factor into multi-factor models leads to a higher model quality compared to the Fama-French and Carhart target portfolio regression models, expressed by the adjusted coefficient of determination* $\bar{R}^2$.

$\boldsymbol{RH_4}$: *Incorporating an investor sentiment risk factor into multi-factor models leads to a lower alpha range in the Fama-French and Carhart target portfolio regression models.*

4.3 METHODOLOGY

4.3.1 Data Sources and Sample

The multi-factor models used in this analysis were constructed using data obtained from both Thomson Reuters Eikon/Datastream (now known as Refinitiv) and the Deutsche Bundesbank time-series database, which is publicly available. The current examination is built on a meticulously selected and organized collection of data from the German CDAX, which includes total returns (RI), taking into account both dividend payments and stock splits in the calculation.

Financial stocks are also included in the analysis, which is considered justifiable when using equally-weighted returns rather than market-value adjusted returns, and the influence is deemed to be negligible.

The research employs logarithmic returns as a method, which is considered a statistically sound approach, however, it diminishes the ability to compare the results with other studies that employ discrete returns.[13] The information used to obtain the risk-free investment rate was sourced from the Deutsche Bundesbank's publicly accessible time-series database. Additional data, such as book- and market based values (MV, PTBV, etc.) were obtained from Refinitiv (formerly Thomson Reuters Eikon/Datastream).

The limitations of the observation period for this study are determined by the availability of data from the sentiment sources being investigated. To ensure comparability of the investor sentiment indicators within the samples, the research sample is based on returns computed on a monthly basis and encompasses a time frame of 240 months, from January 15th, 2001 to January 15th, 2021.

The yields are calculated using the total return index (RI) as it takes into account stock splits and other corporate actions, in addition to dividends, unlike the price index (P). The analysis includes all CDAX shares. CDAX is a benchmark that measures the performance of companies listed on the Frankfurt Stock Exchange in the General Standard and Prime Standard. It represents the overall development of the German stock market and includes all stocks in those segments.

[13] This is because the simple returns are skewed, so that a log form is considered more normal. Meanwhile, the longer the time interval over which the returns are calculated, the greater the difference between simple and log returns due to the continuous compounding effect. The interval is relatively short in the original Fama-French studies. Furthermore, there appears to be no sound argument in mathematical modeling that precludes the use of log returns. Current studies use the log return variant when examining risk factors (Christoffersen and Langlois 2013; Zhao et al. 2019), even though the actual interpretation of the values is unlikely to change much in terms of the core message.

Additionally, even though several of the investor sentiment factors analyzed specifically relate to the DAX, the study uses equally-weighted CDAX returns. The reason for this is that the DAX does not contain enough data points to create 25 distinct Fama-French portfolios.[14] Furthermore, incorporating the size risk factor into the multi-factor model for the DAX 30 would be incongruous, as the DAX 30 is composed solely of the largest companies by market capitalization in the German stock market and accounts for approximately two-thirds of the entire German market capitalization.

A comparison of the DAX 30 and CDAX reveals that they have a very strong correlation in terms of their price development, as demonstrated by the beta coefficients. The Pearson's correlation coefficient of the indices is $\rho > .99$ when weekly returns are taken into account (Ziemer 2018). Thus, an analysis of the CDAX can also be considered generally applicable when applied to investor sentiment indicators that specifically relate to the DAX 30. If the CDAX and DAX 30 move in lockstep, it can be inferred that elements that influence stock returns are largely similar for both indices.

Although this particular section of the empirical analysis examines equally-weighted CDAX returns, which primarily includes smaller stocks, it is plausible to assume that investor sentiment plays a significant role in the performance of DAX 30 stocks (blue chips) due to their high media presence (Fang and Peress 2008).

In this study, smaller companies are given more importance in comparison with the index weighted by capital. The use of equal-weighted CDAX values is intended to represent smaller companies, which, according to the results of Baker and Wurgler (2007), are extra susceptible to investor sentiment. Additionally, Kumar and Lee (2006) and Barber and Odean (2008) suggest that less liquid small stocks tend to be more affected by shifts in investor sentiment.

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[14] As of September 20, 2021, the DAX 30 expansion was completed with 10 additional listed companies and has since been referred to as the DAX 40.

The approximate market portfolio, represented by the CDAX index, was redetermined using shares that were given equal weighting. The relationship between this new index and the original CDAX index is very high, expressed by a Pearson correlation coefficient of approximately 1, which is consistent with literature on the topic (Ziemer 2018).

The Euro Interbank Offered Rate (EURIBOR) is utilized monthly as a stand-in for the risk-free asset, r_f, which is a common practice in the German market according to studies by Hanauer et al. (2013), Schrimpf et al. (2007), and Ziegler et al. (2007).[15]

4.3.2 Construction of the Carhart Risk Factors

This study employs a methodology that is consistent with previous research in the field of empirical multi-factor modeling, as outlined by Fama and French (1992, 1993), Ziegler et al. (2007), and Hanauer et al. (2013). The Market Risk Premium ($RMRF$) is determined by subtracting the Risk-Free Rate (R_f) from the estimated Market Portfolio (R_m). Additionally, the size factor "Small Minus Big" (SMB) and the value factor "High Minus Low" (HML) are also calculated using the same methodologies as those employed by Fama and French (1993), which are based on monthly returns.

Therefore, at the conclusion of June (or the initiation of July) for each year y, the median market capitalization is computed, and separately, the 30% and 70% quantile values of the book-to-market ratio for December 31st of each year are determined for all stocks under examination.[16]

[15] For future studies, other reference rates are to be used as a result of the IBOR reform. In the period under consideration, however, EURIBOR is available without any limitations.

[16] In this study, the book value as of December 31st of the preceding year is calculated as a ratio relative to the market capitalization of that day.

In their seminal work, Fama and French (1993) employed the use of balance sheet date as opposed to December 31st as the reference point for their analysis. However, in the present study, this methodology is not adopted due to the assumption that newly released book values are instantly reflected in stock prices.

Conversely, as per the available data, the overwhelming majority of the corporations under examination have established December 31st as their end-of-year closing date.

In this study, the shares are divided into two groups, Group B (Big) and Group S (Small), built on the median market capitalization, with the corporations having the biggest market capitalization being allocated to Group B, and the smallest to Group S. Additionally, the shares are also grouped into three categories built on the book-to-market ratio by utilizing the 30% and 70% quantiles as the threshold points for the classification.

The present study employs a classification scheme that categorizes public corporations based on their book-to-market ratio, with those possessing a high ratio being allocated to Group H (High), those with a medium ratio being assigned to Group M (Medium), and those with a low ratio being assigned to Group L (Low). This classification serves as the foundation for the formation of six equally-weighted share portfolios, denoted as S/H, S/M, S/L, B/H, B/M and B/L, which represent the cross-combination of the five groups.[17]

The shares included in the sample for monthly yields are classified into one of the six portfolios at the beginning of July of the current year y and retained in those portfolios until the end of June of the following year $y + 1$. The process of portfolio recalibration is conducted in July of the following year, utilizing the updated data.

[17] The acronym S/H represents "Small-High," and pertains to firms with a low market capitalization and a high book-to-market value ratio.

All over the duration of the study, the yields of six portfolios $R_t^{S/H}$, $R_t^{S/M}$, $R_t^{S/L}$, $R_t^{B/H}$, $R_t^{B/M}$, and $R_t^{B/L}$ are computed on a monthly basis. The SMB portfolio is determined as the equal-weighted mean of the returns of small firm portfolios subtracted by the returns of large firm portfolios, as outlined in Equation 9.

$$SMB_t = \frac{\left(R_t^{S/L} - R_t^{B/L}\right) + \left(R_t^{S/M} - R_t^{B/M}\right) + \left(R_t^{S/H} - R_t^{B/H}\right)}{3} \tag{9}$$

HML is defined analogously (Equation 10).

$$HML_t = \frac{\left(R_t^{S/H} - R_t^{S/L}\right) + \left(R_t^{B/H} - R_t^{B/L}\right)}{2} \tag{10}$$

In the final step, the momentum risk factor "Winners Minus Losers" (WML) is determined using the methodology outlined by Carhart (1997). Specifically, for every month t ranging from July of year y to June of year $y + 1$, equities are arranged according to their performance from the start of month $t - 12$ to the start of month $t - 2$.[18]

In order to classify shares, the 30% and 70% quantiles were calculated by utilizing the ranked list of the previous year's performance stocks.

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[18] The performance for July of year y is based on the time period starting from the first day of July in the previous year (year $y - 1$) and ending on the first day of June in the current year (year y). This method, which was proposed by Ziegler et al. (2007), excludes the last month in order to eliminate any issues related to market microstructure, such as the bid-ask bounce (Fama and French 1996). These issues can lead to a negative correlation in one-month returns, which would negatively impact the momentum effect and reduce its ability to explain market trends (Asness 1995).

Subsequently, the shares with the highest performance from the previous year were designated to the "winners" group (W), those with medium performance from the previous year were assigned to the "neutral" group (N), and those with the lowest performance from the previous year were placed in the "losers" group (L).

Similar to the computation of HML, the construction of the six portfolios S/W, S/N, S/L, B/W, B/N, and B/L is achieved by utilizing the cross-product method with the market capitalization groups.[19]

The returns associated with each portfolio are calculated as the equal-weighted returns of the corporations comprising the portfolio. The Winner Minus Loser (WML) return is determined as the equal-weighted average of the yields generated by portfolios of corporations with favorable previous-year performance, subtracted from the yields of portfolios of corporations with inferior previous-year performance, as outlined in Equation 11.

$$WML_t = \frac{\left(R_t^{S/W} - R_t^{S/L}\right) + \left(R_t^{B/W} - R_t^{B/L}\right)}{2} \tag{11}$$

The design of this factor construction guarantees that $RMRF$, SMB, HML, and WML exhibit regularly only minimal correlation. This presumption was verified through the examination of a monthly cross-section sample, as depicted in Figure 8.

[19] The acronym S/W denotes "Small-Winners" and refers to a category of stocks that possess a relatively low market capitalization and have demonstrated positive performance in the preceding year.

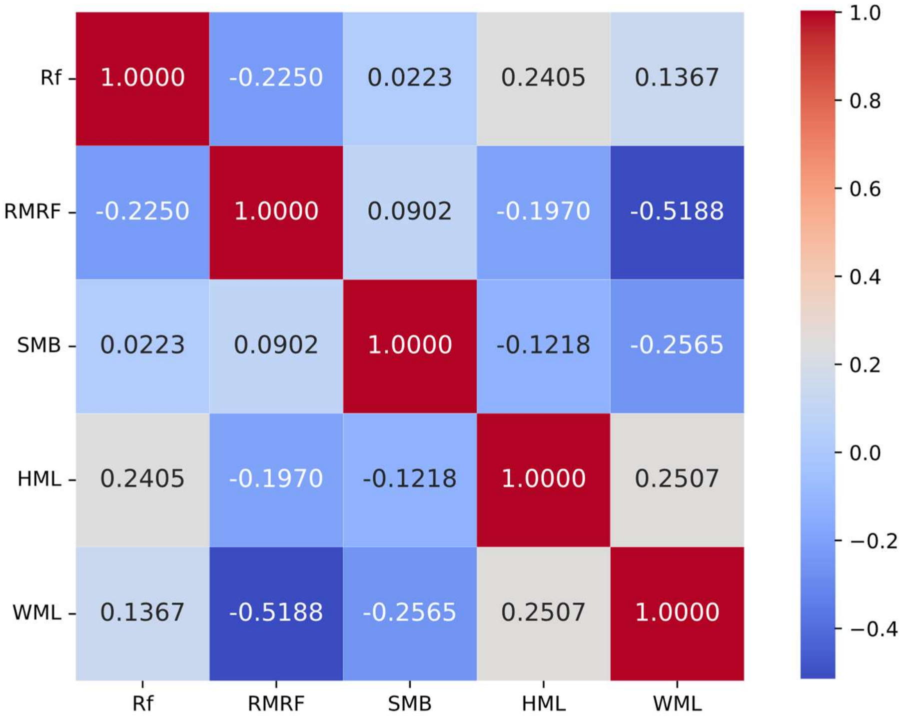

Figure 8: Correlation Matrix of the Multi-factor Model Components.

[Source: Author's representation]

Figure 8 shows the respective Pearson correlation coefficients. However, it must be noted that the cross-sectional correlation between the market risk premium $RMRF$ and the momentum factor WML is negative with a value of $-.5188$. In comparison, the observed correlation in Hanauer et al.'s (2013) study is also negative, with a value of $-.324$. However, the absolute correlation is considerably lower than in this present study.

Unlike Hilliard et al. (2016, 2020) and Fama and French (2015), no particularly high correlation is found between HML and other risk factors and, therefore, HML cannot be considered redundant.[20]

However, since the present analysis is a cross-sectional analysis, it does not consider that the risk factors examined are variable over time (see Chapter 5).

In addition to the cross-sectional view, a longitudinal view of the independent variables is also of interest. The graphical representation depicted in Figure 9 illustrates the cumulative performance of the Carhart factors, specifically $RMRF$ ($R_m - R_f$, representing the excess return of the stock market portfolio), SMB (Small Minus Big, denoting the differential return based on market capitalization), HML (High Minus Low, representing the differential return based on the book-to-market ratio), and WML (Winners Minus Losers, indicating the differential return based on the previous year's performance), in the German stock market, covering the period from January 15, 2002 to January 15, 2021.

Of interest are the years around 2008 (financial crisis), when the momentum premium WML becomes strongly positive, and the market risk premium $RMRF$ becomes strongly negative. The observed negative correlation, which was also found in the cross-section, becomes particularly stunning and visible.

At the end of the time series, using the example of WML and $RMRF$, it is again interesting to see that, unlike at the beginning and middle of the time series, $RMRF$ increases and WML decreases accordingly.

[20] The profitability and investment factors from Fama and French (2015) were deliberately not taken into account because, on the one hand, they are hardly considered, even in current studies in the investor sentiment context (a rare example for a study where the new factors are applied is Habibah et al. (2021)) and, on the other hand, the new risk factors were not included, in order to maintain comparability with other studies on the German stock market, such as Ziegler et al. (2007) and Hanauer et al. (2013).

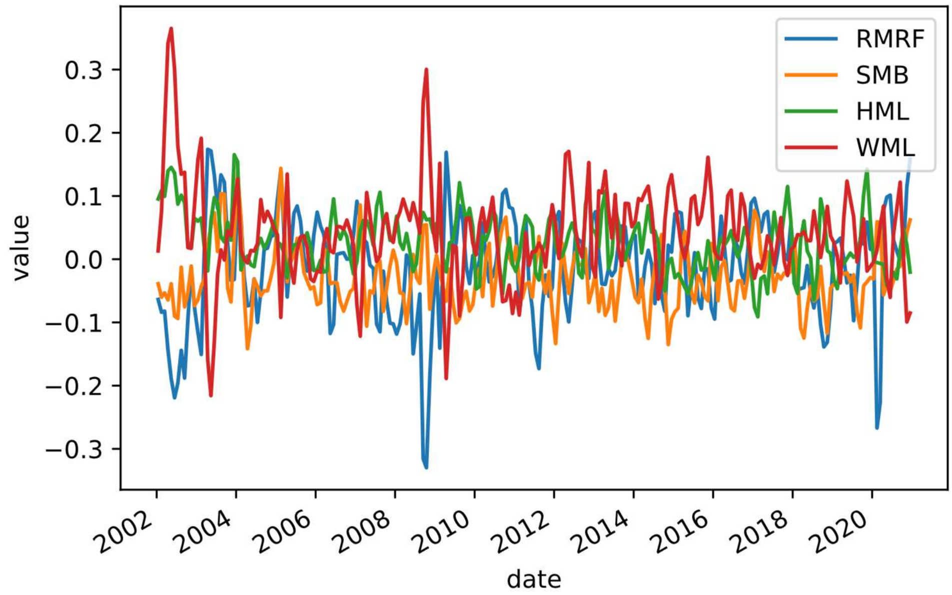

Figure 9: Cumulative Progression of the Four Carhart Risk Factors.

[Source: Author's representation]

4.3.3 Construction of the Investor Sentiment Risk Factor

A method of PCA was employed to incorporate an investor sentiment risk factor into the multi-factor models utilized in this study. This was accomplished by first ranking all shares included in the CDAX index in relation to their correlation with the primary factor gained from the PCA analysis that was performed using 73 different sentiment indicators sourced from both market-implied and survey-based investor sentiment datasets. To ensure consistency and comparability, all investor sentiment factors were determined using the same procedure outlined by Carhart (1997) and Hilliard et al. (2016, 2020). The specific sentiment indicators included in this study can be found in the appendix (Annex 1).

Based on the ranking of stocks according to their correlation with the principal component, three equally weighted portfolios were formed for each observation period.

These portfolios were defined based on the 10% and 90% quantiles and represent the return differential of the 10% of stocks that were most strongly or positively correlated with the principal component, the 10% of stocks that were weakest or negatively correlated, and a third portfolio representing the shares that were not or neutrally correlated with the sentiment source. These portfolios are designated as "High-Sentiment" (HS), "Low-Sentiment" (LS), and "Neutral-Sentiment" (NS), respectively.

At every point in time t, the sentiment measure was calculated as the difference in returns between the high sentiment (HS) and low sentiment (LS) portfolios.

This allows for the examination of the effect of investor sentiment on stock market returns. Even after inserting the investor sentiment-based risk factor (with and without lag), the risk factors remain largely uncorrelated when considering the Pearson correlation coefficient. (see Figure 10).

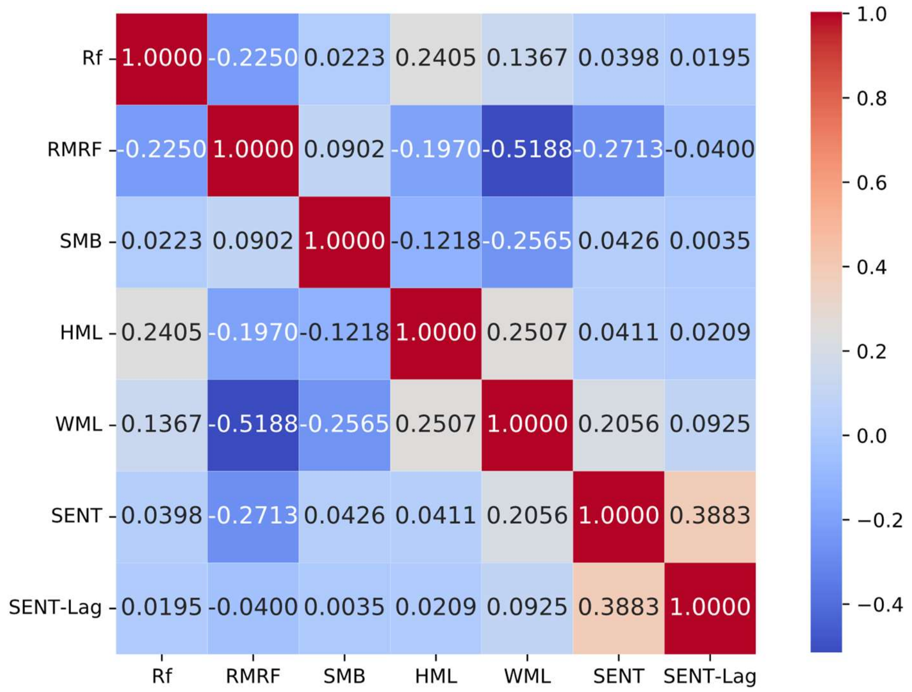

Figure 10. Correlation Matrix of the Multi-factor Model Components Including Investor Sentiment.

[Source: Author's representation]

The cumulative trend of the PCA-based sentiment risk factor also shows that it is not stable over time (see Figure 11). It is also interesting to note that the value appears to be elevated in the financial crisis period from 2008 onward and shows sharp swings at the end of the time series.

However, in a calm market environment, the investor sentiment risk factor appears to be relatively unspectacular. This can be taken as a first indication that such a risk factor can be very relevant, especially in times of crisis, while no clear signals can be perceived in typical market phases.

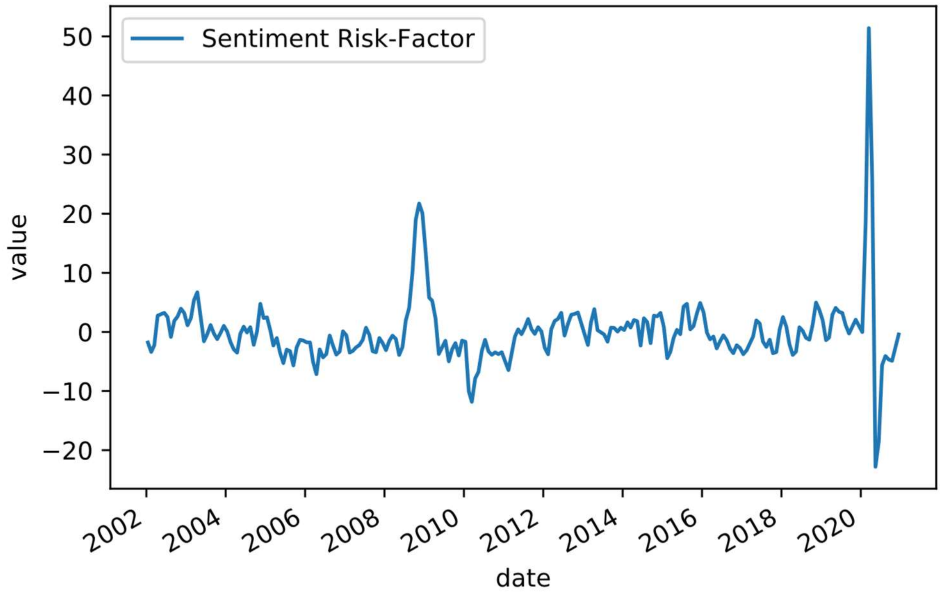

Figure 11. Cumulative Progression of the Four Carhart Risk Factors.

[Source: Author's representation]

4.3.4 Construction of the Fama-French Portfolios

In order to conduct the analysis, the construction of portfolios was initiated by taking into account factors such as market capitalization and book-to-market ratios. The returns generated by these portfolios were subsequently evaluated through linear regression analysis.

In accordance with the methodologies outlined by Ziegler et al. (2007) and Hanauer et al. (2013), 16 (= 4 x 4) Fama-French portfolios were created instead of 25, as proposed by Fama and French (1993). The division of stocks into quartiles based on market capitalization and the book-to-market ratio was utilized as the foundation for the formation of these portfolios.

This approach guarantees that each portfolio comprises of an adequate number of shares, thereby rendering the multi-factor models more analogous to other studies conducted on the German stock market. Descriptive statistics on the Fama-French portfolios are described and explained in Section 4.4.1. The 16 Fama-French portfolios are sorted based on their market value and their book-to-market value ratio in the following manner:

1-1 ("Small-Low"),… ,1-4 ("Small-High"),… ,4-1 ("Big-Low"),… ,4-4 ("Big-High").

4.3.5 Operationalization

The study conducts an examination of the ability of investor sentiment indicators that are based on market expectations to predict future returns within a one-month time horizon. This investigation is illustrated in Figure 12.

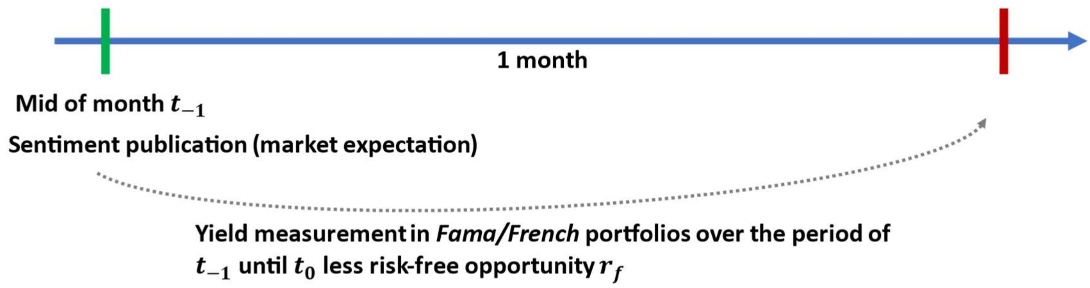

Figure 12. Regressive Investor Sentiment Analysis on a Monthly Basis.

[Source: Author's representation]

In the present analysis, it is presumed that an investor sentiment publication is released on the 15th of a given month, regardless of whether it is announced within the first half of the month under examination. This approach places a greater emphasis on the timing of the investor sentiment survey than on the timing of the publication; thus, it prioritizes the synchrony of the investor sentiment survey over the chronology of its dissemination.

Additionally, in order to examine the hypothesis that risk factors can explain share yields, all Carhart risk factors are integrated into the empirical CAPM, and the factor weights of the single-factor model outlined in Equation 12 are estimated for the 16 portfolios through the application of ordinary least squares regression based on the Capital Asset Pricing Model.

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + \varepsilon_{it} \tag{12}$$

In the next step, the single-factor model is expanded to incorporate the additional factors of Small Minus Big (SMB) and High Minus Low (HML) to form the Fama-French three-factor model, as outlined in Equation 13 in its empirically testable form.

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + \varepsilon_{it} \tag{13}$$

The four-factor model, as proposed by Carhart (1997), is presented in an empirical form in Equation 14. This model includes the incorporation of all four established risk factors in order to check the hypothesis that they can effectively explain share yields.

This represents the classic Fama-French three-factor model extended by an additional momentum risk factor WML.

$$\begin{aligned} R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + w_i \cdot WML_t \\ + \varepsilon_{it} \end{aligned} \tag{14}$$

The integration of the PCA-based investor sentiment factor into the multi-factor models proposed by Fama and French (1992, 1993) and Carhart (1997) is achieved through the use of linear multivariate regression models. Specifically, Equations 15 and 16 are employed in order to incorporate the sentiment factor, which is derived from 73 sentiment indicators from both survey-based and market-implied sentiment categories, into the three- and four-factor models, respectively.

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + \psi_i \cdot SENT_{PCA_t} + \varepsilon_{it} \tag{15}$$

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + w_i \cdot WML_t + \psi_i \cdot SENT_{PCA_t} + \varepsilon_{it} \tag{16}$$

In addition, there is an investigation, in which an investor sentiment-based factor lagged by one period is examined (Equations 17 and 18).

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + \psi_i \cdot SENT_{PCA_{t-1}} + \varepsilon_{it} \tag{17}$$

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + w_i \cdot WML_t + \psi_i \cdot SENT_{PCA_{t-1}} + \varepsilon_{it} \tag{18}$$

If the lagged investor sentiment risk factor also becomes significant in the Fama-French portfolios, it may indicate that investor sentiment information takes time to be priced into the stock market.

4.4 EMPIRICAL RESULTS

4.4.1 Descriptive Statistics of the Fama-French-Portfolios

The process of determining the empirical CAPM involves utilizing bivariate linear regression on the 16 Fama-French portfolios, while the multi-factor models based on the empirical CAPM are concluded by utilizing multivariate linear regression. The software R, in version 4.1.1 (2021), is utilized in combination with the packages "lm-test" by Zeileis and Hothorn (2002) in version 0.9-38, and the package "zoo" by Zeileis and Grothendieck (2005) in version 1.8-9, for the estimation and evaluation of the regression results. The coefficients and their significance are then analyzed.

Additionally, the adjusted coefficient of determination $\bar{R}^2$ is examined, whereby the calculation rule for the adjusted $\bar{R}^2$ (Equation 20) can be derived from that for R^2 (Equation 19) with

SSR: $Regression\ Sum\ of\ Squares,$

SST: $Total\ Sum\ of\ Squares,$

SSE: $Error\ Sum\ of\ Squares,$

N: $Number\ observations\ in\ sample,$

K: $number\ of\ independent\ regressors,$ that is, the number of variables in the model, excluding the constant.[21] Detailed information on global model goodness and inference can be found in Gehrke (2019).

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[21] The corrected coefficient of determination $\bar{R}^2$ is applied instead of the "normal" coefficient of determination R^2 to make different models with different numbers of independent variables more comparable (the "normal" R^2 cannot deteriorate with increasing numbers of independent variables; the $\bar{R}^2$'s "penalty" increases with the number of independent variables). Adding a new variable can improve the model in terms of $\bar{R}^2$ only if the additional explanatory content more than offsets the penalty term. Furthermore, $\bar{R}^2$ is applied to ensure better comparability with the work of Ziegler et al. (2007) and Hanauer et al. (2013).

$$R^2 = \frac{SSR}{SST} = \frac{\sum(\hat{y}_i - \bar{y})^2}{\sum(y_i - \bar{y})^2} = 1 - \frac{SSE}{SST} = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2} \tag{19}$$

$$\bar{R}^2 = 1 - \frac{\frac{SSE}{N} - K - 1}{\frac{SST}{N} - 1} = 1 - (1 - R^2) \cdot \frac{N - 1}{N - K - 1} \tag{20}$$

In this study, a thorough examination of the α-constant is conducted within the framework of regression analysis.

Utilizing the Durbin-Watson and Breusch-Pagan tests, instances of autocorrelation and heteroskedasticity are identified and taken into consideration in the computation of standard errors and the presentation of results.

To rectify for these issues, the Newey and West (1987) estimator of order p is employed in estimating standard errors, which accounts for both autocorrelation and heteroskedasticity.

Additionally, in line with the methodology employed by Hanauer et al. (2013) and Liew and Vassalou (2000), a total of three periods are chosen for the parameter p in order to maintain the comparability of findings. Statistical significance is determined by comparing the results to zero using a two-tailed t-test with a significance level of at least 10%.

Table 5 contains the monthly excess returns for the 16 Fama-French portfolios for the German stock market, covering the period from January 15th, 2002 to January 15th, 2021.

Table 5. Monthly Surplus Yields of the $i = 1, \dots, 16$ Fama-French Investment Portfolios.

Variables to be explained, i.e., excess returns $R_{it} - R_{ft}$ of the $i = 1, \dots, 16$ portfolios				
	Book-to-market ratio			
Market Value	1 (Low)	2	3	4 (High)
	Arithmetic mean (standard deviation)			
1 (Small)	−3.742%*** (8.986%)	−1.378%** (9.115%)	−0.618% (8.171%)	−3.137% *** (9.435%)
2	−2.756%*** (6.933%)	−0.572% (6.173%)	−0.101% (5.692%)	−0.413% (6.736%)
3	−1.081%*** (6.053%)	−0.187% (6.104%)	0.164% (5.559%)	0.592% (6.881%)
4 (Big)	0.023% (5.653%)	0.373% (5.441%)	0.603% (6.017%)	0.872% * (7.636%)

Significance levels: 10% (*), 5% (**), and 1% (***).

Note. The following table presents the data on the excess returns generated by the 16 portfolios defined by the Fama-French methodology for the German market, in addition to the corresponding standard deviations. Each year, starting in July, the stocks are divided into four distinct groups based on their market capitalization as of the end of June. Additionally, these stocks are further classified into four groups based on their book-to-market ratio as of the end of the previous year. The combination of these two classifications results in the formation of 16 distinct portfolios. It is important to note that all calculations utilized in this analysis were based on monthly returns from the CDAX index, spanning from the year 2001 to 2020.

[Source: Author's representation]

The findings indicate a significant spread in the average monthly excess yields, with values ranging from −3.742% (Portfolio 1-1) to 0.872% (Portfolio 4-4) in the cross-section. Other studies, such as Ziegler et al. (2007), Schrimpf et al. (2007), and Hanauer et al. (2013) also show a range of excess returns, with values ranging from 0.002% to 0.668%, −0.329% to 0.472%, and −0.656% to 1.094%, respectively. Additionally, Fama and French (1993) also reported a range of excess returns, with values between 0.32% and 1.02%.

It is noteworthy that the excess returns of the Fama-French portfolios exhibit significant yields in six out of the sixteen portfolios, as observed by Hanauer et al. (2013), who found the same phenomenon in three out of the sixteen portfolios.

Furthermore, an analysis of the standard deviations of the returns in comparison to the returns themselves via the use of Pearson's correlation coefficient reveals a negative correlation coefficient of -0.658, which contradicts the assumption that the correlation should be positive, as proposed by $\mu - \sigma$ theory.

This finding is in contrast to the results obtained by Hanauer et al. (2013) who did not detect any relationship between the two variables.

4.4.2 Empirical Multi-factor Models

The formal CAPM (Equation 21), according to Hanauer et al. (2013), is transformed into the empirical form (Equation 22).

$$E[r_i] - r_f = \beta_{CAPM,i} \cdot \left(E[r_m] - r_f\right) \tag{21}$$

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + \varepsilon_{it} \tag{22}$$

The data set shows that the average coefficient β_i is 1.036 and is statistically significant at the 1% level across all 16 portfolios. Additionally, the average $\bar{R}^2$ for the sample is 0.685. Detailed results can be obtained from Table 6.

Table 6. Linear Regressions of Monthly Surplus Yields of the i = 1,…,16 Fama-French Investment Portfolios (Empirical CAPM).

$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + \varepsilon_{it}$				
	Ratios of book value to market value			
	1 (Low)	2	3	4 (High)
Market Value	Approximated value for β_i			
1 (Small)	1.170***	1.005***	1.079***	1.274***
2	1.009***	0.977***	0.930***	1.059***
3	0.879***	1.032***	0.980***	1.119***
4 (Big)	0.903***	0.934***	1.017***	1.203***
	Corrected coefficient of determination $\overline{R}^2$			
1 (Small)	0.486	0.351	0.471	0.508
2	0.673	0.781	0.822	0.726
3	0.732	0.862	0.859	0.748
4 (Big)	0.720	0.805	0.759	0.655
Significance levels: 10% (*), 5% (**), and 1% (***).				

Note. This table documents the estimates of the regression analysis of the CAPM-compatible single-factor model. In addition to the coefficients of the $RMRF$ risk factor, the corrected coefficient of determination is shown.

[Source: Author's representation]

It is interesting to note that the values for the CAPM beta coefficient in the portfolios containing shares with an exceptionally high book-to-market ratio tend to be higher than in the other portfolios, averaging 1.164. This observation can presumably be explained by the fact that in the case of shares with a high book value compared with their market value (value stocks), general market risk has a powerful impact on the variance of returns.

Furthermore, it is interesting to note that the model goodness of fit (measured by the corrected coefficient of determination $\bar{R}^2$) seems to be lower, especially for the group of smallest stocks, compared to the groups with a higher value measured by their market capitalization.

This can possibly be interpreted as an indication that general market risk plays a less prominent role for small stocks than larger stocks when explaining stock returns.

As a result, it can also be observed that in the group of corporations with the highest market value, the average of the corrected coefficient of determination $\bar{R}^2$ is 0.725, which is considerably higher than the average of the total consideration (0.685). The returns of highly capitalized corporations are comparatively well explained by the simple CAPM model. In the group of the smallest companies, on the other hand, the corrected coefficient of determination $\bar{R}^2$ is only 0.454.

This is also consistent with the literature. Thus, Kumar and Lee (2006) and Barber and Odean (2008) suggest that small shares are often considered to have lower liquidity compared to larger shares. The widely cited work of Amihud (2002) showed that expected market illiquidity over time positively affects the ex-ante excess return of stocks, suggesting that the expected excess return of stocks is partly an illiquidity premium.

This complements the cross-sectionally positive relationship between returns and illiquidity. The Fama-French size risk factor should pick up this premium of smaller corporations.

To verify this, the formal (Lübbering et al. 2018, p. 10) Fama-French three-factor model (Equation 23) is converted into the empirical form (Equation 24) according to Hanauer et al. (2013).

$$E[r_i] - r_f = \beta_{FF,i} \cdot E[RMRF] + s_i \cdot E[SMB] + h_i \cdot E[HML] \tag{23}$$

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + \varepsilon_{it} \tag{24}$$

The average CAPM coefficient β_i in the Fama-French three-factor model sample is 1.033 and is statistically significant at the 1% level for all 16 portfolios. A small decrease is discernible compared with the empirical CAPM (1.036).

Despite this, the literature suggests that the CAPM coefficient should be $\beta_i = 1$. However, the empirical value of the Fama-French coefficient is slightly different and has marginally decreased compared to the single-factor model. Additionally, the coefficient of the size factor SMB averages out at -0.040 and is statistically significant at the 1% level in 11 out of 16 portfolios, and only statistically significant at the combined 5% and 10% levels in 4 out of 16 portfolios.

As expected, the size factor in the four portfolios with the smallest stocks measured in market capitalization is consistently positive and highly significant (the average in the four portfolios as mentioned above, is 0.771).

The size risk premium in the four portfolios with the largest stocks measured by market capitalization is consistently highly significant and negative, averaging -0.603. The phenomenon can also be observed in the work of Hanauer et al. (2013).

The average coefficient for the HML value factor is -0.041 and it is only statistically significant in eight out of the 16 portfolios analyzed. However, the value-risk factor HML is consistently highly significant and negative in the four portfolios with the lowest book-to-market ratios.

In the stated four portfolios, an average coefficient of -0.400 is obtained. The opposite picture emerges in the four portfolios with the highest book-to-market ratio. The coefficient consistently shows a positive trend, with an average of 0.280; however, it is only statistically significant from zero at the 1% level in two out of the four portfolios. The study of Hanauer et al. (2013) draws a similar picture concerning the algebraic sign.

The $\bar{R}^2$ for the multi-factor model averages at 0.765, which is, as expected, a higher value than that of the single-factor model.

Compared with the simple CAPM, the average coefficient for the group of smallest companies is now 0.600 (instead of 0.454 in the CAPM). The multi-factor model thus manages to explain the returns of smaller companies much better. Detailed results can be obtained from Table 7.

Table 7. Linear Regressions of Monthly Surplus Yields of the i = 1,…,16 Fama-French Investment Portfolios (Empirical Three-factor Model).

$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + \varepsilon_{it}$				
	Ratios of book value to market value			
	1 (Low)	2	3	4 (High)
Market Value	Approximated value for β_i (CAPM)			
1 (Small)	1.022***	0.938***	1.074***	1.289***
2	0.976***	0.970***	0.928***	1.067***
3	0.871***	1.055***	1.000***	1.143***
4 (Big)	0.909***	0.964***	1.059***	1.265***
	Approximated value for s_i (SMB)			
1 (Small)	1.061***	0.566***	0.286**	1.169***
2	0.094	-0.093*	-0.087**	0.120*
3	-0.356***	-0.357***	-0.318***	-0.306***
4 (Big)	-0.602***	-0.520***	-0.600***	-0.688***
	Approximated value for h_i (HML)			
1 (Small)	-0.896***	-0.369**	0.078	0.647***
2	-0.263***	-0.101	-0.058	0.120
3	-0.232***	0.056	0.042	0.086
4 (Big)	-0.207***	0.050	0.216***	0.267***

	Corrected coefficient of determination $\overline{R}^2$			
1 (Small)	0.781	0.417	0.482	0.720
2	0.689	0.784	0.825	0.730
3	0.794	0.909	0.900	0.773
4 (Big)	0.865	0.919	0.884	0.765
Significance levels: 10% (*), 5% (**), and 1% (***).				

Note. This table documents the regression analysis estimates of the Fama-French-compatible three-factor model. In addition to the coefficients of the $RMRF$ risk factor, the size and book/market ratio factor, as well as the corrected coefficient of determination, are shown.

[Source: Author's representation]

The four-factor model, as outlined in (Lübbering et al. 2018, p. 12) and attributed to Carhart (1997) , is represented in Equation 25.

$$E[r_i] - r_f = \beta_{Carhart,i} \cdot E[RMRF] + s_i \cdot E[SMB] + h_i \cdot E[HML] + w_i \cdot E[WML] \tag{25}$$

By utilizing the method outlined by Hanauer et al. (2013) to convert Equation 25 into its empirical form, Equation 26 can be derived.

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + w_i \cdot WML_t + \varepsilon_{it} \tag{26}$$

In the empirical Carhart model, the average CAPM ($RMRF$) coefficient β_i is 1.050 and it is statistically significant at the 1% level in all 16 portfolios. It is notable that the market risk coefficient in this model deviates further from the ideal value of 1 compared to the CAPM and the three-factor model.

Additionally, the size factor SMB coefficient averages out at -0.029 and is statistically significant in 14 out of the 16 portfolios. The picture already observed in the Fama-French three-factor model of a highly significant and consistently positive coefficient in the quartile of the smallest firms by market capitalization remains.

The picture is also strengthened in the quartile with the most prominent corporations by market capitalization, where the coefficient is consistently negative and significantly different from zero at the 1% level.

In comparison, the coefficient of the value factor HML averages out at -0.010 and is significantly different from zero in only 10 of the 16 portfolios. Nevertheless, the picture already observed in the Fama-French three-factor model of a highly significant and consistently negative coefficient in the quartile of companies with the lowest book-to-market ratio (growth stocks) persists.

Similarly, the picture strengthens in the quartile with the companies with the highest book-to-market ratio (value stocks). A consistently positive coefficient can be observed in these portfolios, which, unlike in the Fama-French three-factor model, is significantly different from zero in three out of four portfolios.

The coefficient of the momentum factor WML, which is now being considered for the first time, averages out at 0.041 and is significant in 11 of the 16 portfolios. Of great interest is that the quartile with the corporations that have the highest ratios of book-to-market value (value stocks) consistently has a negative coefficient that is significantly different from zero. This is possibly an indication that the momentum effect in the cross-section works for value stocks with opposite signs, and poorly performing value stocks perform well according to the law of regression to the long-run mean.

$\bar{R}^2$ averages out at 0.774, which is slightly higher compared to the three-factor model in this study, apart from the fact that Carhart risk factors can explain a high degree of variation, as already shown in other studies. If only the eight portfolios containing the largest companies by market value are considered here, the corrected coefficient of determination $\bar{R}^2$ is 0.861 on average, which leaves hardly any room for further improvement.

Therefore, it can be concluded that the returns of large companies, in particular, can be explained quite well by the Carhart factors. Looking at the eight portfolios with the smallest companies in terms of market value, the average corrected coefficient of determination is still 0.688, which is nonetheless a creditable value. Detailed results can be obtained from Table 8.

Table 8. Linear Regressions of Monthly Surplus Yields of the i = 1,…,16 Carhart Investment Portfolios (Empirical Four-factor Model).

$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + w_i \cdot WML_t + \varepsilon_{it}$				
	Ratios of book value to market value			
	1 (Low)	2	3	4 (High)
Market Value	Approximated value for β_i (CAPM)			
1 (Small)	1.017***	1.084***	1.055***	1.142***
2	1.075***	1.032***	0.936***	0.986***
3	0.977***	1.124***	1.016***	1.080***
4 (Big)	0.985***	1.007***	1.076***	1.213***
	Approximated value for s_i (SMB)			
1 (Small)	1.058***	0.656***	0.254**	1.077***
2	0.155**	-0.054	-0.082*	0.069
3	-0.290***	-0.314***	-0.309***	-0.345***
4 (Big)	-0.555***	-0.493***	-0.589***	-0.720***
	Approximated value for h_i (HML)			
1 (Small)	-0.893***	-0.443***	0.088	0.722***
2	-0.313***	-0.132**	-0.063	0.162**
3	-0.286***	0.021	0.034	0.118
4 (Big)	-0.246***	0.028	0.117**	0.294***
	Approximated value for w_i (WML)			
1 (Small)	-0.011	0.344***	-0.046	-0.346***
2	0.234***	0.146***	0.020	-0.191***
3	0.250***	0.163***	0.037	-0.148***
4 (Big)	0.180***	0.103***	0.041	-0.124*

	Corrected coefficient of determination $\bar{R}^2$			
1 (Small)	0.780	0.438	0.480	0.740
2	0.708	0.793	0.824	0.742
3	0.826	0.921	0.900	0.780
4 (Big)	0.881	0.924	0.885	0.768
Significance levels: 10% (*), 5% (**), and 1% (***).				

Note. This table documents the regression analysis estimates of the Carhart-compatible four-factor model. In addition to the coefficients of the $RMRF$ risk factor, the size and book/market ratio factor and the momentum factor, as well as the corrected coefficient of determination, are shown.

[Source: Author's representation]

4.4.3 Sentiment-factor Integration

Incorporating a sentiment factor derived from PCA into the Fama-French three-factor model resulted in an average CAPM beta coefficient of 1.028, which is significant for all 16 portfolios and has the closest value to 1 among the other models. The size factor had an average coefficient of -0.038, which is slightly closer to zero than in the original three-factor model.

Similarly to the three-factor model, the size-coefficient s_i is found to be significant in 14 portfolios. The observation from the other multi-factor models of consistently highly significant positive premium for the smallest firms measured by market value and consistently highly significant negative premium for the largest firms measured by market value remains. Consequently, the size factor is highly persistent in the present sample.

The value factor h_i has a coefficient of -0.041, which remains the same as in the three-factor model. Nine out of the 16 portfolios show a significant difference from zero for the factor coefficient, an increase of one portfolio compared to the three-factor model. Again, the persistence of the value factor is confirmed, which shows highly significant negative premiums in the quartile of portfolios that have low book-to-market ratios (growth stocks) and a consistently positive premium in the quartile of companies with the highest book-to-market ratios (value stocks).

As in the other models, the value premium is not as pronounced for the actual value stocks as for the growth stocks. This can be seen because only two of the four coefficients are significantly different from zero.

The coefficient for sentiment factor ψ_i has an average of $-2.67 \cdot 10^{-4}$ and is statistically significant in seven portfolios. The average for the $\bar{R}^2$ is 0.767, which is slightly higher than the original three-factor model (0.765). However, it is still lower than in a similar Carhart four-factor model (0.774).

Interestingly, the quartile with the corporations that have the highest book-to-market ratio has the highest rate of significant portfolios, three out of four. In particular, companies with high book-to-market ratios (value stocks) that are also part of the quartile of the smallest companies measured by market value are significantly explained by the investor sentiment risk factor.

This is unusual, as the companies concerned are few and far between and do not fit into a traditional category. Small companies tend to be growth stocks, that is, with a low book-to-market ratio, while value stocks tend to be long-established on the market and have already left the phase of strong corporate growth behind them. Detailed results can be obtained from Table 9.

Table 9. Linear Regressions of Monthly Surplus Yields of the i = 1,…,16 Fama-French Investment Portfolios (Investor Sentiment Enhanced Empirical Three-factor Model).

$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + \psi_i \cdot SENT_{PCA_t} + \varepsilon_{it}$				
	Ratios of book value to market value			
	1 (Low)	2	3	4 (High)
Market Value	Approximated value for β_i (CAPM)			
1 (Small)	1.039***	0.928***	1.028***	1.349***
2	0.960***	0.940***	0.911***	1.066***
3	0.883***	1.050***	0.990***	1.118***
4 (Big)	0.933***	0.959***	1.055***	1.239***

	Approximated value for s_i (SMB)			
1 (Small)	1.056***	0.569***	0.302***	1.148***
2	0.099	-0.082	-0.082*	0.120*
3	-0.360***	-0.355***	-0.315***	-0.298***
4 (Big)	-0.611***	-0.518***	-0.599***	-0.679***
	Approximated value for h_i (HML)			
1 (Small)	-0.896***	-0.369**	0.077	0.649***
2	-0.263***	-0.101*	-0.059	0.120
3	-0.232***	0.056	0.042	0.085
4 (Big)	-0.207***	0.050	0.126***	0.267***
	Approximated value for ψ_i ($SENT_{PCA}$)			
1 (Small)	0.001	-0.001	-0.002**	0.003***
2	-0.001	-0.002***	-0.001*	0.000
3	0.001	0.000	-0.001	-0.001**
4 (Big)	0.001***	0.000	0.000	-0.001*
	Corrected coefficient of determination R_k^2			
1 (Small)	0.781	0.415	0.490	0.733
2	0.690	0.793	0.827	0.728
3	0.795	0.909	0.901	0.777
4 (Big)	0.871	0.918	0.884	0.768
Significance levels: 10% (*), 5% (**), and 1% (***).				

Note. This table documents the regression analysis estimates of the Fama-French-compatible three-factor model enhanced by a PCA-based investor sentiment risk factor. In addition to the coefficients of the $RMRF$ risk factor, the size and book/market ratio factor, the corrected coefficient of determination, and the sentiment factor are shown.

[Source: Author's representation]

In summary, the findings indicate that incorporating the PCA-based investor sentiment risk factor into the three-factor model results in a slight enhancement of the model when compared to the traditional Fama-French model; however, it does not surpass the model performance of the four-factor model as proposed by Carhart.

Incorporating the investor sentiment risk factor into the four-factor model, as proposed by Carhart, resulted in an average CAPM beta factor of 1.045 for all 16 portfolios. This value is statistically significant at a 1% level and indicates that the CAPM beta is slightly lower than in the original four-factor model, which had a value of 1.050.

The average of the size factor coefficient is -0.026 and it is closer to zero compared to the original Carhart model (-0.029). Similar to the Carhart four-factor model, the coefficient of SMB is significant in 14 portfolios.

The observation that the size risk factor in the quartile of the four portfolios with the smallest companies in terms of market capitalization represents a consistently positive and highly significant coefficient is still observable. Meanwhile, a diametrical picture is visible in the quartile of the four portfolios with the largest companies in terms of market capitalization. The coefficient here is consistently highly significant and negative.

In 10 out of the 16 portfolios, the coefficient of the value factor h_i, also known as the factor coefficient of HML, is found to be significantly different from zero, similar to the original Carhart model. However, the coefficient in this case is -0.050, which is lower in comparison to the -0.010 value found in the Carhart model.

Again, the observation that the value risk factor in the quartile of the four portfolios with the highest quotients of the book-to-market ratio (value stocks) represents a consistently positive and highly significant coefficient in three out of four portfolios is still observable.

Likewise, conversely, a diametrical picture is visible in the quartile of the four portfolios with the companies that have the lowest quotients of book-to-market value ratios. The coefficient here is consistently highly significant and negative.

The momentum factor's coefficient is 0.044, which is slightly higher compared to the Carhart model's coefficient of 0.041.

It is interesting to observe here, as in the previous models with momentum factor, that in the quartile with the four portfolios containing companies with the highest ratios of book-to-market value, the coefficient is negative in each case and significantly different from zero in three out of four cases.

The average value of the investor sentiment factor coefficient, ψ_i, is $-3.15 \cdot 10^{-4}$. This factor is found to be significant in nine portfolios, indicating an increase in negativity and importance when compared to the extended three-factor model.

As before, the highest highly significant value is found in the noteworthy portfolio, which, on the one hand, contains the intersection of companies with high quotients of book and market value (value stocks) and, on the other hand, contains the smallest companies.

The $\bar{R}^2$ value of the modified four-factor model, which includes the sentiment risk factor determined through principal component analysis, is slightly higher at 0.777 compared to the original Carhart model's $\bar{R}^2$ value of 0.774.

This suggests that the integration of the sentiment risk factor improves the overall performance of the model. Detailed results can be obtained from Table 10.

Table 10. Linear Regressions of Monthly Surplus Yields of the i = 1,…,16 Carhart Investment Portfolios (Investor Sentiment Enhanced Empirical Four-factor Model).

$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + w_i \cdot WML_t + \psi_i \cdot SENT_{PCA_t} + \varepsilon_{it}$				
	Ratios of book value to market value			
	1 (Low)	2	3	4 (High)
Market Value	Approximated value for β_i (CAPM)			
1 (Small)	1.031***	1.070***	1.018***	1.198***
2	1.057***	1.005***	0.923***	0.989***
3	0.982***	1.117***	1.007***	1.062***
4 (Big)	1.002***	1.002***	1.072***	1.194***

	Approximated value for s_i (SMB)			
1 (Small)	1.050***	0.665***	0.296**	1.045***
2	0.165**	-0.038	-0.074*	0.067
3	-0.293***	-0.310***	-0.304***	-0.335***
4 (Big)	-0.565***	-0.489***	-0.587***	-0.709***
	Approximated value for h_i (HML)			
1 (Small)	-0.891***	-0.445***	0.083	0.730***
2	-0.316***	-0.136**	-0.065	0.162**
3	-0.285***	0.020	0.033	0.115
4 (Big)	-0.244***	0.027	0.116**	0.291***
	Approximated value for w_i (WML)			
1 (Small)	-0.019	0.352***	-0.024	-0.378
2	0.244***	0.162***	0.028	-0.193***
3	0.247***	0.168***	0.042	-0.138**
4 (Big)	0.170***	0.106***	0.044	-0.113*
	Approximated value for ψ_i (SENT)			
1 (Small)	0.001	-0.001	-0.002**	0.004***
2	-0.001*	-0.002***	-0.001**	0.000
3	0.000	0.000	-0.001*	-0.001*
4 (Big)	0.001***	0.000	0.000	-0.001*

	Corrected coefficient of determination R_k^2			
1 (Small)	0.780	0.437	0.488	0.757
2	0.710	0.804	0.827	0.741
3	0.826	0.921	0.901	0.782
4 (Big)	0.885	0.924	0.884	0.770
Significance levels: 10% (*), 5% (**), and 1% (***).				

Note. This table documents the regression analysis estimates of the Carhart-compatible four-factor model enhanced by a PCA-based investor sentiment risk factor. In addition to the coefficients of the RMRF risk factor, the size and book/market ratio factor, the corrected coefficient of determination, and the sentiment risk factor are shown.

[Source: Author's representation]

An examination of the integration of the investor sentiment risk factor with a one-period lag reveals a noteworthy level of significance in six of the 16 portfolios (seven out of 16 without time-lag) within the Fama-French model. The CAPM ($RMRF$) beta coefficient β_i averages out at 1.031 and stays significantly different from zero to the 1% level in all 16 Fama-French portfolios. This is slightly further from the ideal of 1, compared to the study of the investor sentiment risk factor without lag, where the value is 1.028. Unlike in the non-lagged extended three-factor model, the size coefficient s_i is significantly different from zero in 13 portfolios (previously 14).

However, the observation from the other multi-factor models that the premium is consistently highly significantly positive for the smallest firms as measured by market value and consistently highly significantly negative for the largest firms as measured by market value remains. The coefficient of the value factor h_i is -0.039, which corresponds to a slight meltdown compared to the three-factor model with non-lagged investor sentiment risk factor.

The factor coefficient is significantly different from zero in eight (previously: nine) of the 16 portfolios. Again, the persistence of the value factor is confirmed, with highly significant negative premiums in the quartile of portfolios with low book-to-market ratios (growth values) and a consistently positive premium in the quartile of companies with the highest book-to-market ratio values.

As with the other models, the value premium is not as pronounced for the actual value stocks as for the growth stocks. This can be seen from the fact that only two of the four coefficients still deviate significantly from zero.

The sentiment risk factor coefficient ψ_i has an average of $-2.20 \cdot 10^{-4}$ and is found to be significantly different from zero in six portfolios, down from seven. The average $\bar{R}^2$ is 0.766, which is slightly higher than the original three-factor model (0.765), but lower than a similar model with a non-lagged investor sentiment risk factor (0.767). Detailed results can be obtained from the Appendix (Annex 2).

Including the lagged risk factor of investor sentiment into the empirical four-factor model resulted in an average CAPM beta factor of 1.049, which is statistically significant at the 1% level across all 16 portfolios. This indicates that the CAPM beta coefficient is slightly higher when incorporating the investor sentiment risk factor with a lag, compared to the four-factor model without it (1.045).

The coefficient on the size factor s_i is -0.030 on average, slightly further from zero in the empirical Carhart model extended by a non-lagged investor sentiment risk factor (-0.026). Unchanged, the coefficient of SMB is significantly different from zero in 14 portfolios. The observation that the size risk factor is a consistently positive and highly significant coefficient in the quartile of the four portfolios with the smallest companies by market capitalization remains. In the quartile of the four portfolios with the largest companies in terms of market capitalization, an opposed picture emerges. The coefficient here is consistently highly significant and negative.

In the modified Carhart model, the impact of the value factor h_i on the portfolio's returns is not trivial, as it statistically differs from zero in 11 out of the 16 portfolios. The coefficient for this factor is -0.048, which is relatively closer to zero compared to the model that includes a non-lagged investor sentiment risk factor (-0.050). Again, it can be observed that the value risk factor in the quartile of the four portfolios with the highest ratios of the book-to-market ratio (value stocks) has a consistently positive and highly significant coefficient in two out of four portfolios.

Similarly, a diametrical picture emerges in the quartile of the four portfolios with the companies that have the lowest book-to-market ratio values. Here, the coefficient is consistently highly significant and negative.

The weight assigned to the momentum factor w_i is 0.041, which is in line with the average value found in the original Carhart model.

It is interesting to note here, as in the previous models with momentum factor, that in the quartile with the four portfolios containing companies with the highest book-to-market ratios, the coefficient is negative in every case and significantly different from zero in all cases.

However, upon incorporating the lagged risk-factor investor sentiment into the Carhart four-factor model, only five out of the 16 Fama-French portfolios exhibit a statistically significant deviation from zero for the risk-factor investor sentiment coefficients, as opposed to the previous nine portfolios.

This can be seen as an indicator that investor sentiment has to accept losses in terms of the explanatory contributions of returns after a certain amount of time. Detailed results can be obtained from the Appendix (Annex 3).

The findings of this study indicate that investor sentiment plays a role in predicting future returns. However, it is also apparent that incorporating the sentiment risk factor into the cross-section of the model does not have a large impact on the overall performance of the model.

When the investor sentiment risk factor is integrated in lagged form, model performance degrades slightly and the number of significantly non-zero coefficients decreases. Therefore, the corresponding pricing over time or an increasing irrelevance of investor sentiment over time can be assumed, although it must be clearly stated that this is a cross-sectional study.

Since the structure of the study results does not differ significantly between the non-lagged investor sentiment risk factor and the lagged investor sentiment risk factor, the remainder of this analysis will mainly refer to the study with the non-lagged investor sentiment risk factor.

Similar to Hilliard et al. (2020), the preliminary conclusion is that the investor sentiment risk factor is significant despite relatively small explanatory contributions in standard asset pricing models.

The findings indicate that the Fama-French and Carhart risk factors play a significant role in accounting for variances in returns on the German stock market.

4.5 REGRESSION DIAGNOSTICS

4.5.1 Risk Premia

The data presented in Table 11 illustrates the statistical characteristics of the five risk factors studied, namely those put forth by Carhart and investor sentiment. The table also illustrates the correlation coefficients between these factors and the returns yielded by the stock market portfolio R_m and a risk-free investment R_f. Furthermore, the table contains the means and standard deviations of the risk factors being analyzed, as well as the Pearson's correlation coefficients.

Table 11. Descriptive Statistics of the Monthly Sample Variables.

	Mean	SD	Pearson's correlation coefficient ρ				
			$RMRF$	SMB	HML	WML	$SENT$
R_m	-0.505%	5.195%					
R_f	0.096%	0.129%					
$RMRF$	-0.601%	5.223%	1				
SMB	-1.431%	3.476%	0.090	1			
HML	1.357%	2.960%	-0.197	-0.122	1		
WML	2.056%	4.711%	-0.519	-0.257	0.251	1	
$SENT$	-0.433%	356.050%	-0.271	0.043	0.041	0.206	1

Note. This table displays the descriptive statistics for the risk premiums associated with the Carhart factors in the German stock market, specifically the $RMRF$ ($R_m - R_f$, excess return of the equity market portfolio), SMB (Small Minus Big, differential return based on market capitalization), HML ("High Minus Low," differential return based on book-to-market ratio), WML ("Winners Minus Losers," differential return based on prior-year performance), and $SENT$ ("Investor Sentiment," differential return based on stock correlation to the investor sentiment principal component). Furthermore, the returns and their corresponding standard deviations of the equity market portfolio R_m and the risk-free investment R_f are also presented. The data utilized in these calculations were derived from monthly CDAX returns from the period of 2001 to 2020.

[Source: Author‘s representation]

Over the period spanning from January 2002 to January 2021, the stock market portfolio exhibited an average monthly return of -0.505% when accounting for excess returns, and -0.601% when not accounting for excess returns. Statistically, this deviation from zero is deemed significant at the 1% level.

The 0.096% average monthly risk-free rate is considered statistically significant at a 1% level of significance. Among the three risk factors, SMB, HML, and WML, the momentum effect has the highest average value at 2.056% per month. The size effect has a negative premium of -1.431%, while the value effect has a positive premium of 1.357%. All risk factors, except for $SENT$, are statistically significant. The investor sentiment factor has a negative premium of -0.433%, which is not statistically significant. Additionally, the high standard deviation in investor sentiment indicates a high level of volatility.

4.5.2 Analysis of the α-Constant and GRS Test

Subsequently, the analysis proceeds to the examination of the estimates for the constants α_i and their corresponding levels of significance. The results of the following section may be interpreted as follows: In an idealized model, all the variance of the model is explained by the independent variables that are applied. However, if independent variables are missing from the model (omitted variables), they may be (partially) reflected in a constant ordinate intercept (here: autonomous return). Furthermore, if a missing variable correlates with one of the independent variables (here: risk factors), as a result, the residuals correlate with the independent variables. This results in a skewed or distorted estimate of the independent variables (Gehrke 2019).

As the number of unexplained constants increases and they are close to zero, the models align more closely with CAPM and the Fama-French and Carhart models. This was evaluated by analyzing the α_i values in both singular and combined settings.

The joint test, also known as the GRS statistics, utilizes the F-test proposed by Gibbons et al. (1989), which examines the significance of all α_i within a set of portfolios against the hypothesis that they are all equal to zero.

It is important to investigate whether the α-constants are substantially different from zero, as in an idealized scenario, no constants should exist, as all return components are explained by the risk factors (as discussed in Chapter 2). The presence of α_i constants that are significantly different from zero may imply the existence of autonomous return components that are not explained by the risk factors employed as independent variables.

The figures for the intercept alpha in the single-factor model that align with the Capital Asset Pricing Model (CAPM) vary from -2.896% to 1.510% (according to Hanauer et al. (2013), the range is -1.050% to 0.562%). A total of eleven constants (Hanauer et al. (2013) states it's three in his sample) are found to be significantly different from zero. This outcome implies that it may be beneficial to expand the CAPM.

The range of a_i for the three-factor model becomes even more narrowed, falling between -1.687% and 0.832% as compared to Hanauer et al. (2013) who reported a range of -0.513% to 0.537%.

The results of the joint GRS test indicate that the null hypothesis of all alpha values being equal to zero ($\alpha_i = 0\ \forall i$) is rejected for the models that are proposed by Fama and French (1992, 1993) and Carhart (1997), suggesting that there is potential for additional explanatory power in not yet applied risk factors. Moreover, the current study determined that the assumption $\alpha_i = 0\ \forall i$, in the GRS statistic, is not supported by the data in any of the sentiment risk-factor extended three- and four-factor models.

However, the t-test also reveals that there are six and seven portfolios with significant alpha values in the three- and four-factor models, respectively. As stated, this result suggests that there may be autonomous return components in the alpha values that are not explained by the independent variables, indicating a potential need for additional risk factors to be considered.

The extended Fama-French model with the $SENT$ factor has the lowest alpha range, spanning from -1.680% to 0.831%. This finding unequivocally illustrates the positive impact of the investor sentiment risk factor on the overall model quality. Despite the limitations identified by the GRS test, the integration of this risk factor into the model yields a substantial improvement in performance.

The results of this analysis indicate that the inclusion of the investor sentiment risk factor into the extended three- and four-factor models does not lead to a deterioration in model performance. In fact, a relatively similar number of portfolios exhibit significant alpha values, with six portfolios in the extended model compared to seven in the original Carhart model. These findings suggest that the factors currently employed in the model do not fully account for all return components, leaving room for further improvement in the model through the incorporation of additional risk factors.

4.5.3 Diagnostics for Multicollinearity

In order to assess the potential existence of multicollinearity resulting from the correlation between explanatory variables, a variance inflation factor (VIF) analysis is employed in this study. Specifically, when analyzing the monthly sample, the correlation between $\rho_{RMRF,WML}$ is of particular interest, as it exhibits a Pearson's correlation coefficient of -0.519. The results of the VIF analysis indicate that the hypothesis of multicollinearity is not supported in any of the models under examination. As a general rule of thumb, multicollinearity is assumed to be present when VIF values exceed 4; however, this is not the case in any of the models considered in this study.

4.5.4 Test for Misspecification

The RESET tests, developed by Ramsey (1969) were conducted on all the models examined in order to assess whether there were any specification errors present. In this study, all models were evaluated using the RESET tests for fitted values and the second to the fourth power as proposed by Ramsey (1969). The null hypothesis (H_0) posits that there is no specification error present, and if the F-test statistic is significant at a level of $p < 0.1$, the null hypothesis is rejected in favor of the alternative hypothesis (H_1), which asserts the presence of specification error. The results of this evaluation are presented in Table 12.

Table 12. Outcomes of the RESET Test for the Monthly Data Set.

Model	Regression models that exhibit indications of specification errors
1F-Model	11/16
3F-Model	9/16
3F+SENT	9/16
4F-Model	5/16
4F+SENT	4/16

Note. This table illustrates the examination of various models and the frequency at which the RESET test suggests the presence of inaccuracies in the specifications of each of the 16 regression analyses. A notable trend of decreasing likelihood of misspecification is observed as an increasing number of risk factors are utilized to explain the respective excess returns.

[Source: Author's representation]

The incorporation of the investor sentiment risk factor, $SENT$, into multi-factor models has been shown to result in a reduction of the misspecification rate when compared to the Carhart model. This finding suggests that the high misspecification rate in the Carhart model may be due to non-linear correlations, as evidenced by the observations in Tiwari et al. (2018). Therefore, it can be inferred that the integration of the investor sentiment risk factor into Carhart's four-factor model can effectively mitigate the misspecification rate. Overall, the integration of the investor sentiment risk factor appears to improve the robustness and accuracy of the models.

4.6 INTERIM RESULTS

According to traditional theory, the market reaches equilibrium when rational investors compete and diversify their portfolios. This results in share prices reflecting the discounted, rationally predicted cash flows. As per this theory, the expected returns of different stocks are only affected by the systematic risks present (Gomes et al. 2003).

Even though there is an understanding of specific risk premiums in the stock market and overall equity risk premiums, the CAPM is still a commonly used standard and foundation for multi-factor models. It is used to calculate the cost of equity in uncertain situations Ziemer (2018). However, recent research has suggested that sentiment can be turned into risk factors and explain variations in yields. This present study also supports this idea to some extent for the German stock market. The main findings are summarized in Table 13.

Table 13. Summary of Key Findings.

Model	$\bar{R}^2$	α -Range	Sig. α	Sig. ψ	$\emptyset\ \psi$
1F-Model	0.685	0.0441	11/16	./.	./.
3F-Model	0.765	0.0252	6/16	./.	./.
3F+SENT	0.767	0.0241	7/16	7/16	−0.027%
4F-Model	0.774	0.0251	7/16	./.	./.
4F+SENT	0.777	0.0243	7/16	9/16	−0.032%

Note. This table summarizes the key results of this empirical study. It shows the average level of fit for each model examined, as measured by the adjusted $\bar{R}^2$ coefficient and the corresponding range of alpha values. The table also displays the percentage of Fama-French portfolios that exhibit significant alpha under different scenarios. Additionally, for models that include a sentiment risk factor, the table illustrates the frequency and average values of significance.

[Source: Author's representation]

The results of this study suggest that investor sentiment can play a role in explaining stock market returns through the use of cross-sectional linear regression analyses. The examination of various sources of investor sentiment revealed that sentiment appears to have an influence on large cap shares. The findings of this study indicate that it is possible to convert abstract concepts, such as aggregate investor sentiment, into measurable factors that can aid in explaining yields and enhancing the goodness of fit of a model. However, it is important to note that the contribution of investor sentiment risk factors to the overall explanatory power of yields is relatively small, particularly when compared to the established Fama-French and Carhart risk factors.

However, the regression diagnostics also show that the model quality is improved with respect to the Ramsey (1969) RESET misspecification rate.

The Fama-French three-factor model demonstrates a significant improvement in explaining the variance in cross-sectional returns compared to the CAPM, with an $\bar{R}^2$ of 0.765 compared to 0.685 for the CAPM. Additionally, the inclusion of the momentum factor in Carhart's (1997) four-factor model leads to a slight increase in the model's explanatory power, resulting in an $\bar{R}^2$ of 0.774.

Integrating the investor sentiment factor into the Carhart model can further increase the model's goodness of fit ($\bar{R}^2 = 0.777$).

The results of this study support research hypothesis $\boldsymbol{RH_1}$ and demonstrate that the models of the APT that incorporate the Fama-French and Carhart factors are more effective in explaining stock market returns in the German market than the CAPM. This is evident in the results obtained from the analysis.

$\boldsymbol{RH_1}$: *Investor sentiment contributes to explaining return variances in the German stock market.*

The investor sentiment risk factor lagged by one period was able to show significant coefficients in considerably fewer Fama-French portfolios.

The practical implications of the findings of this study are limited in nature, and further research is required in order to establish a more robust understanding of the significance of investor sentiment factors in the German stock market. In particular, additional studies with larger sample sizes and different time frames will be necessary to confirm the validity of the results.

Additionally, it is crucial to determine the optimal duration of the time lag between the detection of sentiment and the implementation of investment decisions, as highlighted by Sul et al. (2017).

In order to further evaluate the results, backtesting techniques can be employed to validate trading strategies. However, it is important to exercise caution when incorporating multiple risk factors within a single regression model, as there exists the potential for an exacerbation of correlation during extreme market conditions, thus undermining the diversification benefits sought.

As an interim result, this part of the study demonstrates that integrating an investor sentiment risk factor into multi-factor models is achievable and satisfactory. The investor sentiment risk factor generated through Principal Component Analysis fulfills the necessary criteria for logical inclusion within multi-factor models and consistently exhibits a negative return premium.

This empirical study lends credence to the research hypothesis $\boldsymbol{RH_2}$ that a inverse relationship exists between the sentiment risk factor and future yields, as demonstrated by the cross-sectional analysis conducted.

$\boldsymbol{RH_2}$: *Investor sentiment is a contra-indicator for stock market developments.*

The level of fit of this sentiment enhanced cross-sectional model, however, falls short of that exhibited by a comparable Carhart model. The examination of the premia associated with the investor sentiment risk factor yielded a negative result, which is consistent with previous research, thereby reinforcing the existing hypothesis that investor sentiment acts as a counter-indicator (Du and Hu 2018, p. 207).

The examination of the VIF diagnostic revealed no signs of multicollinearity among the independent variables. Additionally, the RESET test showed no indication of increased inaccuracies in the specification of the sentiment enhanced cross-sectional model in comparison to the three- and four-factor models.

The hypothesis $\boldsymbol{RH_3}$ is only moderately substantiated as the observed advancements in model quality are minimal or insignificant. Nevertheless, the underlying effect of investor sentiment on excess returns in the German stock market can be established, which is in alignment with recent international studies (Gutierrez and Perez-Liston 2021; Al-Nasseri et al. 2021; Zaremba et al. 2020).

$\boldsymbol{RH_3}$: *The integration of an investor sentiment risk factor into multi-factor models leads to a higher model quality compared to the Fama-French and Carhart target portfolio regression models, expressed by the adjusted coefficient of determination* $\bar{R}^2$.

The methodology employed in this investigation for the inclusion of sentiment factors in multi-factor models presents a beneficial enhancement in contrast to traditional portfolio theory.

However, it can be clearly shown that the alpha range in the model extended by investor sentiment risk factors is smaller than in comparable models and, also, the significant rate of the alpha constants did not rise. $\boldsymbol{RH_4}$ can, therefore, be considered confirmed.

$\boldsymbol{RH_4}$: *Incorporating an investor sentiment risk factor into multi-factor models leads to a lower alpha range in the Fama-French and Carhart target portfolio regression models.*

Despite this study and other investigations on the German stock market, the definitive confirmation of the significance of investor sentiment risk factors can only be established through cross-country studies. However, the substantial pertinence of the Fama-French and Carhart factors for the German stock market can be empirically demonstrated. They continue to exhibit a substantial enhancement in model quality in comparison to the CAPM.

Particularly striking is the finding that both the size factor SMB and the value factor HML show very persistent and primarily highly significant results in their respective quartiles, which are hardly influenced by an investor sentiment risk factor.

Here, one might have suspected that the investor sentiment risk factor would at least provide some explanatory contribution to the size risk factor since investor sentiment could presumably play a greater role for these more difficult-to-value shares. This cannot be confirmed in this cross-sectional study. Since long-term cross-sectional data are observed here, this observation can clearly indicate profitable trading utilizing long-term investment in value stocks.

In conclusion, the main findings of this study, which extend beyond the testing of research hypotheses, are succinctly reiterated:

Firstly, the study contributes to the existing academic literature by utilizing a unique set of indicators to evaluate investor sentiment in the German stock market. By employing a Principal Component Analysis-based investor sentiment risk factor that combines both survey-based and market-implied sentiment, it addresses the limitations of previous studies and provides a more comprehensive assessment of investor sentiment.

Secondly, the research demonstrates the impact of investor sentiment as a risk factor on stock returns, even when accounting for other commonly recognized risk factors such as size, value, and momentum.

Thirdly, this study demonstrates that investor sentiment plays a noteworthy role in explaining stock yields in the German market.

Additionally, when evaluating these findings in relation to a broader perspective, it becomes apparent that the EMH cannot be entirely validated by this empirical data. This is due to the fact that, based on the information currently available in the market, the returns of future periods can be partially predicted using systematic methods.

However, the older the investor sentiment is that is taken into account, the poorer the explanatory power, which is clearly evident in the cross-section when examining the investor sentiment risk factor lagged by one period.

Finally, the findings of this research have practical implications for stakeholders in the German stock market. Investors can leverage the statistical significance of the variables in the research models as a benchmark for selecting equities to invest in, thus providing valuable information for investment decision-making.

Furthermore, portfolio- and risk managers can expect that high investor sentiment is likely to have a negative effect on future share prices and vice versa.

Nevertheless, the indication from the literature (see Chapter 3) suggests that even better results could be obtained if the time-varying component of investor sentiment is captured in a model.

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5 LONG SHORT-TERM MEMORY-BASED STUDY

In this second of three empirical studies of this dissertation on investor sentiment in the German stock market, the most critical shortcomings of the first multi-factor model-based study are addressed through an artificial RNN with LSTM neurons. Moreover, unlike in the previous study, investor sentiment is considered to explain other risk measures in addition to German stock market yields (Hövel and Gehrke 2019, 2020, 2022b).

In this section of the study, the approach is to consider portfolio management risk measures, such as variance and higher-order statistics of the return distribution, such as skewness and kurtosis, in addition to return to explain and predict using investor sentiment.

Yu et al. (2013) already considered risk as a measure of performance in addition to returns in their research. Since the previous study already showed that investor sentiment seems to be particularly suitable in times of crisis and strong market fluctuations (see Chapter 4.3.3), in which market movements cannot always be explained by fundamental data alone, the present study focuses on the COVID-19-induced price decline on the German stock market in 2020. It uses out-of-sample tests to validate the empirical investor sentiment-based predictability.

5.1 INTRODUCTION TO THE EMPIRICAL ANALYSIS

The unprecedented economic disruptions caused by the COVID-19 pandemic have strengthened interest in research on the influence of investor sentiment on capital markets (Mensi et al. 2020; Jiang et al. 2021; Singh and Yadav 2021). However, the efficient market hypothesis (EMH), which has traditionally been the foundation of academic literature and economic models, has been called into question by recent empirical studies, particularly in the context of the US market (Baker and Wurgler 2006, 2007; Malandri et al. 2018). The EMH was initially propounded by Fama (1970) but has since been challenged by the aforementioned research (see Chapter 2.2).

Previous chapters have acknowledged the growing body of research that is uncovering limitations and shortcomings in markets, such as arbitrage limits, and also casting doubt on the assumption of rational investors (Kahneman and Tversky 1979; Lakonishok et al. 1994). The efficient market hypothesis (EMH) holds that only new information can cause changes in market prices. However, the emergence of new discoveries that challenge prevailing paradigms also presents opportunities for the advancement of novel explanatory frameworks.

While the EMH, along with the assumptions of rationally acting investors and the random walk theory (Fama 1995), previously precluded the possibility of accurate and reliable trend forecasts in the share market, recent studies that rely on empirical evidence have demonstrated increasing predictability of returns (Gao and Liu 2020; Yousra Trichilli et al. 2020; Zaremba et al. 2020; Al-Nasseri et al. 2021; Szczygielski et al. 2021).

The COVID-19 pandemic has led to substantial movements in international stock markets, and this second empirical study specifically examines the German stock market as a case in point. Germany, as the EU's most robust exporting economy, is highly dependent on global developments such as the COVID-19 pandemic. Due to its focus on exports and consistent trade surpluses, the German economy is particularly susceptible to changes in future economic expectations, as reflected in investor sentiment. The COVID-19 pandemic has also caused disruptions in the German stock market, particularly in 2020, making it a relevant subject for research in this context.

Previous research has shown that investor sentiment factors are fundamentally valuable for explaining differences in returns in multi-factor models on the German stock market (Hövel 2018; Shen et al. 2018; Hövel and Gehrke 2022a).

Recent years have seen the publication of critical evaluations of the applicability of neural networks in explaining statistical moments related to investor sentiment, such as the work by Malandri et al. (2018). These innovative methodological approaches, including the use of artificial neural networks, have yielded promising results.

Specifically, the use of Long Short-Term Memory (LSTM) neurons is considered highly effective in this scenario, as they have the ability to identify the expected dynamic effects of investor sentiment on stock market yields (Nakagawa et al. 2019; Li et al. 2020).

The second part of this empirical study shows the initial findings of this analysis on the German stock market, utilizing LSTM neurons during the market turmoil caused by the COVID-19 pandemic as an out-of-sample test period. Given that previous studies have observed the impact of investor sentiment on stock returns in international markets and specifically the German stock market (Hövel 2018), this study aims to demonstrate that an LSTM-based neural network can be employed to forecast trends in the German stock market during the COVID-19 pandemic.

In contrast to the cross-sectional study discussed in detail in Chapter 4, this study also takes into account the time-varying influence of investor sentiment. What renders the volatility instigated by Covid-19 particularly intriguing for research on investor sentiment is that the abrupt price decline brought about by the pandemic was unprecedented in that it did not persist and was instead succeeded by an impromptu and robust recovery driven by investor sentiment and optimism.

Furthermore, this research not only delves into the reasons behind the price drops brought about by COVID-19, but it also pioneers in identifying and forecasting other vital risk indicators for portfolio management, such as portfolio volatility and asymmetry.

The results of the out-of-sample tests indicate that in addition to providing an excellent explanation of the dependent variables, the trend predictions also appear to be accurate.

Lower and higher-order statistics play a crucial role in assessing financial risk (Ebrahimi and Pirrong 2018; Kim et al. 2018; Khan et al. 2020). Kim et al. (2018) argue that the "risk" of a portfolio comprises of three components: variance, skewness and kurtosis. Most previous studies have focused only on variance. Skewness is a risk that is considered particularly challenging to diversify. Khan et al. (2020) confirm that portfolios optimized by incorporating skewness and kurtosis are more sustainable and significantly different from portfolios optimized for mean-variance, which have asymmetric risk and fat-tail risk.

This study provides substantial validation for the hypothesis that these events can be attributed to investor sentiment. Nevertheless, a significant portion of the kurtosis in the return distribution remains unexplained by the study.

5.2 TIME-VARYING RISK PREMIA

To better understand the study design, it is crucial to describe the study's underlying assumptions about the relationship between return and risk. It can be stated that the concept of return is not directly inferred from underlying data such as commercial profits or projected cash flows, but rather is remuneration for the ability to assume risk (see also Chapter 2.1).[22] Portfolio risks include variances and higher statistical moments, such as skewness and kurtosis.[23]

Aside from the traditional and widely accepted risk measurements, such as variance, standard deviation, shortfall risk, and so on, there is also the subjective perception of risk that is influenced by factors like time, context and the investor's perspective, which could have implications for the established risk metrics.

Furthermore, risk tolerance varies over time and is referred to as "time-varying risk premia" (Kommer 2018; Chaieb et al. 2018). Thus, return and risk are linked, so the risk is a compelling condition for return.

This principle holds true for both individual investors as well as the market as a whole. Quantifiable and objective characteristics of securities, known as risk premia, have been demonstrated to possess a strong correlation to the past and prospective yields and risks of various asset classes.

Regression analysis can be utilized to evaluate the impact of specific factors on market or portfolio returns and determine the probability that the observed causal relationship is not random.

[22] Although not every risk on the stock market is compensated by returns (see Chapter 2.4).
[23] Other risk metrics, such as Value-at-Risk and Expected Shortfall are also applied.

This method of analysis can account for a large proportion of the yield disparities between the portfolio under examination and the benchmark in diversified portfolios. The most widely recognized risk factors, which have undergone the most extensive empirical testing, include the size effect, which refers to the return premium that smaller stocks, as measured by market capitalization, exhibit relative to larger stocks. This phenomenon was also clearly evident in the empirical study presented in Chapter 4.

Furthermore, there is solid data that confirms the presence of a value effect, where value stocks, which have a high book-to-market ratio, tend to yield higher returns than growth stocks, which have a low book-to-market ratio (Cakici and Topyan 2014). This effect is also strongly present in the German stock market, as the results in Sections 4.4.2 and 4.4.3 show.

Furthermore, the momentum effect illustrates the inclination for stocks to continue their favorable or unfavorable performance compared to the overall market. The previous study also showed that this effect seems less meaningful on the German stock market than the size and value risk factor. In Chapter 4.3.2, it can also be seen that the risk factors are not stable over time. In addition, there are other risk factors not considered in the models, such as profitability and liquidity.[24]

Recent studies have demonstrated that investor sentiment is a significant risk factor that can be integrated into multi-factor models (Hövel and Gehrke 2022a). In the context of the German stock market, it has been observed that investor sentiment can contribute more meaningfully to the explanation of returns than the momentum factor in certain instances (Hövel 2018).

[24] Whereby it must be noted that these risk factors may be part of other risk factors. The size risk factor, for example, which states that smaller companies pay a return premium, is certainly also due to the fact that shares in smaller companies are generally less liquid. As such, it can be inferred that the size premium encompasses an illiquidity premium as well.

As risk factors are nation-specific, partially undiscovered, and subject to change over time (Merville and Xu 2002), this study employs the use of a neural network to predict future moments of yield distributions based on current investor sentiment. By utilizing the weightings generated by the neural network, it is possible to theoretically identify the prevailing factors at the time of observation. Figure 13 illustrates the assumed interrelationships.

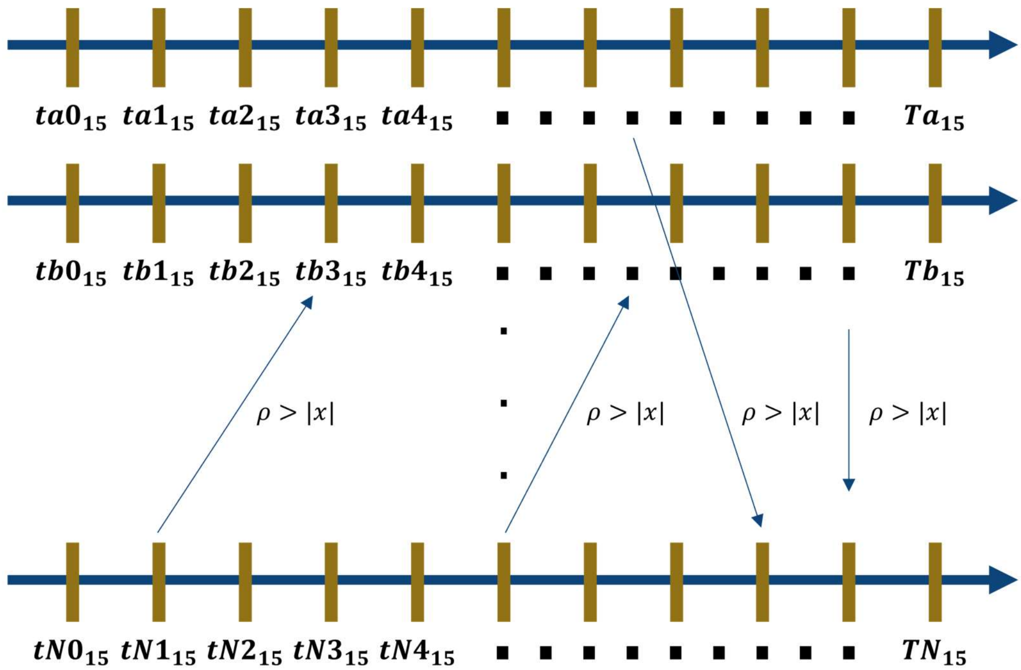

Figure 13. Intertemporal Dependencies Among Investor Sentiment Indicators.

[Source: Author's representation]

Figure 13 illustrates the concept of time-varying risk premia based on various hypothetical investor sentiment indicators that form temporary correlations. These dependencies may also manifest with a delay.

If time-varying risk premia are taken into account, it is advisable not to perform a dimensional reduction via a PCA, for example, as in the study from Chapter 4. This makes the calculation of such a model significantly more complex.

5.3 RELEVANCE OF HIGHER-ORDER STATISTICS

When it comes to distributing investments within a portfolio, assessing financial risk involves utilizing measures such as higher statistical moments and variance (Ebrahimi and Pirrong 2018; Kim et al. 2018; Khan et al. 2020). Auer (2015) notes that in recent years researchers and practitioners have developed new performance measures for mutual fund portfolios that take into account mean, variance, and the higher moments of the return distribution (skewness, kurtosis). The concept of skewness is presented in Figure 14.

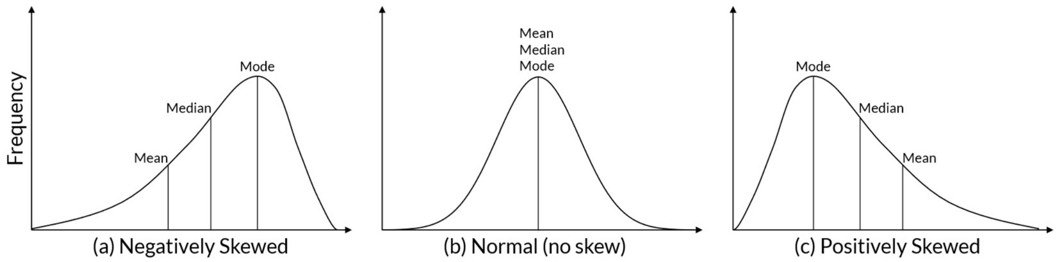

Figure 14. Density Plots of Skewed Distributions.

[Source: Author's representation]

Skewness is a measure of the asymmetry of a unimodal probability distribution from the Gaussian normal distribution (Grimmer 2014). According to Fechner's position rule for distributions (Eckey et al. 2008), the following relationship regularly applies:

$negatively\ skewed: \bar{x} < \tilde{x}_{0,5} < \bar{x}_{mod} \qquad positively\ skewed: x_{mod} < \tilde{x}_{0,5} < \bar{x}.$

While a symmetric distribution always has a skewness of 0, right skewed (left skewed) distributions are positive (negative). In terms of return distributions, a right-skewed distribution represents the probability of many losses and some gains, with potential losses being low and potential gains being high. This distribution is also frequently observable in gambling and lotteries.

The (excess) kurtosis, meanwhile, indicates the curvature of a distribution. A distinction is made between positive (leptokurtic distribution) and negative (platykurtic distribution) kurtosis. The kurtosis is the fourth central moment of the statistics. The kurtosis of a normal peaked (mesokurtic) distribution amounts to 3. With the excess kurtosis, the mesokurtic distribution is normalized on the value 0.

Accordingly, an excess kurtosis with a value greater than 0 is leptokurtic and a negative sign is platykurtic (see Figure 15). While platykurtic distributions scatter relatively uniformly, the scatter in leptokurtic distributions results more from rare but extreme events (Eckstein et al. 1994). The formulas used in each case can be found in the Appendix (Annex 4).

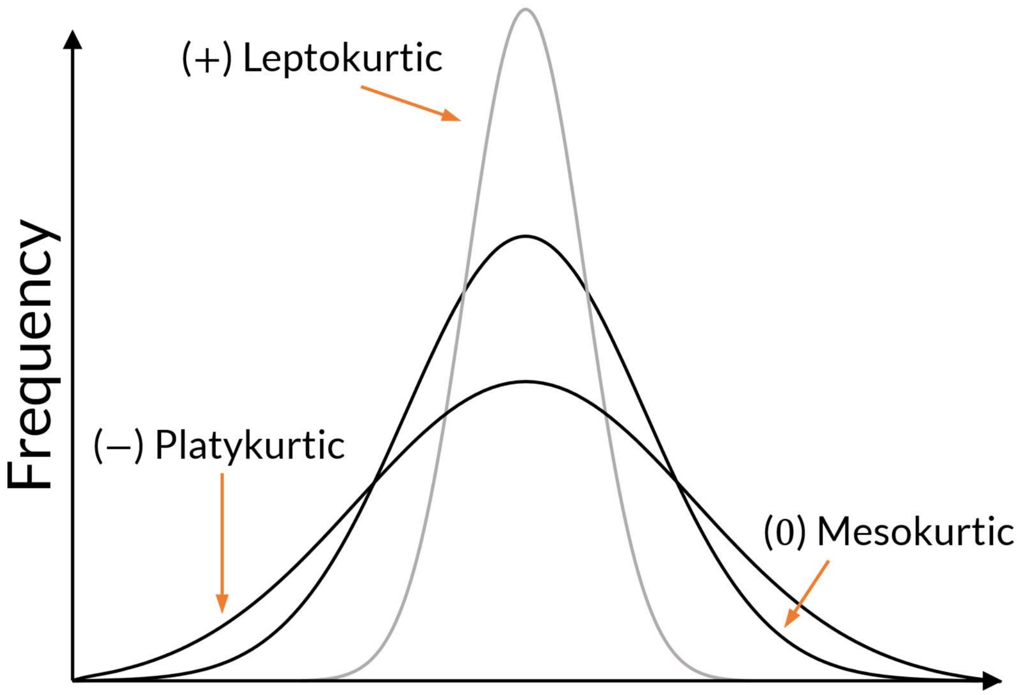

Figure 15. General Forms of Excess Kurtosis.

[Source: Author's representation]

The use of the Sharpe ratio is not appropriate in cases where empirical return data exhibits high skewness and kurtosis as it violates the theoretical conditions required for its application, as argued by certain authors. An interesting presentation of these higher-order risk measures in the context of portfolio management can be found in Bergh and van Rensburg (2008).

In this context, Gormsen and Jensen (2022) find that even and odd higher-order moments are strongly negatively correlated, leading to periods when the return distribution is riskier because it is more left-skewed and has a fat tail.

Such higher moments risk is negatively correlated with past variance and returns, which means that it peaks in quiet times. Based on the explanations presented, it can be inferred that the utilization of higher statistical moments, such as kurtosis, serve as valuable indicators of risk in the realm of portfolio- and risk management.

Likewise, recent evidence suggests that consideration of higher statistical moments shows further insight. In their study, Agarwalla et al. (2021) show how COVID-19 affects the higher statistical moments in the Indian market.

They have observed that while fiscal and monetary policy actions have contributed to a near-normalization of skewness and kurtosis, volatility levels have remained elevated. This highlights the significance of considering higher statistical moments in capturing uncertainty during a pandemic.

In particular, there may be a link to investor sentiment in consideration of skewness, as there is empirical evidence for investors' skewness preference for portfolio returns, although it appears to be irrational. Zhang (2013b) notes, in addition to the problem that investors tend to misprice securities due to fundamental cognitive biases, such as overconfidence, that risks vary over time (time-varying risk premia). One of the key discoveries in his research is that skewness exhibits a notable, negative correlation with subsequent stock returns. Conversely, no significant correlation between kurtosis and mean return has been established.

One of the significant findings in relation to the connection between investor sentiment and skewness in this context is that companies with relatively extreme positive (negative) skewness in their return distributions are more likely to be found among stocks with high (low) prices relative to their fundamentals (Zhang 2013b).

These stocks with high skewness are also called glamour stocks, while those with negative skewness are regularly classified as value stocks. It has been previously established that the degree of skewness in a stock return distribution reflects the level of its asymmetry, with positive (negative) skewness indicating a longer or thicker right (left) tail, representing the likelihood of exceptionally high gains (losses) in the share.

Zhang's analysis also illustrates that a considerable portion of the premium (discount) investors apply to glamour stocks (value stocks) is a result of investors' preference for positive skewness in the return distribution. A similar inclination towards positive skewness has also been observed in consumer behavior in relation to lottery purchases and gambling activities, according to Zhang (2013b).

The behavior is irrational, as although investors prefer glamour stocks, that is, stocks with positive skewness, in their return distribution, the literature and empirical evidence shows that value stocks generate higher returns than glamour stocks (in the long run). Furthermore, the prospect theory posits that investors place a greater emphasis on the extremes of the return distributions (Tversky and Kahneman 1992). This preference for skewness is in line with the idea that investors have inverse S-shaped utility functions (see Figure 16).

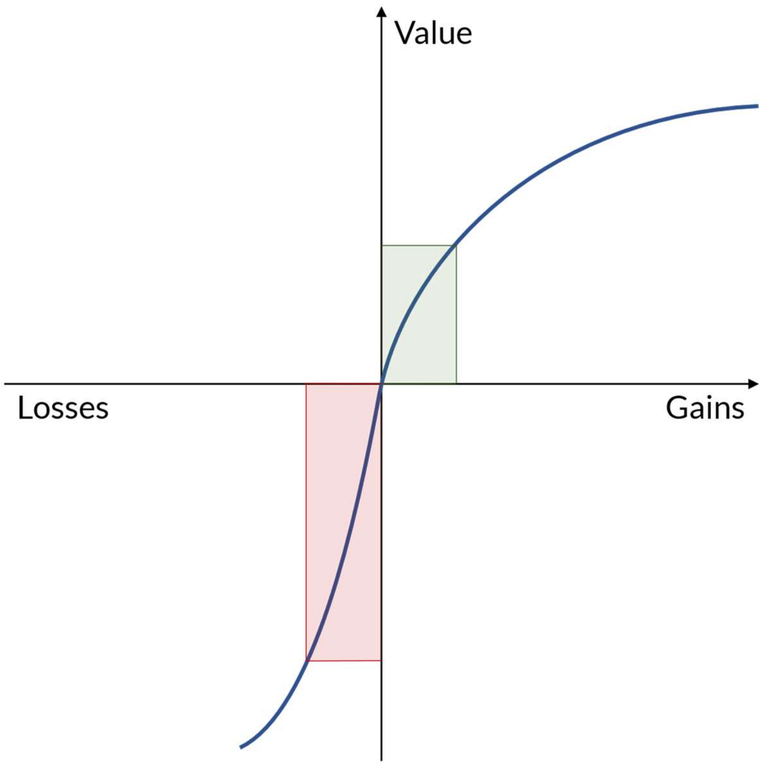

Figure 16. Inverse S-utility Function with Preference for Positive Skewness.

[Source: Author's representation based on Kahneman and Tversky 1979, p. 279]

The utility function passes through the origin, is S-shaped, and weights losses more heavily than gains. The probability weighting function expresses that probable outcomes are underweighted and improbable outcomes are overweighted.

Prospect theory posits that investors attach greater importance to the extremes of a return distribution. Zhang (2013b) notes that the causal research for irrational investor behavior (affinity for positive skewness) requires a change from the expected value-variance model of asset valuation. All previous research on this subject has focused on examining individual behavior in lottery games and gambling, and it has been consistently observed, as mentioned, that investors have a preference for right-skewed return distributions.

Nevertheless, research has implemented skewness affinity variously in stock pricing models. A number of studies introduce skewness-affine utility functions with the assumption that all investors are risk-averse and consequently hold diversified portfolios. In such a scenario, only the co-skewness is priced.[25] Another set of studies substitutes the assumption of universal risk aversion with the implementation of convexity in the utility function.

Behind this is Kahnemann and Tversky's assumption that increasing wealth beyond a specific limit can substantially increase utility. An example of this would be a situation in which an individual's wealth level is tangent to the mountain base of the utility function, and investment provides the opportunity for a substantial increase in utility without excessive downside risk. Then, the individual's affinity to invest increases. The individual will not diversify (completely) as this would reduce the skewness effect.

Barberis and Huang (2008) show that some investors in equilibrium do not fully diversify, but hold more stocks with positive return distribution skewness.

[25] Co-skewness measures the skewness of the portfolio's return distribution relative to the return distributions of the individual assets. It captures the non-linear relationship between the returns of the portfolio and the returns of the individual assets.

The model assumes that the more positive the return distribution skewness, the lower, ceteris paribus, the average return. Zhang (2013b) approaches the topic by linking investor preference for skewness to risk-taking behavior inherently.

Since higher statistical moments are relatively less frequently the subject of empirical studies than, for example, variance as a measure of risk, this part of the empirical study also examines them in the context of investor sentiment for the German stock market. The results are promising and represent a valuable research contribution to the academic community.

5.4 DEVELOPMENT OF THE RESEARCH HYPOTHESIS

Given the temporal variation of risk premia and the implication of higher statistical moments, such as skewness and kurtosis, in portfolio management, this empirical study utilizing LSTM methodology, as a sub-study within the broader dissertation, aims to investigate the following research hypothesis:

$\boldsymbol{RH_5}$: *By taking the time-varying characteristics of investor sentiment into account, the explanatory power of investor sentiment increases perceptibly compared to traditional cross-sectional analyses.*

Moreover, in addition to the research hypothesis, the predictability of the dependent variables is also examined, which seems to make a good impression in the chosen COVID-19 scenario, especially in terms of predictive power in trend forecasting.

5.5 EMPIRICAL ANALYSIS

5.5.1 Database

The focus of this study is to examine the monthly returns of a German CDAX equities portfolio that is equally weighted. The CDAX's makeup is evaluated every month to guarantee that there is no survivorship bias, in contrast to other research.

In this research, the investor sentiment indicators adopted are derived from the framework proposed by Finter et al. (2012) and supplemented with a subset of risk indicators from Refinitiv's Key Indicator List for Germany, resulting in a total of 73 monthly sentiment indicators used as independent variables. The data, including specific information obtained from the open time-series database of the Deutsche Bundesbank, is detailed in Annex 1.

As previously established in the first empirical study of Chapter 4, the sentiment indicators utilized in this research exclusively comprise of survey-based and market-implied sentiment categories.

Similar to the first study in Chapter 4, the yields and associated statistical moments are calculated from mid-month to mid-month. This has to do with the fact that this procedure guarantees that the investor sentiment publications do not lie so far back in time. Calendar effects at the beginning or end of the month are also minimized as possible confounding factors.

As previously discussed, the CDAX is an index that is computed by Deutsche Börse, which includes all shares listed on the Frankfurt Stock Exchange in the General Standard and Prime Standard.

The CDAX represents a substantial part of the German market capitalization, excluding a limited number of shares listed on local stock exchanges. It is worth mentioning that the total number of shares listed on the CDAX has been declining in recent times, as shown in Table 1 in Section 2.3.

The present study employs an equal-weighted CDAX index to represent smaller stocks, which according to Baker and Wurgler (2007) are more sensitive to changes in investor sentiment. This observation is supported by the findings of Kumar and Lee (2006) and Barber and Odean (2008) who suggest that small stocks have lower liquidity, making them more susceptible to changes in investor sentiment. This study examines the period from March 2006 to March 2020, spanning a total of 169 months. On average, each period takes into account 537 individual shares.

5.5.2 Descriptive Statistics

An examination of the monthly log return for the equally weighted CDAX portfolio reveals a decline of -0.3115 within the time frame of February 15, 2020 to March 15, 2020. The massive drop in prices is subsequently attributed to the rapid spread of COVID-19.[26]

Simultaneously, the variance of the portfolio experiences an uptick, rising from 0.0677 to 0.1204 in comparison to the preceding period. Additionally, the skewness of the return distribution within the portfolio increases from 0.6601 to 4.1543 in relation to the previous period. Furthermore, the kurtosis of the portfolio also increases from 54.1343 to 77.6483. The computations for the individual moments of the portfolio can be found in the appendix (Annex 4).

This COVID-19-induced event is predicted in this study as an out-of-sample test. However, training data is needed to build the prediction model presented in this descriptive statistics chapter. The statistical moments of the CDAX return distribution are computed on a monthly basis. Figure 17 illustrates the distribution of return data for the purpose of this study. In the subsequent summary, all statistical moments for the whole period under investigation are described using histograms and boxplots.

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[26] For the purpose of consistency and ease of interpretation within the context of the scales employed in the figures, this section of the thesis opts to abstain from using percentage notation for all values presented.

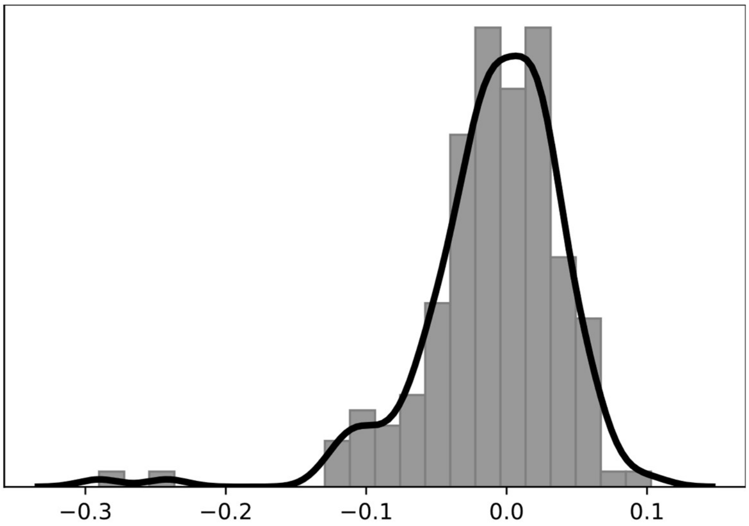

Figure 17. Distribution of the First Moment.

[Source: Author's representation]

With reference to the distribution of the first moment (as depicted in Figure 17), which reflects the distribution of the monthly returns, it can be observed that the distribution has a mean of −0.0092, a variance of 0.0025, a skewness of −1.7889, and a kurtosis of 7.1536. Also of interest is the corresponding boxplot (Figure 18), which shows that outliers are mainly to be found in the negative yield range.

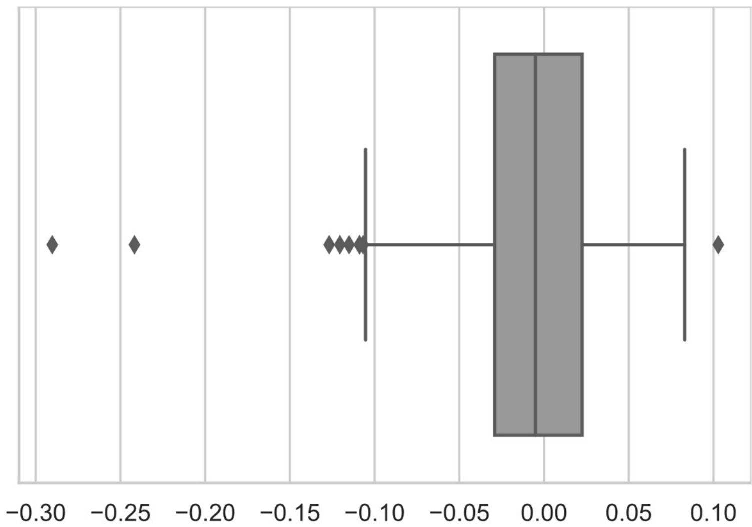

Figure 18. Boxplot of the First Moment.

[Source: Author's representation]

In contrast, the second moment (as displayed in Figures 19 and 20) reveals the following values: Mean 0.0328, variance 0.0003, skewness 2.4898, and kurtosis 9.5424. It is important to note, however, that from a technical standpoint, negative variance is not a viable concept.

Outliers in the variance band are, thus, only visible in the strong positive range.

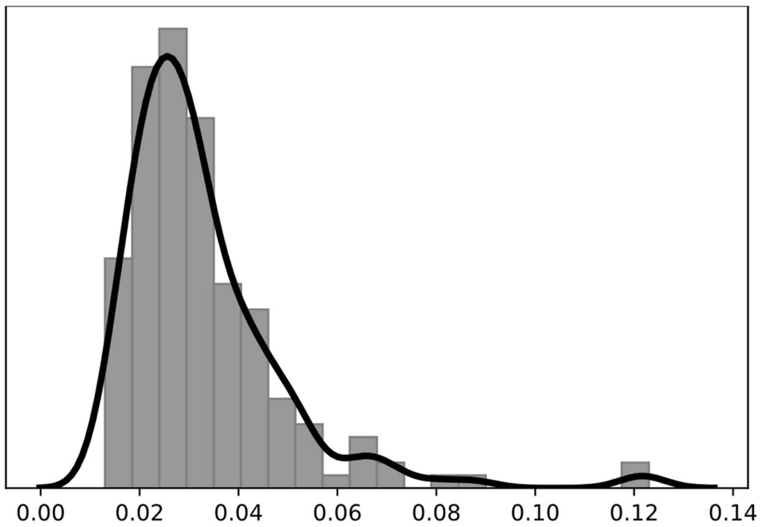

Figure 19. Distribution of the Second Moment.

[Source: Author's representation]

It is uncommon to observe exceptionally high variances, which are typical of stock market returns. Additionally, as depicted in Figure 19, a relatively high kurtosis can be observed, resulting in a leptokurtic distribution.

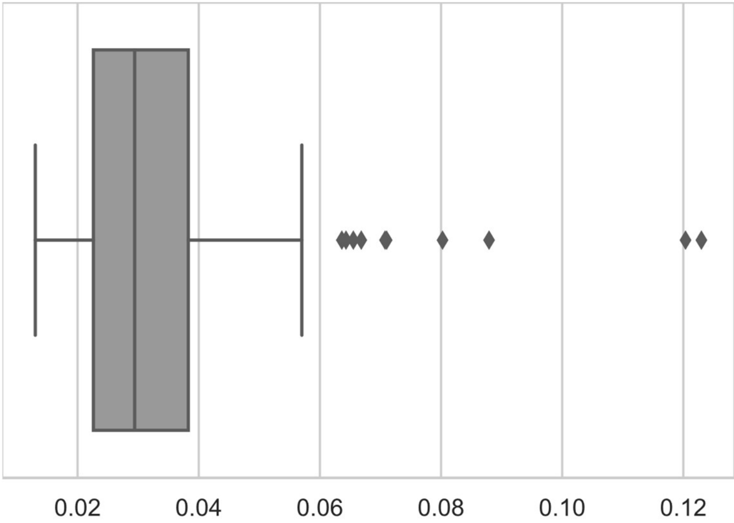

Figure 20. Boxplot of the Second Moment.

[Source: Author‘s representation]

The third statistic depicts the variation in the skewness of the monthly portfolio over the entire observation period (as depicted in Figures 21 and 22).

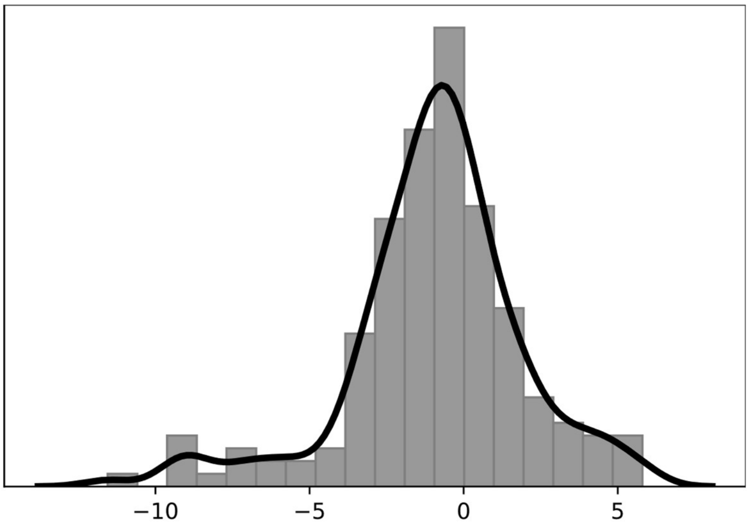

Figure 21. Distribution of the Third Moment.

[Source: Author‘s representation]

The mean, variance, skewness, and kurtosis of the third moment cross-sectionally are −0.9190, 8.1194, −0.7261, and 2.0271, respectively.

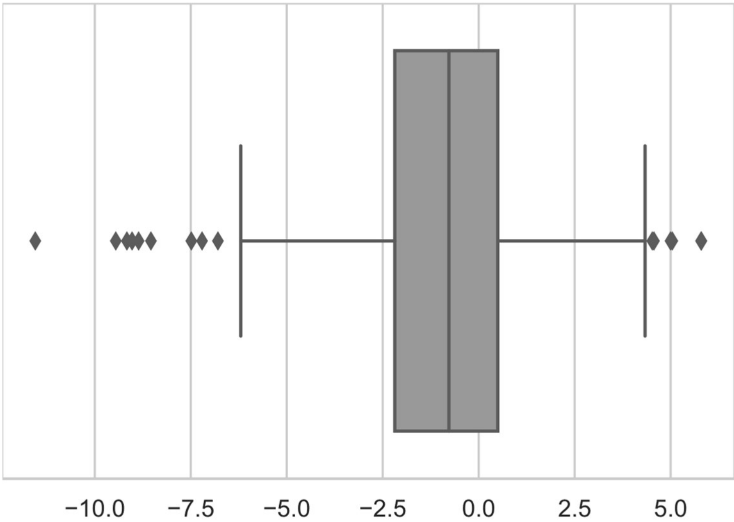

Figure 22. Boxplot of the Third Moment.

[Source: Author's representation]

The cross-sectional distribution of the kurtosis, as illustrated in Figures 23 and 24, has the following values: Mean 30.2095, variance 966.8614, skewness 2.9025, and kurtosis 10.7539.

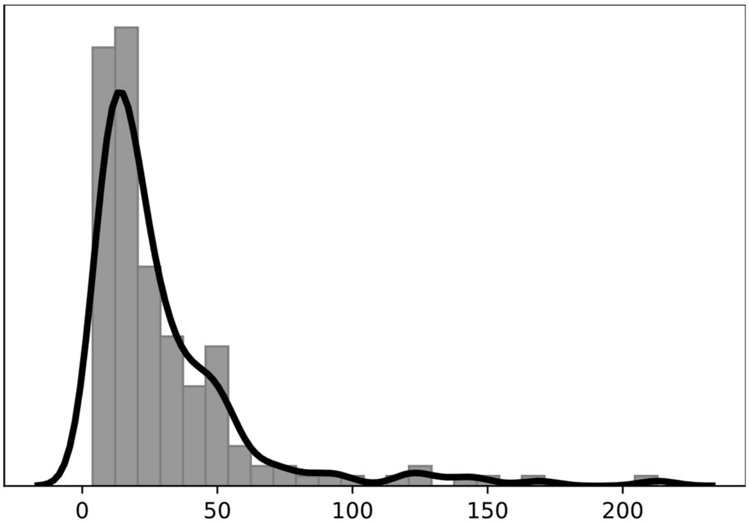

Figure 23. Distribution of the Fourth Moment.

[Source: Author's representation]

It is evident that the scales on which the moments operate are vastly divergent. By comparing Figures 23 and 24 to the other graphs, it is clear that the scales on which the moments operate are vastly dissimilar.

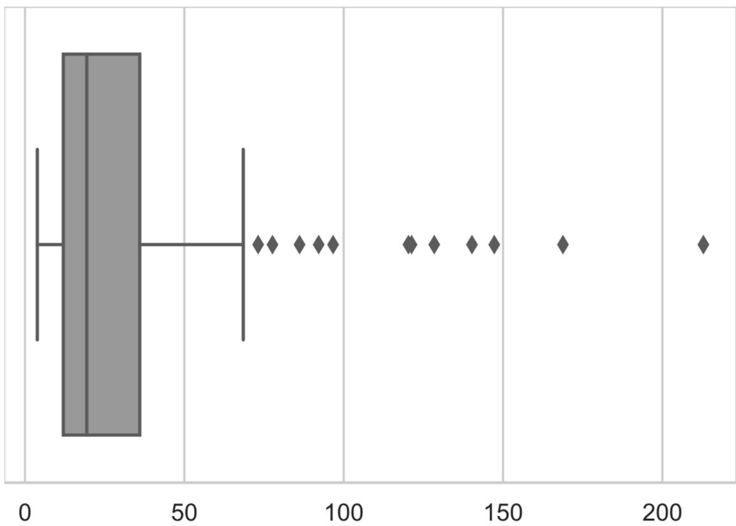

Figure 24. Boxplot of the Fourth Moment.

[Source: Author's representation]

In Figure 25, the results are again clearly summarized in a pair plot. It is also clear from the pair plot that the different moments are largely statistically independent.

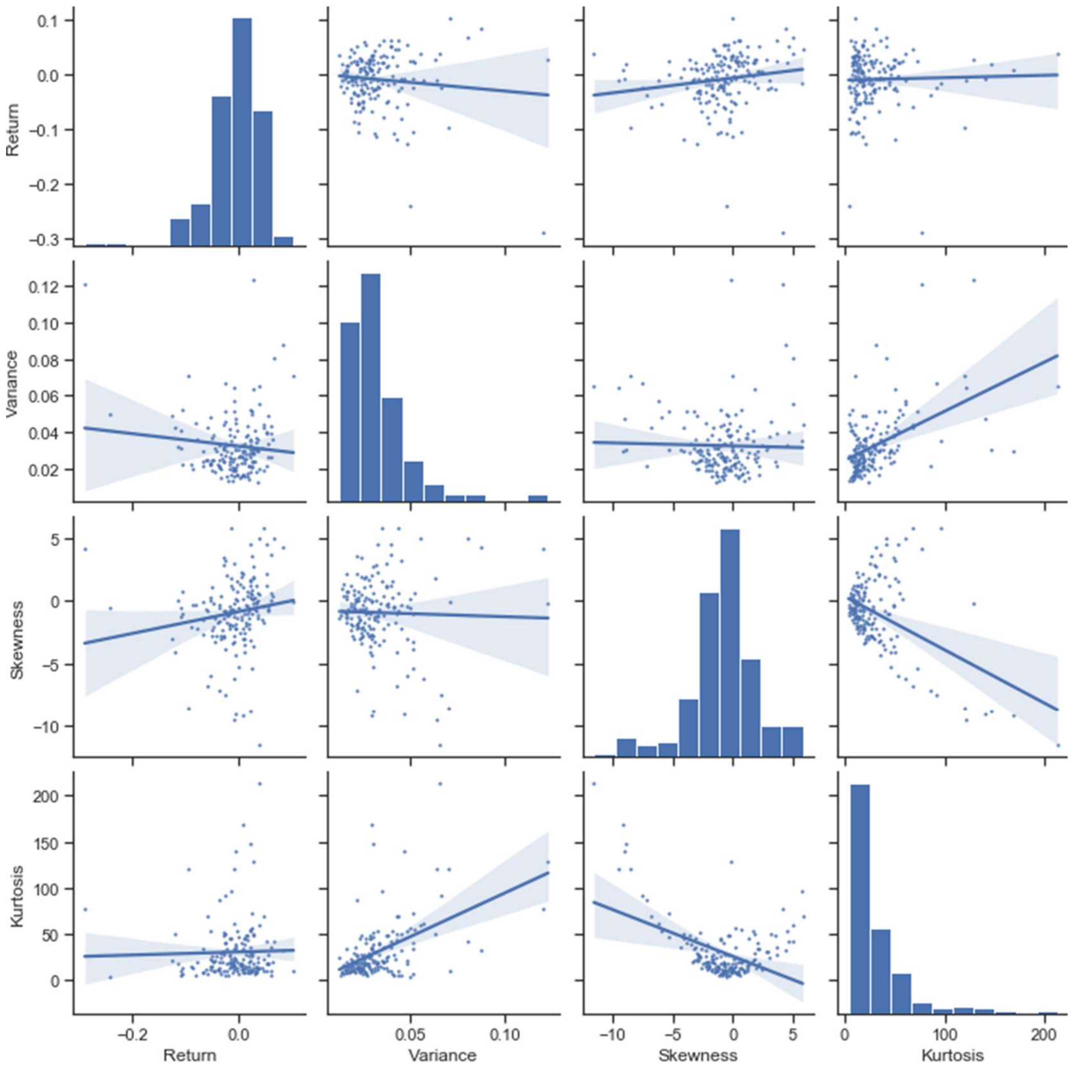

Figure 25. Descriptive Overview of the Statistical Moments.

[Source: Author's representation]

The correlation matrix, as presented in Figure 26, illustrates that the moments display comparatively weak correlation in the cross-section, excluding instances of variance and kurtosis, which exhibit a significant positive correlation with a Pearson correlation coefficient of $.5042$.

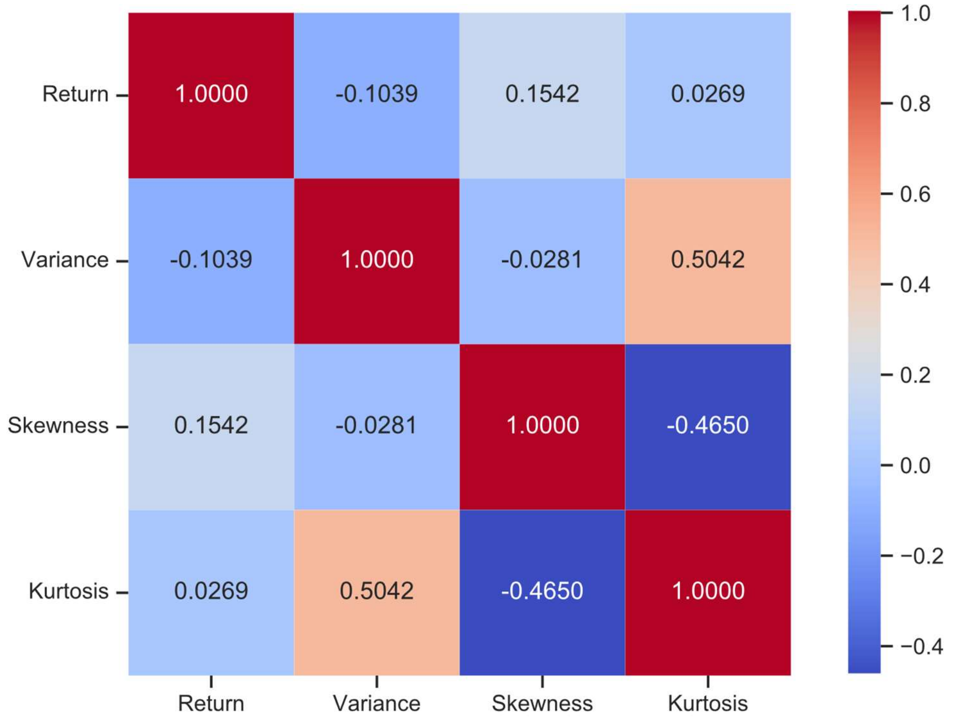

Figure 26. Cross-sectional Correlations Among Statistical Moments.

[Source: Author's representation]

It is also worthy of note that a negative correlation exists between variance and return. The uncertainty represented by variance is inversely proportional to return, which confirms the hypothesis that there is a correlation between risk and return, even though, as in the first part of the study in Chapter 4, this relationship should be positive according to μ-σ theory.

The cross-section reveals a mild positive association between skewness and yield. Additionally, a negative correlation between kurtosis and skewness is demonstrated by a Pearson correlation coefficient of $-.4650$. The correlation between investor sentiment indicators in the cross-section is also analyzed.

This correlation is present in some cases, but it is not necessarily consistent throughout the analysis. However, no significant correlation clusters were detected (as seen in Figure 27).

The figure shows the correlation matrix of all sentiment indicators in the cross-section, where the more blue a correlation is, the more negative it is, and the more positive it is, the more intense the red color is. The decision was made to not perform a dimensional reduction through PCA in order to give the RNN the fullest possible range of information.

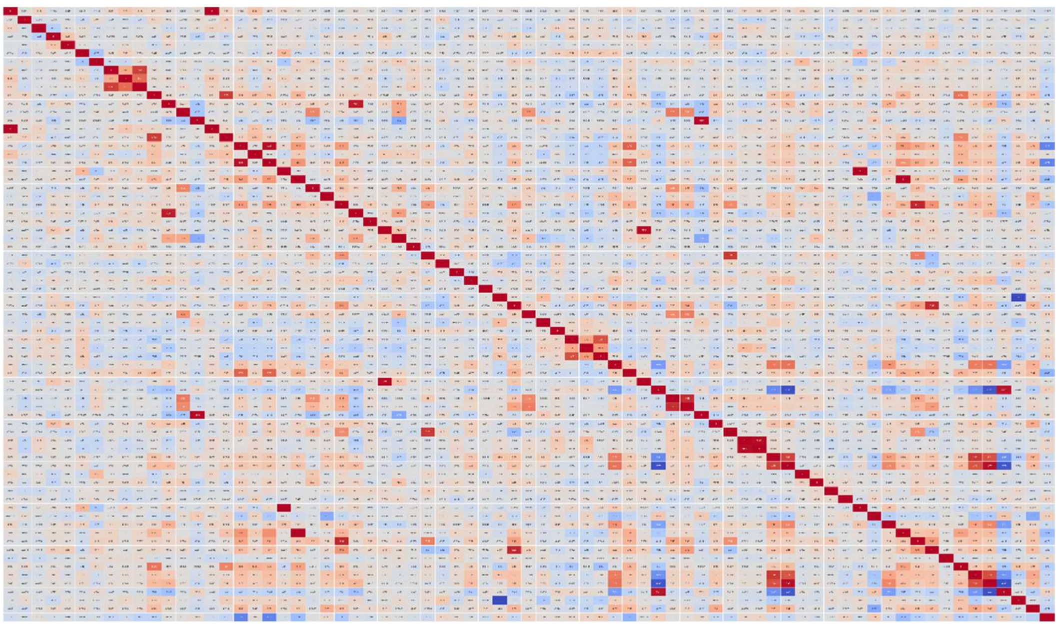

Figure 27. Cross-sectional Correlations among Investor Sentiment Indicators.

[Source: Author's representation]

However, the possibility of dimensional reduction through the use of autoencoders could be explored in future research. Given that the data demonstrate that all moments operate on dissimilar scales, direct comparison in the cross-section is not possible. In order to establish equivalence between the various distribution moments, scaling to a consistent numerical space is essential (see Figure 28).

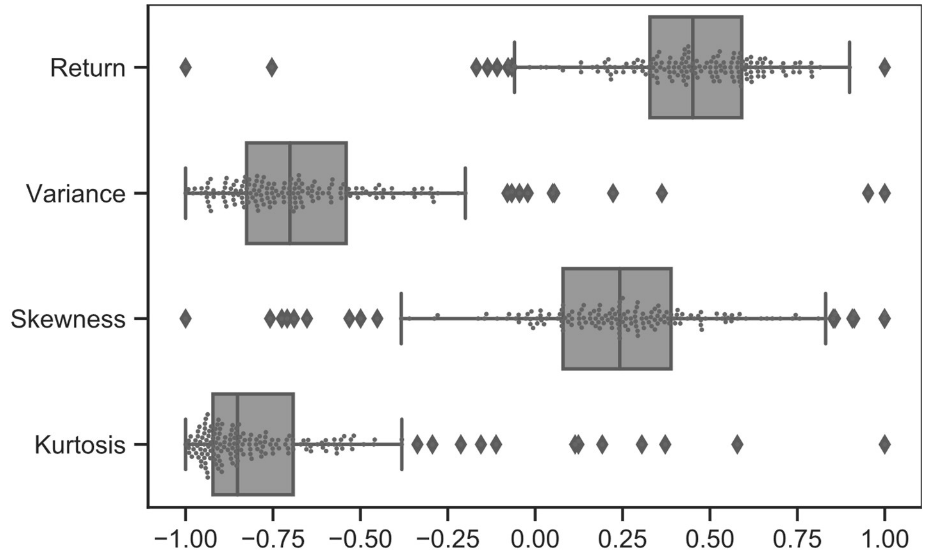

Figure 28. Boxplot of Scaled Statistical Moments.

[Source: Author's representation]

The scaled boxplots suggest that the median values for the odd moments, return, and skewness are mostly positive, whereas the median values for the even moments, variance, and kurtosis are mostly negative. The use of a scale from −1 to +1 is suitable as the output of the model is also scaled to this range as a result of the hyperbolic tangent activation used within the LSTM cell (as depicted in Figure 30 in Chapter 5.5.3).

5.5.3 Methodology and Model Selection

Figure 29 presents the general configuration of this section of the empirical analysis. The assumption is that the change in investor sentiment in the previous period plays a role in the evolution of the statistical moments in the subsequent month.

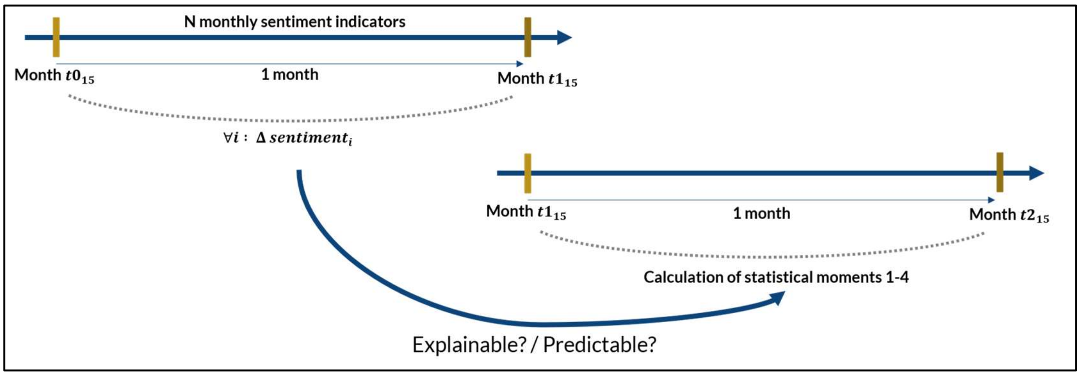

Figure 29. Hypothesis Test in the LSTM-based Empirical Study.

[Source: Author's representation]

Given that investor sentiment can fluctuate over time (Kozak et al. 2018) and that, without dimension reduction, it has a relationship that is intricately intertwined and not following a straightforward pattern with the statistical moments of yield distributions, an LSTM-based neural network is utilized to take into consideration the characteristics of investor sentiment.

As a result, studies using cross-sectional methods to investigate the correlation between sentiment and stock returns often produce inconclusive results, as discussed in Chapter 4.

In order to address the inherent shortcomings of traditional approaches, an artificial neural network with LSTM cells, which are capable of processing data sequentially and maintaining its hidden state over time, is applied to anticipate future instances of the monthly yield distribution for a CDAX portfolio that is equally distributed among its assets.

This dissertation posits that LSTM-based models possess a high level of pertinence and are superior to other model architectures when gaining knowledge from prolonged dependencies between individual investor sentiment indicators.

One of the key advantages of LSTM over standard RNNs is its capacity for selective memory retention and information updating. Figure 30 presents a detailed examination of the mechanisms of an LSTM cell within the context of a neural network.

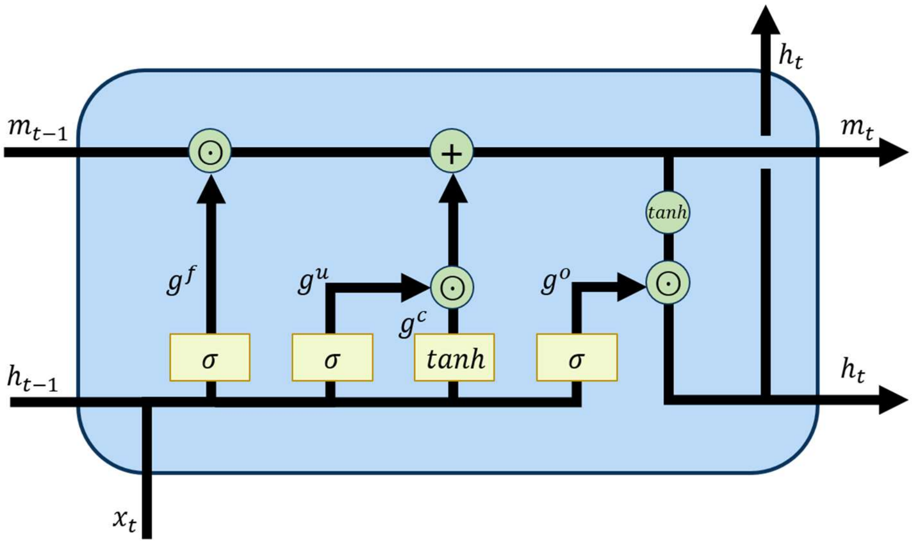

Figure 30. Long-Short-Term-Memory Cell.

[Source: Author's representation based on Graves (2012) and Olah (2015)]

Initially proposed by Hochreiter and Schmidhuber (1997), LSTM cells have undergone continuous improvement.

As illustrated in Figure 30, the yellow containers depict neural network layers, while the green circles depict pointwise operations. The arrows symbolize the movement of vectors. Typically, two activation functions are utilized within an LSTM neuron, specifically, a logistic sigmoid activation (Equation 27; Figure 31) represented by σ and the hyperbolic tangent ($tanh$) activation (Equation 28; Figure 32).

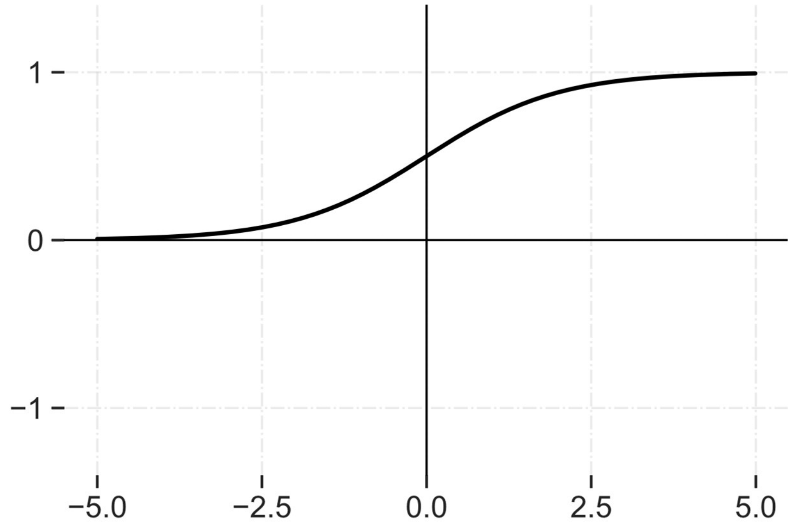

Figure 31. Logistic Sigmoid Activation.

[Source: Author's representation]

$$\sigma(x) = \frac{1}{1 + e^{-x}} \tag{27}$$

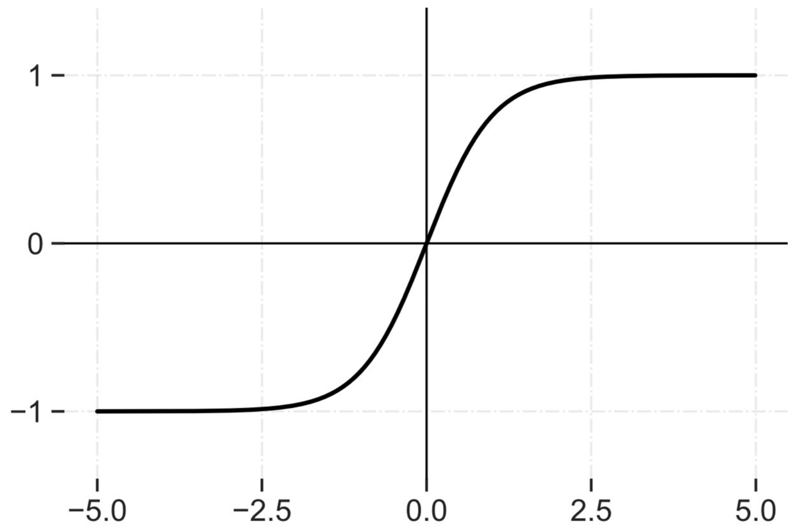

Figure 32. Hyperbolic Tangent Activation

[Source: Author's representation]

$$tanh(x) = \frac{e^{2x} - 1}{e^{2x} + 1} \tag{28}$$

The LSTM cells are interconnected in a chain-like structure and can link previous information to the current task, as well as pass it on to future tasks (see Figure 33).

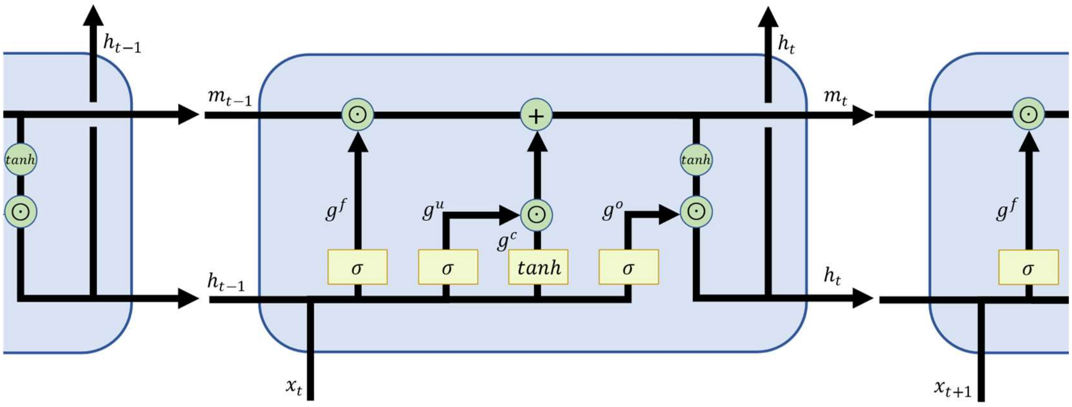

Figure 33. Chain-like Structure of Long-Short-Term-Memory Cells.

[Source: Author's representation based on Graves (2012) and Olah (2015)]

As depicted in the lower-left corner of Figure 30, the decision of which information from the memory vector or cell state m_{t-1} should be disregarded is made. The layer responsible for making this decision using the logistic sigmoid function is called the *forget-gate layer*.

It is important to note that if an investor sentiment indicator is no longer found to be a considerable factor in determining the moment of the return distribution at a given point in time, it is necessary to take into account other factors such as input values x_t and maybe to some extent the previous hidden vector h_{t-1}. By doing so, a value between 0 and 1 can be calculated for each element in the memory vector, as illustrated in Figure 34.

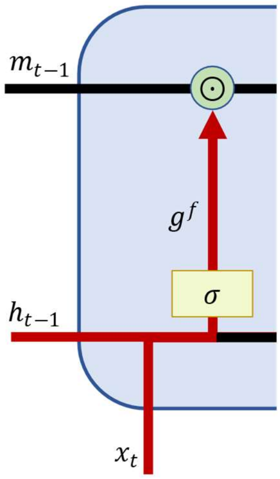

Figure 34. LSTM Cell Forget-Gate Layer (red).

[Source: Author's representation based on Olah (2015)]

Thereafter, the determination of which new information is to be recorded in the cell state is made. The *input-gate layer*, which comprises a sigmoid activation, determines which values will be altered or modified (g^u). Furthermore, utilizing a hyperbolic tangent activation layer results in the creation of a vector containing potential new values (g^c). Both the determined values and the candidate values are combined through a pointwise operation as illustrated in Figure 35.

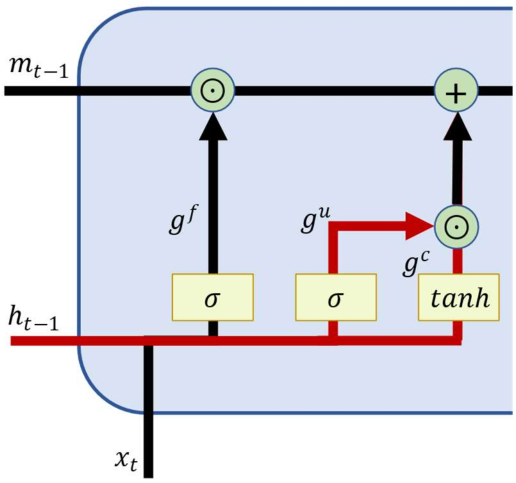

Figure 35. LSTM Cell Input-Gate Layer (red).

[Source: Author's representation based on Olah (2015)]

The output of the LSTM cell is derived from the updated cell state m_t. The update from m_{t-1} to m_t is done by pointwise multiplying the old cell state m_{t-1} by g^f.

Then $g^u \odot g^c$ is added, that is, the new candidate values, each scaled to the extent that the model has determined the degree of updating (see Figure 36).

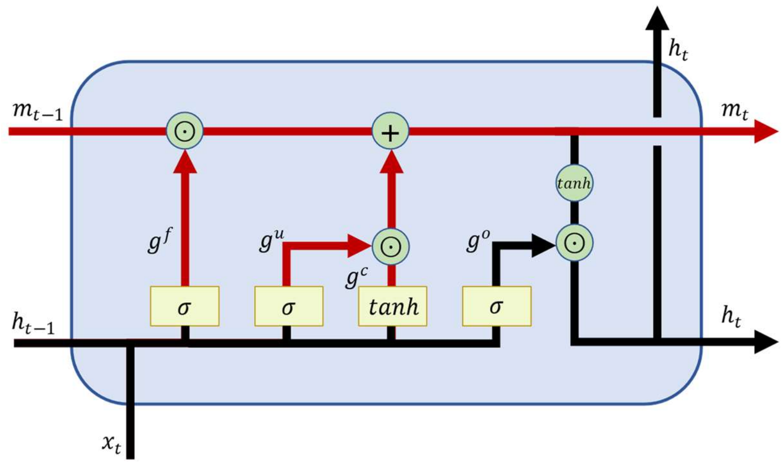

Figure 36. LSTM Cell State Update (red).

[Source: Author's representation based on Olah (2015)]

Finally, it must be determined which filtered output of the cell state m_t is done by the LSTM cell. For this purpose, the output gate layer g^o runs through a sigmoid activation, while the cell stat m_t runs through a $tanh$ activation and scales the output values to a scale from -1 to 1. Then, both results are combined in a pointwise multiplication. The cell state undergoes a transformation through a hyperbolic tangent layer, which scales the values to fall within a range of -1 to 1. This process, combined with pointwise multiplication, results in a normalized and filtered version of the cell state (h_t).

The individual steps and also special features of other LSTM variants, such as Gers and Schmidhubers "peephole connections" (Gers 1999; Gers and Schmidhuber 2000; Gers and Schmidhuber 2001a; Gers and Schmidhuber 2001b) Depth Gated RNNs by Yao et al. (2015) can be read up in Olah (2015) very well.

However, he also points to the study by Greff et al. (2017), who tests several popular LSTM architectures and concludes that "they're all about the same."

Graves (2012) and Karim et al. (2018) explain the procedure at time step t as depicted in Equations 29 to 34:

$$g^u = \sigma(\boldsymbol{W^u} h_{t-1} + \boldsymbol{I^u} x_t) \tag{29}$$

$$g^f = \sigma(\boldsymbol{W^f} h_{t-1} + \boldsymbol{I^f} x_t) \tag{30}$$

$$g^o = \sigma(\boldsymbol{W^o} h_{t-1} + \boldsymbol{I^o} x_t) \tag{31}$$

$$g^c = \tanh(\boldsymbol{W^c} h_{t-1} + \boldsymbol{I^c} x_t) \tag{32}$$

$$m_t = g^f \odot m_{t-1} + g^u \odot g^c \tag{33}$$

$$h_t = \tanh(g^o \odot m_t) \tag{34}$$

with $\odot$ for pointwise multiplication, $\boldsymbol{I^u}$; $\boldsymbol{I^f}$; $\boldsymbol{I^o}$; $\boldsymbol{I^c}$ for projection matrices, and $\boldsymbol{W^u}$; $\boldsymbol{W^f}$; $\boldsymbol{W^o}$; $\boldsymbol{W^c}$ for recurrent weight matrices. The present assessment demonstrates that this methodology, which is outlined in a simplified manner here, is effective in utilizing investor sentiment as a means of explaining the distribution of returns, due to its ability to analyze temporal dependencies within sequences. Further information can be referred to in Graves (2012) and Karim et al. (2018).

As an initial primary step, it is assessed if the artificial LSTM based recurrent neural network architecture can sufficiently adapt to the aspects of investor sentiment. The sequential model utilizes RMSprop, a gradient descent optimization algorithm, as it is effective for optimizing non-convex objectives (Soydaner 2020).

This initial investigation encompasses all observations and demonstrates that a highly adaptable two-level LSTM neural network with a 20-period rolling look-back window can perform effectively over 10,000 iterations, as reflected by the strong predictive ability of the labeled training data based on investor sentiment.

5.5.4 Implementation and Empirical Findings

The key outcome of the preliminary training, in the context of pre-training and initial evaluations, is the demonstration of the capability of constructing a model with an exceptional fit for explicating each statistical moment using investor sentiment. This is a crucial initial discovery, as it has been established that neural networks can approximate any mathematical function when designed for the specific application. Additionally, in order for the model to work efficiently, there must be a corresponding function that it can adjust to.

The study found that there is a clear connection between investor sentiment and the future distribution of returns, and that this connection cannot be random. By analyzing data from a labeled training scenario that covered all statistical moments and using 10,000 epochs, the research determined that all statistical moments, including mean, variance, skewness, and kurtosis, can be effectively explained by using 73 sentiment indicators and a neural network with two LSTM layers.

Given that the current challenge at hand does not involve classification or categorization, it is not possible to define an accuracy metric at this stage. However, the fit's accuracy can be described using various measures, such as a two-sided t-test, the mean squared error (MSE) and the simple coefficient of determination R^2, which can also be calculated as the square of Pearson's correlation coefficient between the actual values y and the predicted values $\hat{y}$.[27]

[27] The use of the R^2 metric is primarily known from simple regression models, but is also used in the context of the research methodology used at this point, e.g., in Jin et al. (2020).

In regards to the first moment (illustrated in Figure 37), the R^2 value is $.9993$ and the p-value of a two-sided t-test is $.9920$, showing that there is not enough evidence to reject the null hypothesis that the average value of the forecast sample constructed using labeled training data is the same as the average value of the actual sample. The mean squared error (MSE) is $4.6446 \cdot 10^{-5}$, which is a rather small value. These metrics and the corresponding graph demonstrate an outstanding fit of the model for scaled returns. The MSE indicators and representation reveal a remarkable model adjustment for returns. The graph for the actual scaled returns (green) and the returns predicted by the model (black) are visually hardly distinguishable (Figure 37).

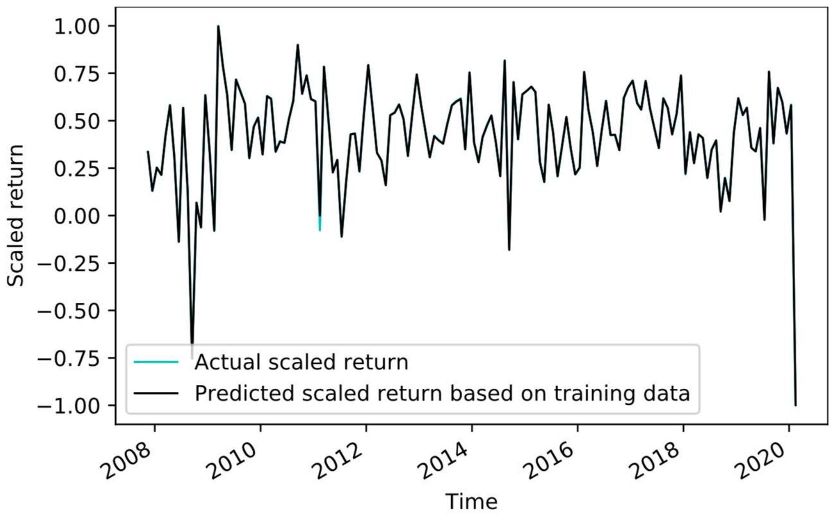

Figure 37. Training of the First Moment.

[Source: Author's representation]

Similar outcomes were obtained during the training process for scaled variance (as depicted in Figure 38). Specifically, the second moment yielded an R^2 value of ≈ 1.0000, and the p-value of the two-sided t-test is $.9894$. Additionally, the MSE is $5.0222 \cdot 10^{-6}$ which is a small value. These results suggest an exceptional model fit for the second moment. Again, the graph for the actual scaled variance (green) and the variance predicted by the model (black) are visually hardly distinguishable.

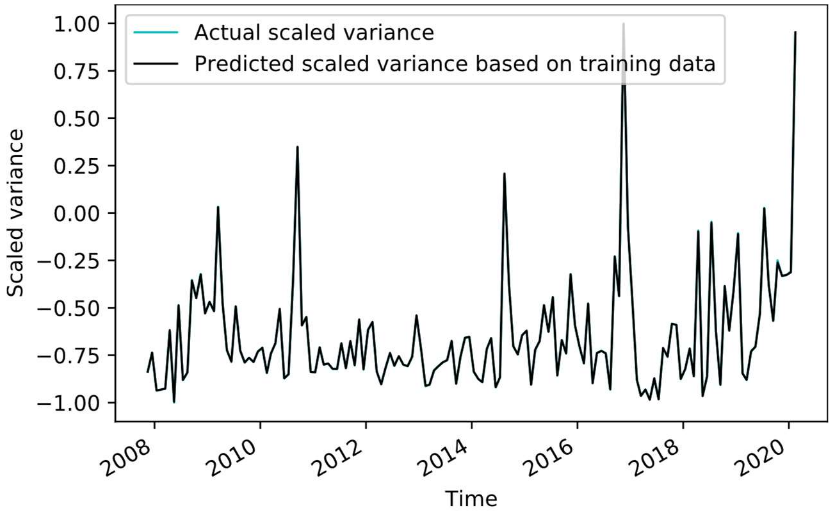

Figure 38. Training of the Second Moment.

[Source: Author's representation]

While the fit to skewness and kurtosis in training may not be as strong as for the first two moments, it can still be considered an adequate model fit. Specifically, for the third moment (illustrated in Figure 39), the R^2 value is ≈ 1.0000 and the p-value of the two-sided t-test is $.9688$. Furthermore, MSE is $5.8810 \cdot 10^{-6}$ which is again a relatively small value. These results suggest a good model fit for the third moment. As with the return and variance, a similar picture emerges. Here, too, the graph for the actual scaled skewness (green) and the skewness predicted by the model (black) can hardly be distinguished visually.

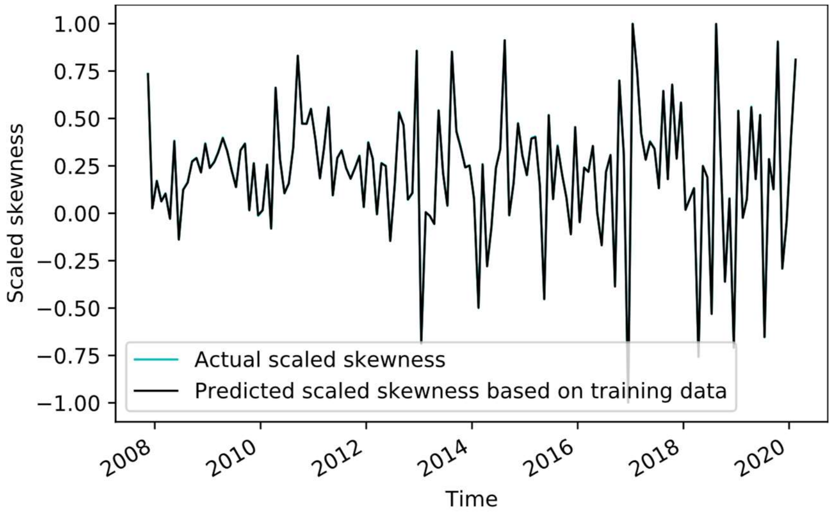

Figure 39. Training of the Third Moment.

[Source: Author's representation]

The fourth statistical measure, kurtosis, yields the least satisfactory results. Specifically, for the kurtosis, the R^2 value is .9897 and the p-value of the two-sided t-test is $.9229$. The mean squared error is $9.3728 \cdot 10^{-4}$ which is a relatively large value compared to the MSE results of the other moments. The weaker fit is also evident in the corresponding graph (illustrated in Figure 40). For example, the period between 2012 and 2014 serves as an illustration of the poorer fit compared to the other moments. In this case, the investor sentiment model is unable to explain the amplitude.

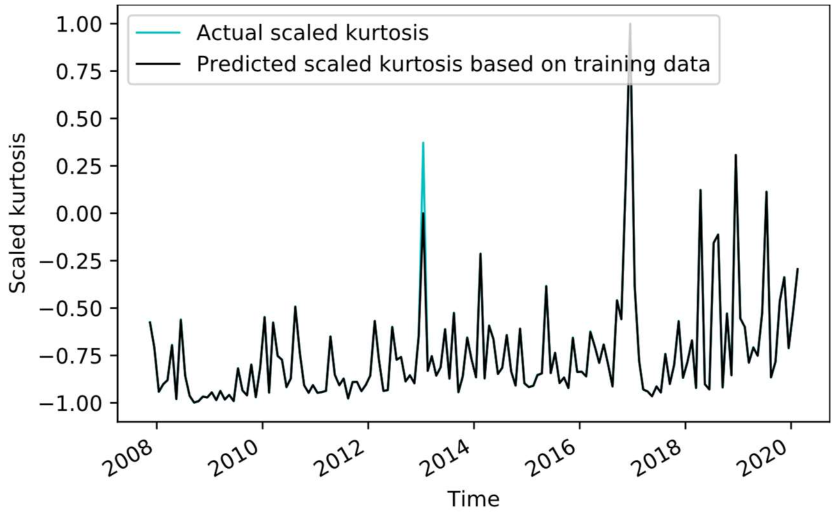

Figure 40. Training of the Fourth Moment.

[Source: Author's representation]

The Mean Squared Error (MSE or loss) for models that are based solely on training data all converge to 0 after 2000 training iterations. The loss indicates the discrepancy between the model's prediction based on investor sentiment and the actual value. The presence of small fluctuations in the loss plots for all moments suggests that none of the loss functions are convex.[28] A gradient descent procedure [here: RMSprop] minimizes the loss over each epoch.

Now that sound conditions have been established and verified, the next step is to train a predictive model and predict based on test data in out-of-sample tests.

[28] A loss function is considered convex if it exhibits a unique global minimum for all points within its domain. In contrast, a non-convex loss function may exhibit multiple local minima or saddle points, resulting in multiple potential solutions. Consequently, optimization algorithms may converge to a suboptimal local minimum rather than the global minimum, leading to subpar model performance.

As the actual loss function is uncertain, a decision has been made to re-train the models for predictions using out-of-sample data, halting at the initial sustainable minimum, recognizing that these could be local minima of the loss functions (since a non-convexity is assumed).

A sustainable minimum is defined as a minimum of the loss that is not caused by an outlier value (negative spike) but instead is consistent with the progression of the loss values in the preceding and following training cycles.

A dropout layer, which randomly eliminates 10% of neurons during training, has been added between the two LSTM layers to construct a more robust predictive model. This dropout layer helps prevent overfitting by reducing the model's reliance on the training data.

As a result, the model's fit to the training data may not be as strong as in earlier tests, but it is believed that its ability to predict test data will be enhanced. Although this may result in less perfect fitting, it makes the models more robust for trend forecasting under different market conditions.

The dataset has been split, with the majority of the data points designated as training data, excluding the final two points which pertain to the COVID-19 time period. Utilizing these two points for prediction offers the benefit of not only assessing the precision of the prediction of specific moments, but also evaluating if the models can accurately predict trends.

The training performance of the individual models using both training and test data can be viewed in Figures 41 to 44.

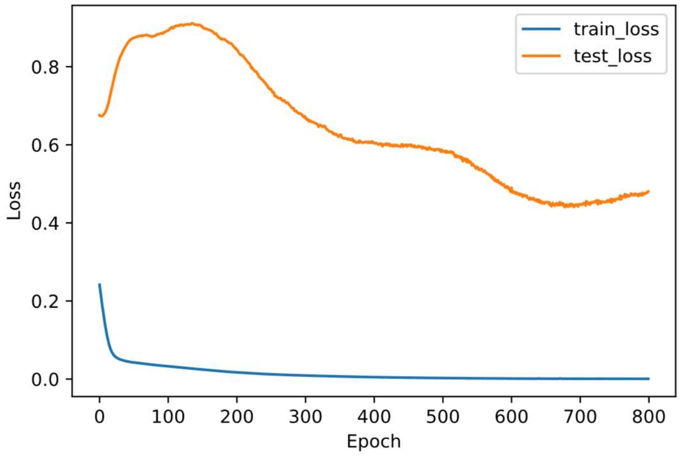

Figure 41. Training and Test history of the First Moment.

[Source: Author's representation]

As depicted in Figure 41, it took approximately 800 training iterations to identify the first sustainable minimum for the first moment, which was located at iteration 678. Similarly, for the second moment (as illustrated in Figure 42), it required 1,200 training iterations to reach the first sustainable minimum.

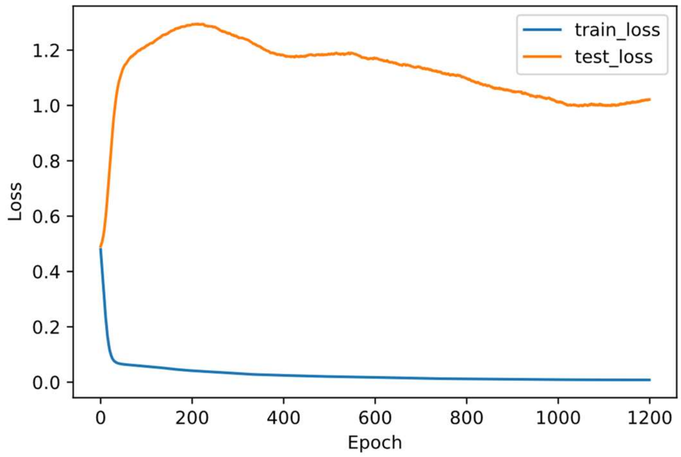

Figure 42. Training and Test history of the Second Moment.

[Source: Author's representation]

The initial sustainable minimum for the seond moment was located at iteration 1,054. It is worth mentioning that the training loss once again approaches zero asymptotically.

In the case of the moments of skewness (Figure 43) and kurtosis (Figure 44), it took only 100 training iterations to identify the first sustainable minima, which were located at iteration 34 for skewness and iteration 23 for kurtosis.

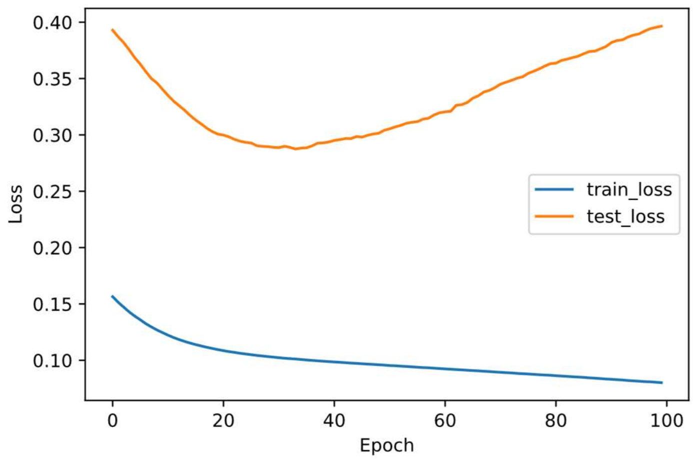

Figure 43. Training and Test history of the Third Moment.

[Source: Author's representation]

However, it should be acknowledged that these minima may be local minima. Additionally, the training loss of Figure 44 does not converge to 0, indicating a relatively strong underfitting to the training data.

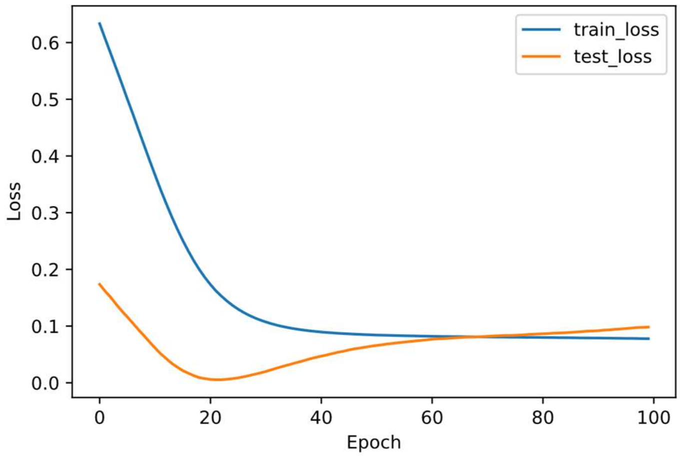

Figure 44. Training and Test history of the Fourth Moment.

[Source: Author's representation]

After the predictive models have been trained using the training data and the lowest point of the loss function has been identified on the test data, it is possible to assess the accuracy and overall effectiveness of the prediction models.

Initially, the coefficient of determination R^2 is recorded as $.9135$, with a p-value of $.9486$. Additionally, the mean squared error (MSE) is $6.1326 \cdot 10^{-3}$.

During the period of the COVID-19 pandemic, the actual decrease in the scaled return was -1.5851, while the model, which was not based on the test data, forecasted a decrease in the scaled return of -0.6876 in the out-of-sample test, as shown in Figure 45.

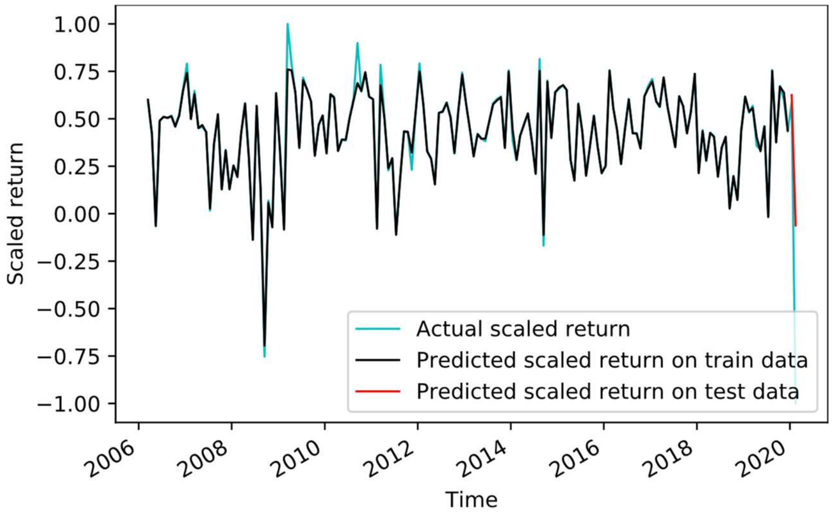

Figure 45. General prediction of the First Moment.

[Source: Author's representation]

Even though the model failed to precisely estimate the extent of the reduction in scaled return, it correctly identified the pattern. Figure 46 shows the section of the out-of-sample test. It is visible that the model can correctly predict the downward trend. However, the predicted downward trend is not as strong as the (real) observed trend.

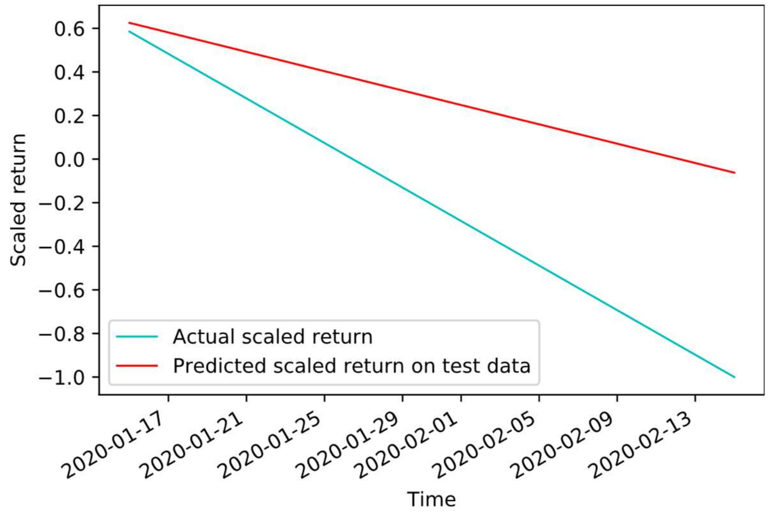

Figure 46. Prediction of the First Moment during the COVID-19 Time Window.

[Source: Author's representation]

For variance, the correlation coefficient (R^2) was found to be $.7899$ and the significance (p-value) was $.5791$. The mean squared error was calculated to be 0.0204. During the COVID-19 period, the observed increase in standardized variance was 1.2308, whereas the model, which was not based on the test data, forecasted a rise in scaled variance of 0.1405 in the out-of-sample evaluation (see Figure 47).

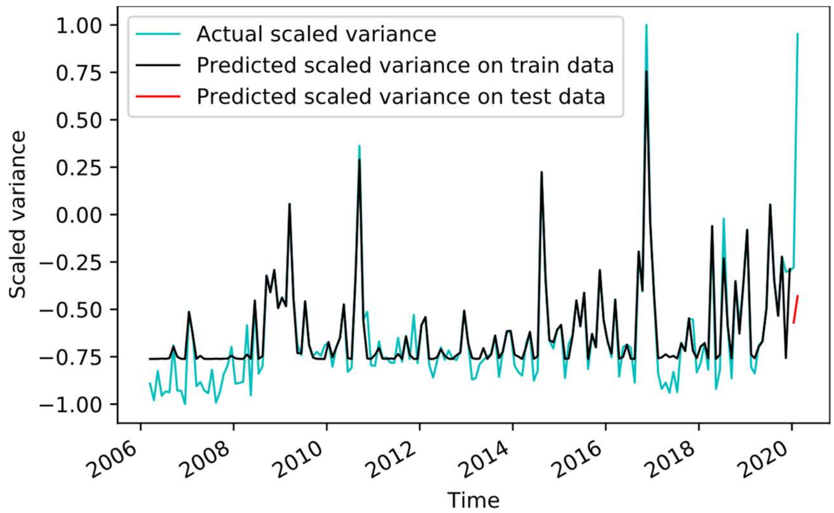

Figure 47. General Prediction of the Second Moment.

[Source: Author's representation]

Once more, the model failed to accurately forecast the magnitude of the increase in normalized return; however, it was able to predict the pattern. Unlike the initial instance, it is evident that the magnitudes in the high and particularly in the low normalized variance ranges are not logical. Apparently, the model also could not comprehend that in the scaling, the variance cannot become more negative than -0.75. Like the first moment, the uptrend predicted is not as strong as the observed uptrend (see Figure 48).

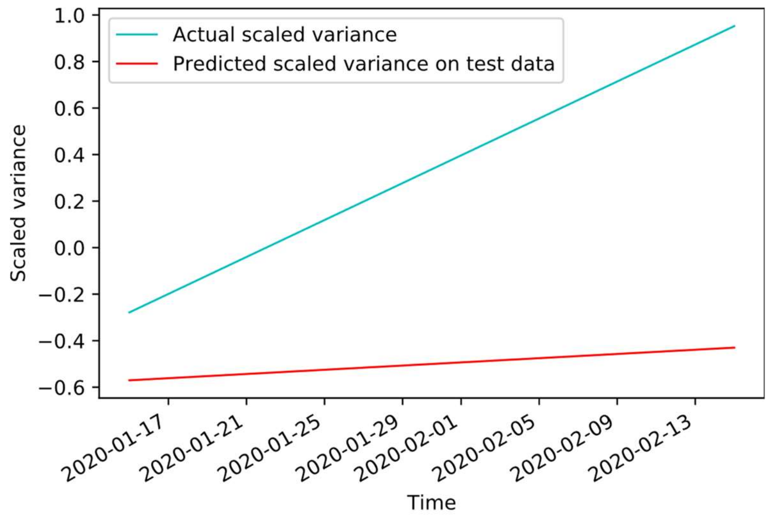

Figure 48. Prediction of the Second Moment during the COVID-19 Time Window.

[Source: Author's representation]

Figure 49 shows that the model's prediction of the rise in scaled skewness during the COVID-19 window was not very accurate, given an R^2 value of 0.0615 and a p-value of 0.3197. The low R^2 value indicates that the model explains only 6% of the variation in the data. The high MSE value of 0.1026 also indicates that the model has a high level of error in its predictions. Additionally, the difference between the actual rise in scaled skewness and the model's prediction is quite large, indicating that the model did not perform well on the out-of-sample test data. Overall, this suggests that the model may not be a reliable tool for predicting changes in scaled skewness during the COVID-19 window.

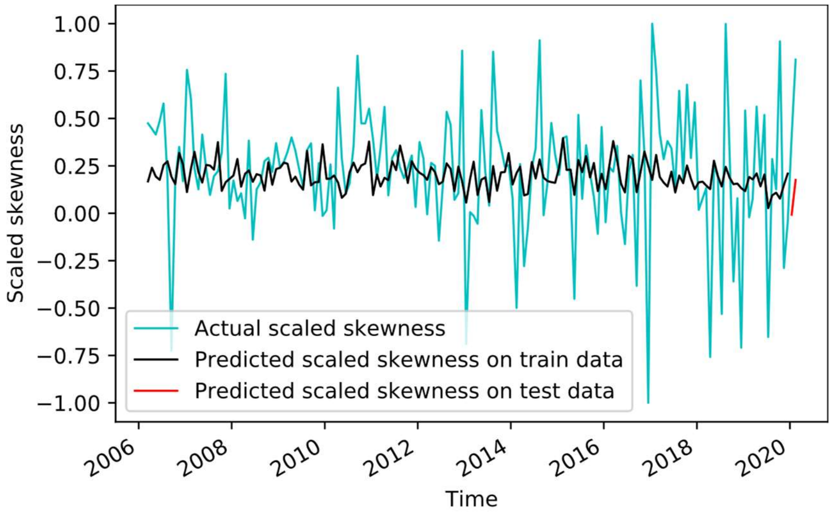

Figure 49. General Prediction of the Third Moment.

[Source: Author's representation]

This observation suggests that the model has a good understanding of the underlying dynamics of the data, but may not be able to capture the exact magnitude of the change in skewness. This could be due to a number of factors, such as limitations in the model or the data itself. However, the ability to correctly forecast the trend during the out-of-sample test is still valuable as it allows for decision-making based on the expected direction of the skewness. Overall, the model's performance on the skewness metric is promising, but further analysis and refinement may be needed to improve its accuracy in predicting the exact magnitude of change. However, the trend slope is estimated quite well this time (see Figure 50).

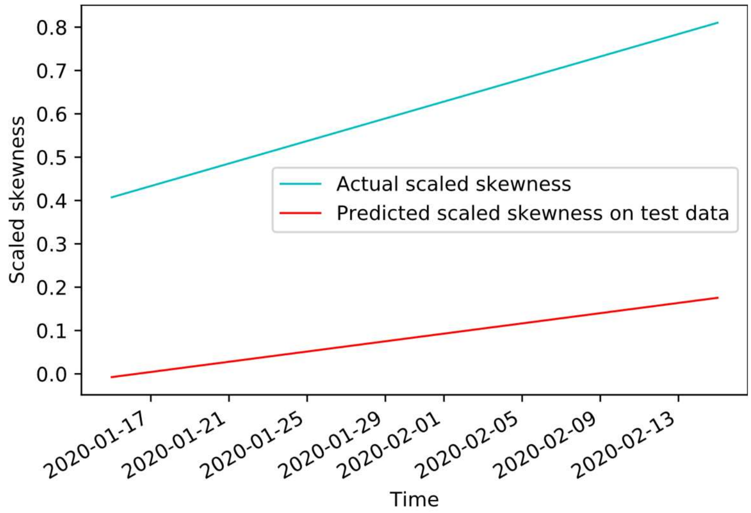

Figure 50. Prediction of the Third Moment during the COVID-19 Time Window.

[Source: Author's representation]

For the scaled kurtosis R^2 is $.0422$ and there is a significant difference between the mean value of the forecast sample fit based on training data and the mean value of the actual sample. The p-value of $8.5121 \cdot 10^{-102}$ indicates that there is a very low probability (essentially zero) that this difference is due to chance.

Therefore, it is likely that the forecast sample fit is not accurately reflecting the actual sample.

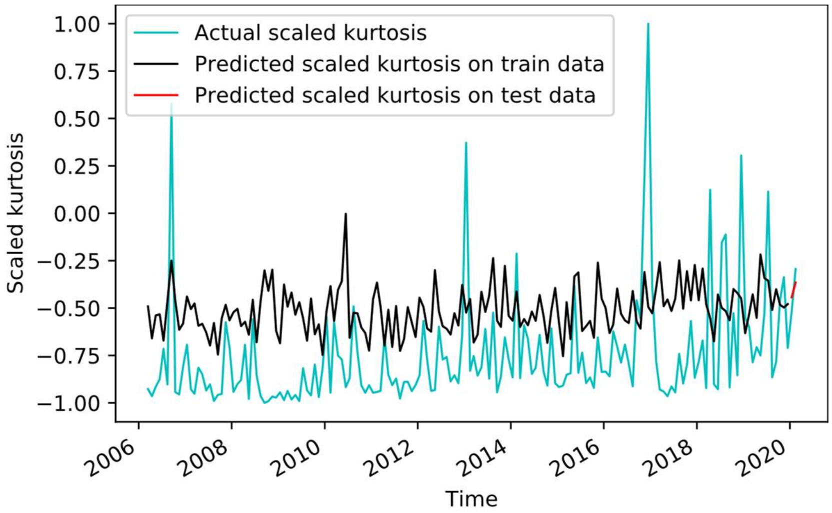

Figure 51. General Prediction of the Fourth Moment.

[Source: Author's representation]

The model has a difference of 0.1470 between the predicted and actual rise in scaled kurtosis during the COVID-19 window measured by MSE. Due to a relatively high MSE, the model's prediction for the rise in scaled kurtosis during this specific time period is not very accurate. This could be due to the unique and unprecedented nature of the COVID-19 pandemic and its effects on the economy and financial markets. Similar to the other predictions of the first three moments, the predicted upward trend is not as strong as the observed upward trend (see Figure 52).

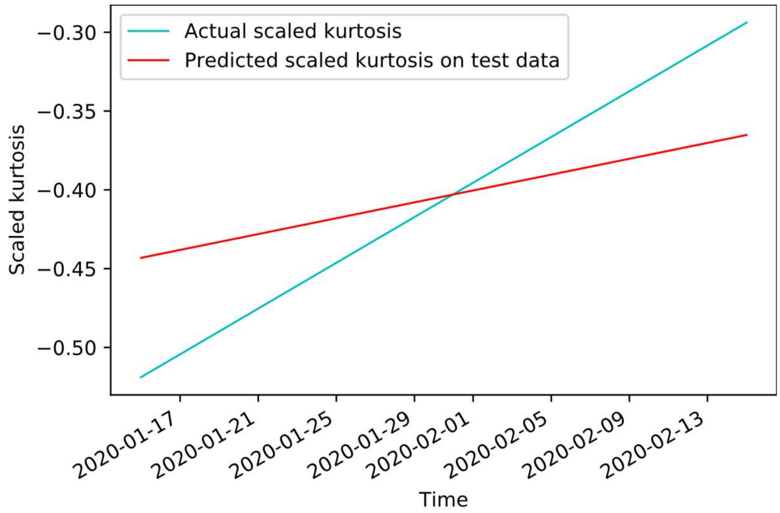

Figure 52. Prediction of the Fourth Moment during the COVID-19 Time Window.

[Source: Author's representation]

In summary, the kurtosis model was able to predict the trend of the data but not the magnitude of the increase in scaled kurtosis. The LSTM method was able to identify the first sustainable minimum of the loss function at both the third and fourth moments for the test data relatively quickly, suggesting these moments may not be well suited to the training data. This is reflected in the forecast graphs and model metrics, with the fourth moment having the worst performance as indicated by the lowest R^2 value.

Additionally, this model is the only one that rejected the null hypothesis based on a two-sided t-test. This suggests that there is a significant difference between the mean of the actual values and the mean of the predicted values, indicating a poor model performance and a lack of accuracy in the predictions made by the model. Further research is needed to determine if the trend prediction holds true in the long term.

5.6 INTERIM RESULTS

The research suggests that by incorporating investor sentiment from various sources into a neural network using LSTM cells, it is possible to predict all aspects of the future yield distribution for a portfolio of German shares on a monthly basis. Additionally, it has been determined with a high level of confidence that investor sentiment can effectively predict future trends in the German stock market on a monthly basis during the COVID-19 period for the first three moments (mean, variance and skewness).

However, it must be noted that the results of the analysis of the fourth moment (kurtosis) are not conclusive, and more research is needed to arrive at a definitive conclusion. Despite this limitation, the study still provides valuable insights into forecasting stock market trends, and the results can be used to inform investment decisions.

Furthermore, in addition to already established knowledge about the Chinese market (Jiang et al. 2021), the US market (Albulescu 2021; Zhang et al. 2020), the Saudia-Arabian market (Hadi and Shabbir 2021), and the Indian market (R et al. 2021), this study adds to the growing body of literature on the connection between investor sentiment and stock market performance, specifically during a crisis.

By providing an understanding of the German stock market, this study adds contemporary perspectives to the ongoing research on the relationship between investor sentiment and stock market yields and volatilities as well as other risk metrics.

Additionally, the use of an RNN-based approach in this research highlights its potential for further study in this field.

Research hypothesis $\boldsymbol{RH}_5$ can be confirmed. In the COVID-19 window under consideration, investor sentiment is well-suited to explain returns and other risk metrics. One could argue that $\boldsymbol{RH}_5$ is exceeded, as even trend prediction works surprisingly well, which is an important finding far beyond the conjectured findings.

$\boldsymbol{RH_5}$: *By taking the time-varying characteristics of investor sentiment into account, the explanatory power of investor sentiment increases perceptibly compared to traditional cross-sectional analyses.*

However, this is also because the typical cross-sectional studies with multi-factor models are probably not very well suited to explain the yields or risk metrics derived from them by investor sentiment. It is important to consider the dynamic nature of risk premia in this context, as the level of risk associated with an investment can fluctuate over time.

The fluctuations in investor sentiment, in particular, can have a significant impact on risk premia and, therefore, the results of forecasting stock market trends. Thus, incorporating time-varying risk premia in the analysis is likely to be crucial in obtaining a more accurate and reliable prediction of stock market trends.

The disturbances in the German stock market due to the worldwide COVID-19 outbreak is examined in this dissertation as an event of interest and serves to confirm the research hypothesis that investor sentiment can be used to explain the moments of yield distributions in the subsequent period. Nevertheless, the lack of sufficient predictive observations on test data hinders the ability further to validate this conclusion with additional metrics.

In terms of prediction, the results are less accurate, and further investigation is needed. Despite this, it is still possible to make accurate predictions about market trends based on investor sentiment by analyzing initial data on returns and variance. These findings can be valuable for portfolio risk management from an academic perspective. It would be intriguing to conduct further studies that extend the examination to various time frames, including weekly and daily indices that encompass all categories of investor sentiment, including investor sentiment from social media.

Furthermore, it would be advantageous to run a portfolio simulation over a prolonged time frame to gauge the effectiveness of investment decisions based on investor sentiment in relation to a benchmark.

The application of RNN and especially LSTM technology to tackle the issue of long-term, varying, and partly lagged correlation between independent variables is not a new concept; however, the emergence of easily accessible computational resources through cloud services has made conducting in-depth research on this topic more attainable.

Lastly, additional investigation is required to ascertain whether the findings from this study on the German stock market can be applied to other nations and crisis events.

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6 EXPLORATIVE SOCIAL SENTIMENT STUDY

The most recent form of investor sentiment research is now taking place on the internet, specifically on social media. Compared to other categories of investor sentiment, such as survey-based sentiment, social sentiment offers a decisive advantage. Like market-implied sentiment, it is instantly available. Some recent study results on this have already been presented in Section 3.2.3.

In this context, the study of social sentiment in microblogging and on the social networking service Twitter remains particularly interesting. Countless short messages (tweets) are written every day, and the current findings that are disseminated there also relate to stock market events. With the help of so-called #hashtags, users can search for articles on specific topics.

In the context of this dissertation, a total of 1,888,407 tweets on the German stock market with the hashtag #DAX were collected through a Virtual Private Server (VPS) specially set up for this purpose for the period from August 1st, 2018 to December 31st, 2021, inclusive, to evaluate them in the context of this dissertation and gain insights for the extraction of investor sentiment and future research in this area.

6.1 EMPIRICAL ANALYSIS

6.1.1 Introduction to the Empirical Analysis

In light of the findings from the multi-factor model-based study (see Chapter 4) and the LSTM-based study (see Chapter 5), it seems reasonable to choose an event window in the exploratory investor sentiment study based on Twitter data, in which an extreme event is considered.

On the one hand, this is due to the fact that in the multi-factor model-based study, it was found that an investor sentiment-based risk factor seems to be particularly suitable for explaining stock market returns when an extreme event is considered (see Figure 11 in Chapter 4.3.3).

In relatively calm market phases, the movement looks more like a random walk, from which little explanatory power seems to emerge. However, it must also be taken into account that in the two studies mentioned above, only investor sentiment from the categories survey-based sentiment and market-implied sentiment was considered. In contrast, in this third empirical study, social sentiment is explicitly and exclusively the subject of the investigation.

As in the LSTM-based study, the COVID-19 time window is also defined as an exemplary object of investigation in the present study. This is because the time period has already proven its worth in illustrating the relationship between investor sentiment and returns on the stock market. In this part of the empirical study, the date range from September 23rd, 2019 to August 27th, 2020 is observed. More than 120,000 minutely tick prices of the CDAX are considered (see Figure 53).

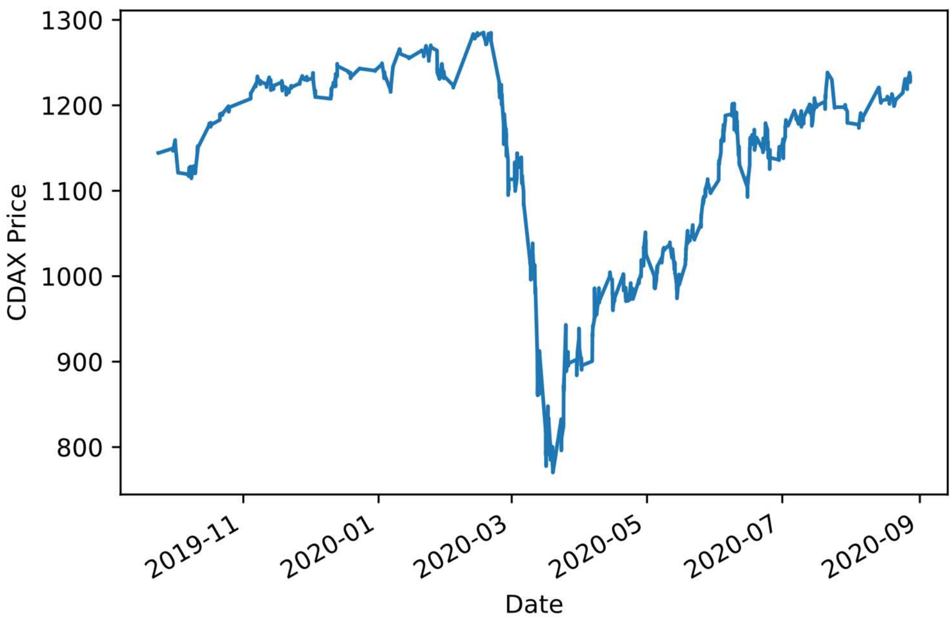

Figure 53. CDAX during the COVID-19 Observation Window (Tick Data).

[Source: Author's representation]

In order to investigate investor sentiment exploratively in the area of social media and correlations with the stock market, extreme events lend themselves to being used, as one can compare the period before the event and the period after the event with the event time and pay special attention to corresponding peculiarities at this point. However, this is more of an exploratory study than one of the classic event studies that can be looked up in, for example, Gehrke (2019).

While this third sub-study of the dissertation also examines the COVID-19 time frame, it diverges in its approach and scope from the studies presented in the previous chapters (Hövel and Gehrke 2022b). This study exclusively examines investor sentiment taken from social media (more precisely: Twitter).

Moreover, this study does not deal with monthly price data as in Chapters 4 and 5, but with intraday tick data on a minute basis, as is common in investor sentiment research in social media. This is because social media sentiment is generally considered to be more erratic than investor sentiment from traditional sources (survey-based and market-implied sources).

The observation window is chosen, so that the market phases before and after the event of the massive price collapse were again somewhat similar, and the intense volatility in the market had calmed down again to a certain extent.

Referring to Figure 53, the CDAX reaches its highest value in the available tick data on a per-minute basis on February 14th, 2020 at 10:16 a.m., before the index reaches its low by March 19th, 2020 at 1:43 p.m. at an index value of 770.27. This corresponds to a price decline of -40.08% within 34 days (see Annex 5).

Empirical evidence for an explanatory contribution by investor sentiment to this price decline has already been suggested in the study in Chapter 5. This section now examines the extent to which there is a connection between this price drop and important investor sentiment metrics in the Twitter sphere. In the context of this analysis, the research hypothesis $\boldsymbol{RH}_6$ also needs to be answered regarding whether social investor sentiment influences the German stock market developments.

$\boldsymbol{RH}_6$*: Investor sentiment-based events on social networks have an impact on market developments in the German stock market.*

Furthermore, whether it is possible to anticipate market developments in Germany based on Twitter investor sentiment alone is also examined.

6.1.2 Data Collection and Database

In order to be able to collect tweets for this explorative analysis over a longer period of time, a VPS was operated on the Digitalocean.com platform over the period from August 1st, 2018 to December 31st, 2021 inclusive.

On this VPS, a virtual RStudio server (v2022.02.1+461) was run, on which a script was executed regularly using a cron agent, in order to collect tweets about the German stock market. To make the script usable and to convert its collections into a suitable data frame structure, the R packages “twitteR” in version 1.1.9 by Gentry et al. (2016) and “plyr” in version 1.8.6 by Wickham (2011) were applied. Gentry's “twitteR” package is also applied in other recent sentiment context studies (Hassan et al. 2021; Bayrak and Alper 2021; Salvatore et al. 2021).

An update of the packages during the usage period was deliberately not done, in order not to endanger the stability of the script and the investigation.

In the period from August 1st, 2018 to December 31st, 2021 inclusive, a total of 1,888,407 tweets were stored in the database. At this point, corresponding meta features were stored for each tweet, which were taken into account by the R package “twitteR” (see Table 14).

Table 14. Features Collected as Part of the Research on Each Tweet

Feature	Description
text	The text of the status
favorited	Whether this status has been favorited (False/True)
favoriteCount	Number of people who have favorited
replyToSN	Screen name of the user this is in reply to
created	When this status was created
truncated	Whether the tweet is truncated in the database (False/True)
replyToSID	ID of the status this is in reply to
id	ID of this status
replyToUID	ID of the user this was in reply to
statusSource	Source user agent for this tweet
screenName	Screen name of the user who posted this status
retweetCount	The number of times this status has been retweeted
retweeted	If this status has been retweeted (False/True)
isRetweet	If the status is a retweet (False/True)

Note. This table shows the meta-features collected as part of the research on each tweet.

[Source: Gentry et al. (2016)]

6.1.3 Methodology

A script on the basis of the R packages "twitteR" and "plyr" was run on a VPS with a virtual RStudio server and executed regularly using a cron agent, in order to collect tweets about the German stock market. The script ran continuously from August 1st, 2018 to December 31st, 2021 inclusive, retrieving tweets about the German stock market with the hashtag #DAX via the Twitter API (authentication via OAuth authentication handshake). The entire theoretical process is shown in Figure 54.

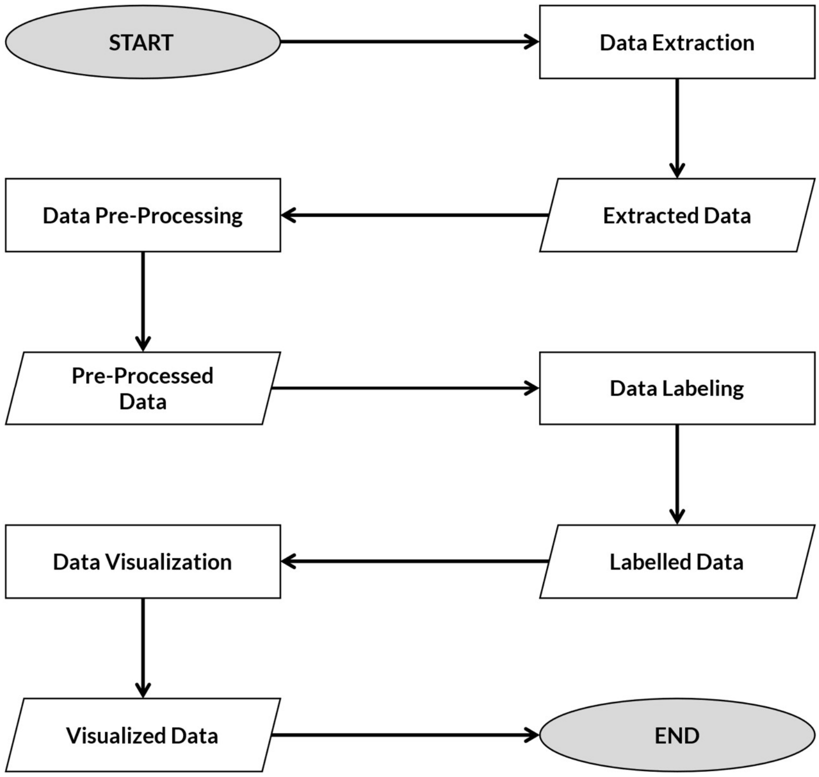

Figure 54. General Tweet-Pre-Processing for the Explorative Study.

[Source: Author's representation based on Pandey et al. (2019)]

The further evaluation was done in a Python version 3.8.5 environment. After the tweets were collected and the time window to be studied was identified, necessary standard cleaning and preparation steps were followed to better analyze the data.

This includes normalization to reduce morphological variation (here: lemmatization), removing duplicates based on the unique ID of each tweet, and cleaning the individual tweet text by removing links and special characters using regex statements.

All tweets were converted into a uniform source language (here: English) to support this and make the tweets ready for the actual investor sentiment analysis. The Python package "googletrans" version 3.0.0 was utilized to translate the tweets. This is also in line with the sentiment analysis methodology, which will be discussed in the following steps.

A reasonable approach can be traced in the work of Ahuja and Dubey (2017) in particular. The specific process used in this work is shown in Figure 55.

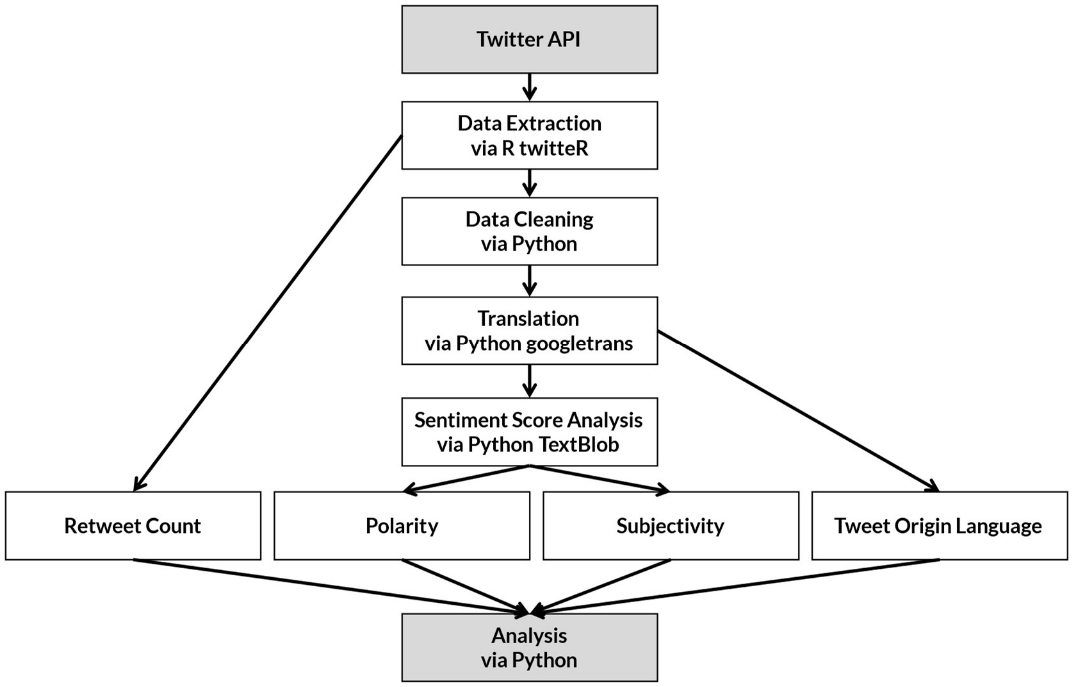

Figure 55. Specific Steps Applied in the Present Twitter Analysis.

[Source: Author's representation]

Processing the tweets through the "googletrans" package not only has the advantage that all tweets are subsequently available in a uniform language for further processing, but also that another valuable feature is added to the data set through the application of automatic speech recognition, the ISO 639-1 language code of the source language.

After the preparatory translation steps have been completed, the Python package "Textblob" in version 0.16.0 is applied to finally perform the sentiment analysis of the tweets after further preparation. Textblob is a ubiquitous package in sentiment analysis (Ahuja and Dubey 2017; Sohangir et al. 2018; Bonta and Janardhan 2019; Laksono et al. 2019; Gujjar and Kumar 2021). Furthermore, Textblob is a Python library for Natural Language Processing (NLP) that takes a lexicon-based approach to sentiment analysis so that a sentiment is defined by its semantic alignment and the intensity of each word in the sentence.

This requires a predefined dictionary that classifies negative and positive words [here: utilizing the Natural Language Toolkit (NLTK)]. By using the Python package "googletrans", all tweets are translated into English to leverage the same lexical source.

The sentiment extraction capability in Textblob returns a tuple of the form sentiment (polarity, subjectivity). Here, the sentiment of the tweet is measured directly. The polarity value is a floating-point number in the range $[-1.0, 1.0]$. Consequently, 1.0 means an absolutely positive statement in polarity and -1.0 means an absolutely negative statement.

The subjectivity is a floating-point number in the range [0.0, 1.0], where 0.0 is very objective, and 1.0 is very subjective. The variable subjectivity measures whether the tweet generally refers to personal opinions, feelings, or judgments, while objective sentences refer to factual information. Consequently, a subjectivity of 0.0 refers more to factual information, while a subjectivity of 1.0 assumes a personal opinion.

Textblob uses the technology of the Python platform NLTK. This makes use of various (> 50) corpora and lexical resources[29] and a suite of text processing libraries for classification to perform the analyses. For a better understanding, here are two examples based on tweets from the data collection:

On May 13th, 2020 at 1:26:12 p.m. a tweet originating from Germany said: "The DAX looks bad" and has a polarity of -0.7 and a subjectivity of 0.67, which means that the statement is negative and it is primarily a public opinion and not factual information.

[29] https://www.nltk.org/nltk_data/

In the wake of the steep recovery, the following tweet, originating from Germany, was posted on May 19th, 2020 at 5:23:56 a.m.: “Yesterday was the 32 best stock exchange day in the history of the DAX The crisis seems to be ticked off”and has a polarity of 1.0 and a subjectivity of 0.3, which means that the statement is totally positive and is rather more factual than a public opinion. Only the aftermath showed that the crisis, at least outside the stock market, was not yet over.

6.2 EMPIRICAL FINDINGS

An important preprocessing step is that tweets are translated using the Python package “googletrans” in version 3.0.0, which uses the Google Translate Ajax API to make calls to such methods as “detect” and “translate”. During the translation process, the Google API automatically detects the source language of the tweets. Within the framework of this process, it can be determined that the tweets on the German DAX stock index are available in a total of 70 different languages during the observation period.

However, the vast majority of tweets are written in English, followed by tweets in German, which should hardly come as a surprise when considering the German stock market.

From a German perspective, the high number of tweets in foreign languages seems logical, as around 55 percent of DAX shares are owned by foreign investors. Sixty-three percent of DAX shares are held by institutional investors. Investors from other European countries hold 26 percent of DAX shares, and 22 percent are held by investors from North America.[30]

Regarding the frequency of the tweets, the German language is followed by the Turkish and Spanish languages. Tweets in other languages combined are rather negligible, as they occur relatively rarely (see Figure 56).

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[30]Ernst & Young GmbH: https://assets.ey.com/content/dam/ey-sites/ey-com/de_de/news/2019/06/ey-wem-gehoert-der-dax-2019.pdf?download

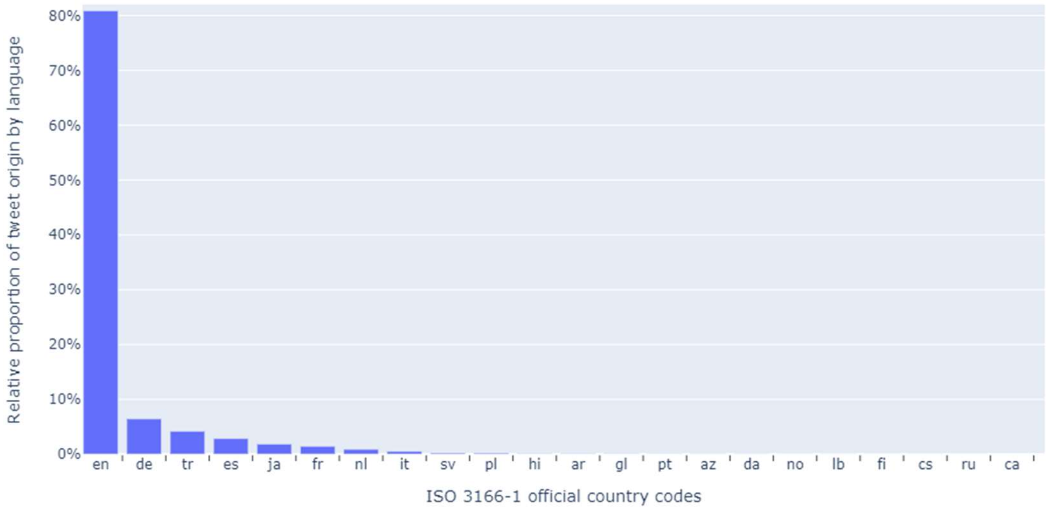

Figure 56. Language Share of Tweets in the Observation Period.

[Source: Author's representation]

First, in this exploratory data analysis, it is interesting to observe how the proportion of tweets with retweets changes. More tweets with retweets can be interpreted as indicating essential news from particularly exposed personalities and cause a furor, which is frequently shared.

Figure 57 visualizes the number of retweets in the COVID-19 window. It is visible that the number of retweets rises exceptionally high when the price drop is at its steepest. One hypothesis that could be derived from this is that the number of retweets is positively related to price development in terms of absolute value. It is observable that immediately before reaching the preliminary trough of the German stock market index CDAX, the number of these retweets increases.

Looking for the correlation between investor sentiment measured on Twitter and the actual market development, it becomes clear that an accumulation of retweets occurs at least during the observed extreme event.

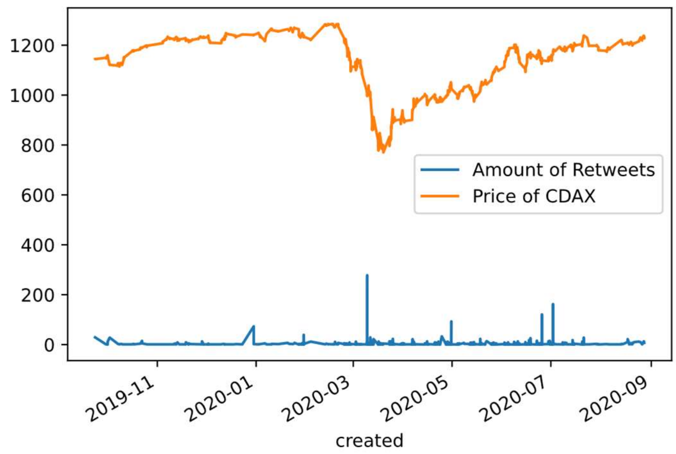

Figure 57. CDAX during the COVID-19 Observation Window (Tick Data).

[Source: Author's representation]

However, the retweet frequency can only be observed when the price trend has already been downward for several days, and the retweet frequency reaches its maximum level only shortly before reaching the market trough. Of course, this needs to be verified in further studies; however, based on the knowledge gained from these observations, it can be assumed that there is a correlation between retweet frequency and extreme sentiment-induced market events.

Nevertheless, looking at the retweet frequency seems to be rather unsuitable for prediction, as this measure only increases after the price decline has solidified to a high degree. Another interesting feature is the subjectivity of the tweets on the hashtag #DAX extracted with an NLP technique (text mining). For better readability, the tweet subjectivity and the CDAX price have been scaled to a standard interval and logarithmized in Figure 58.

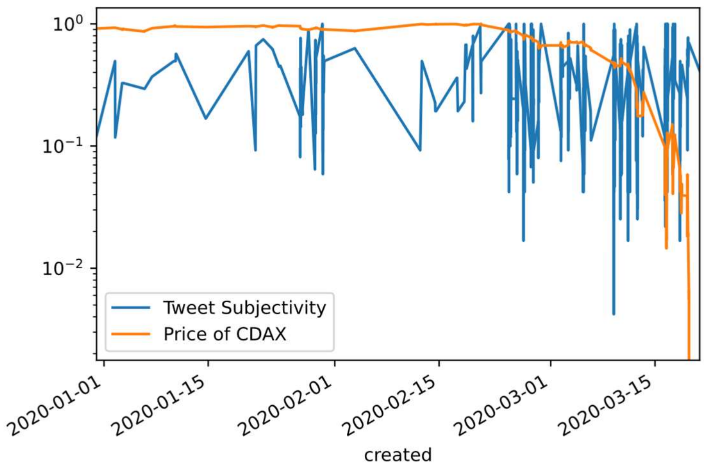

Figure 58. Logarithm of Scaled CDAX and Tweet Subjectivity.

[Source: Author's representation]

Figure 58 provides the first strong evidence that a large upstream increase in variation in Twitter subjectivity occurs before the price decline is actually observable in the market. Although the price decline, which is quite different from normal volatility, does not start until early March 2020 and peaks negatively on March 19th, strong variations in Twitter subjectivity precede this price decline. The strong variance in Twitter subjectivity remains throughout the period leading up to the August 2020 recovery (see Figure 59).

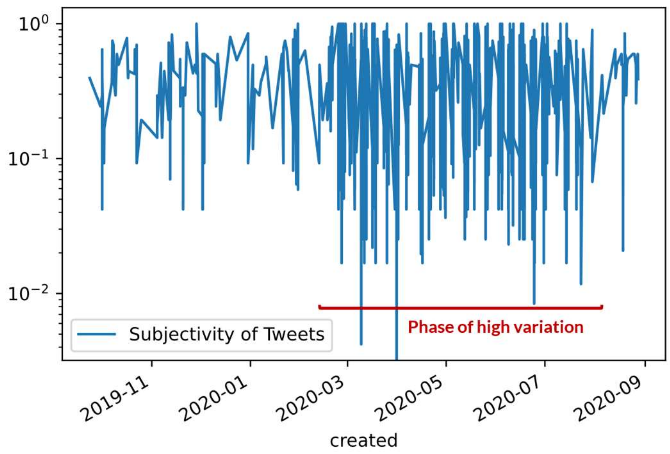

Figure 59. Logarithm of Scaled Tweet Polarity in the Window under Investigation.

[Source: Author's representation]

However, the relationship between investor sentiment on Twitter and the CDAX stock market price index can be observed by looking at the actual investor sentiment, that is, the polarity of tweets during the study period (Figure 60).

In the figure, the CDAX prices and the polarity of the tweets have been scaled for better readability. The graph clearly shows that investor sentiment from Twitter and market prices do not react simultaneously, which has already been confirmed empirically in other studies outside the German stock market.

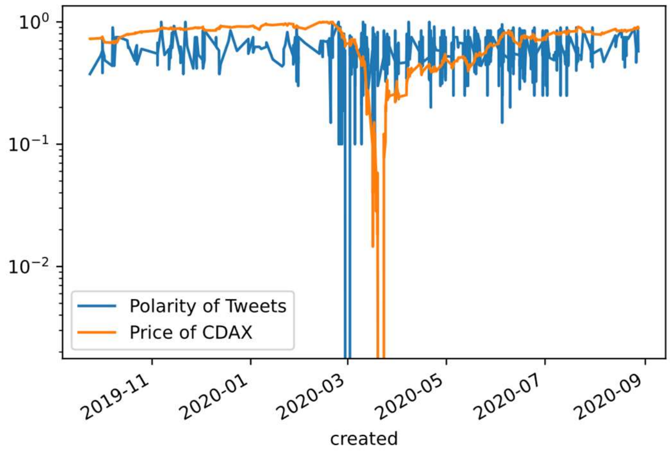

Figure 60. Logarithm of Scaled CDAX and Tweet Polarity.

[Source: Author's representation]

The sharp decline in twitter-based investor sentiment clearly precedes the actual price decline in the market. There are 17 to 20 days between the period with the most negative sentiment (February 28th, 2020 and March 2nd, 2020) and the day with the lowest quotation of the CDAX (March 19th, 2020).

The DAX developed similarly and also had its lowest quotation in the crisis period on March 19th, 2020, closing at 8,610.43 points. These results are consistent with Sul et al.'s (2017) observations that investor sentiment in tweets significantly affects stock market returns, even 20 days after posting.

The defined research hypothesis $\boldsymbol{RH}_6$ can be confirmed. Based on various metrics, such as the number of retweets, the subjectivity of the tweets, and the basis of the polarity of the tweets, it can be shown in the context of the explorative study that there is a connection between Twitter investor sentiment and the German market development.

$\boldsymbol{RH}_6$: *Investor sentiment-based events in social networks have an impact on market developments in the German stock market*

In Figure 60, it is also interesting to observe that the polarity of tweets remains volatile for months until the market price recovers to the pre-crash level.

In a future study, it may be interesting to test an LSTM-based or GARCH model incorporating various metrics of social investor sentiment. Compared to ARCH, the GARCH model also allows the conditional variance to depend on its own lagged values, which can also account for time-lags in investor sentiment.

Depending on the exact research question: If the goal is to model the volatility of a sentiment time series and there is no reason to suspect that the mean of the data changes over time, ARCH models would be the right choice. On the other hand, if there is reason to suspect that the mean of the data changes over time and both the volatility and the mean of the sentiment data are to be modeled, GARCH models would be a better choice.

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7 CONCLUSION

This dissertation addresses the topic of investor sentiment and the lower and higher-order statistical moments of the return distribution of the German stock market. The first part of the empirical study in Chapter 4 examined the extent to which the German stock market is fundamentally efficient and the contributions of established Fama-French and Carhart risk factors. A cross-sectional period of 20 years was considered. In addition, a PCA-based risk factor was tested for the German stock market based on 73 investor sentiment indicators. The following research hypotheses could be tested in the first empirical study:

$\boldsymbol{RH_1}$: *Investor sentiment contributes to explaining return variances in the German stock market.*

$\boldsymbol{RH_2}$: *Investor sentiment is a contra-indicator for stock market developments.*

$\boldsymbol{RH_3}$: *The integration of an investor sentiment risk factor into multi-factor models leads to a higher model quality compared to the Fama-French and Carhart target portfolio regression models, expressed by the adjusted coefficient of determination* $\bar{R}^2$.

$\boldsymbol{RH_4}$: *Incorporating an investor sentiment risk factor into multi-factor models leads to a lower alpha range in the Fama-French and Carhart target portfolio regression models.*

The traditional multi-factor model-based approach was able to show that both the coefficient of the PCA-based investor sentiment risk factor with and without lag were always demonstrating a statistical significance in multiple Fama-French portfolios out of the total 16 portfolios studied. The extended Carhart model's investor sentiment risk factor was significantly different from zero in nine of 16 portfolios. Therefore, the first research hypothesis $\boldsymbol{RH_1}$ can be considered confirmed.

However, it should be noted that the slope of the coefficients was relatively close to zero in some cases.

This is possibly due to the fact that the study was a cross-sectional investigation on the one hand and the assumed relationship in the multivariate-regression model was linear.

Since literature has shown that the explanatory contributions of investor sentiment are probably non-linear, not time-stable, and become relevant in extreme events, the traditional cross-sectional approach may not be so well suited to investigating the influence of investor sentiment on returns on the German stock market.

The study showed that investor sentiment and excess returns on the German stock market are negatively correlated. This applies to all models examined, regardless of whether the extended Fama-French three-factor model or the Carhart four-factor model is considered and whether the investor sentiment risk factor was considered with or without lag. Research hypothesis $\boldsymbol{RH_2}$ can, therefore, be considered confirmed with a high degree of certainty. Investor sentiment and excess returns on the German stock market are negatively correlated.

In research hypothesis $\boldsymbol{RH_3}$, it is hypothesized that integrating the PCA-based investor sentiment risk factor will increase model quality. The answer to this hypothesis is somewhat more ambivalent than the first two hypotheses. In the extended three-factor model an increase of $\bar{R}^2$ after integration of the PCA-based sentiment risk factor can be observed, namely 0.767 compared to 0.765. Also, in the extended four-factor model, $\bar{R}^2$ is increased from 0.774 to 0.777. Looking at these values, one could argue that the model quality is increased based on this metric, although the increase is merely marginal.

However, a direct comparison between the three-factor model with $\bar{R}^2 = 0.767$ and the four-factor model with $\bar{R}^2 = 0.774$ shows that the momentum risk factor seems superior to the investor sentiment risk factor in the cross-sectional analysis. Investor sentiment, thus, seems to be a good complement to the established risk factors, but it is not a substitute.

Regarding the fourth research hypothesis $\boldsymbol{RH_4}$, the evaluation of the empirical results is more straightforward. The alpha range decreases from 0.0252 to 0.0241 in the extended Fama-French three-factor model after integrating the PCA-based investor sentiment risk factor.

The improved drop in the alpha range is also much more visible in a direct comparison with Carhart's four-factor model, where the alpha range is 0.0251.

After integrating the PCA-based investor sentiment risk factor, the alpha range also drops to 0.0243 in Carhart's extended four-factor model. The quantity of stastitical significant alphas in the Fama-French portfolios stays the same and does not fluctuate, remaining at a steady seven. Research hypothesis $\boldsymbol{RH_4}$ can, therefore, be considered confirmed. Nevertheless, it is interesting to see that the three-factor model has only six out of 16 significant alpha constants, which is the lowest value in the entire study.

In the second of three empirical studies discussed in Chapter 5, there was deliberate refrain from a dimensional reduction of the investor sentiment indicators compared to the first study in Chapter 4. This is a necessary condition to address the problem of time-varying risk premia. This time, the model is an RNN architecture based on LSTM neurons to develop a predictive model. Research hypothesis $\boldsymbol{RH_5}$ is investigated in this part of the study.

$\boldsymbol{RH_5}$: *By taking the time-varying characteristics of investor sentiment into account, the explanatory power of investor sentiment increases perceptibly compared to traditional cross-sectional analyses.*

Also, different from the previous study, all statistical moments of the return distribution are considered, as variance is not the only substantial risk measure and it is possible that other essential moments relevant to portfolio management can also be well explained or predicted by investor sentiment.

In addition to variance, skewness and return itself can be well explained and even reliably trend-predicted in out-of-sample tests during the COVID-19 time window. The study also provided a relatively straightforward answer to the research hypothesis $\boldsymbol{RH}_5$. In addition to the very good explainability of all statistical moments of the return distribution, trends could even be predicted reliably and consistently in out-of-sample tests. Higher R^2 values compared to the cross-sectional analysis, even in out-of-sample tests, indicate a superior model quality in terms of goodness of fit.

Finally, the third and last empirical study in Chapter 6 looks at investor sentiment in the third category (social sentiment). As in the second study in Chapter 5, the time window COVID-19 was a suitable object of investigation here as well, as particularly extreme market movements were observed at this point, the cause of which could not initially be explained by the fundamental data.

Almost two million tweets were collected exclusively to answer the research hypothesis $\boldsymbol{RH}_6$ over a multi-year period and formed a sound basis for answering the research hypothesis:

$\boldsymbol{RH_6}$: *Investor sentiment-based events in social networks have an impact on market developments in the German stock market.*

Utilizing the metrics polarity and subjectivity extracted by the NLP technique text mining and the purely quantitative metric "retweetCount", it could be shown that investor sentiment from Twitter on the German stock market has a substantial influence on the market development. Based on polarity, it could be shown that the change in sentiment on Twitter preceded the actual market movement, indicating suitability with regard to the use of these findings in explanatory and predictive models. In part, results were shown in the area of prediction quality that go beyond the answers to the research hypotheses. Thus, it remains to be stated that all research hypotheses could be confirmed in the dissertation.

As to the question of why investor sentiment-based models work despite the EMH, it is evident: Certain patterns occur in time series. These patterns can occur (almost) randomly or are difficult to predict, but investors react to them. At a glance, the results of each research hypothesis can be reviewed in Table 15.

Table 15. Overview of the Key Results of the Thesis.

Hypothesis	Content	Empirical Study	Evaluation
RH_1	Investor sentiment contributes to explaining return variances in the German stock market	Chapter 4: Risk Factor Integration in Multi-factor Models	The hypothesis is confirmed
RH_2	Investor sentiment is a contra-indicator for stock market developments	Chapter 4: Risk Factor Integration in Multi-factor Models	The hypothesis is confirmed
RH_3	The integration of an investor sentiment risk factor into multi-factor models leads to a higher model quality compared to the Fama-French and Carhart target portfolio regression models, expressed by the adjusted coefficient of determination $\bar{R}^2$	Chapter 4: Risk Factor Integration in Multi-factor Models	The hypothesis is confirmed
RH_4	Incorporating an investor sentiment risk factor into multi-factor models leads to a lower alpha range in the Fama-French and Carhart target portfolio regression models	Chapter 4: Risk Factor Integration in Multi-factor Models	The hypothesis is confirmed
RH_5	By taking the time-varying characteristics of investor sentiment into account, the explanatory power of investor sentiment increases perceptibly compared to traditional cross-sectional analyses	Chapter 5: LSTM Based Study	The hypothesis is confirmed and even partially exceeded, as trend prediction also yields promising results for several statistical moments
RH_6	Investor sentiment-based events in social networks have an impact on market developments in the German stock market	Chapter 6: Explorative Social Sentiment Study	The hypothesis is confirmed

Note. This table presents a summary of the findings of the research hypotheses examined in this dissertation.

[Source: Author's representation]

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8 DISCUSSION

In this chapter, the most important assumptions and results of the thesis will be discussed critically, and the question of why, for example, the application of generalized autoregressive conditional heteroskedasticity or autoregressive (Integrated) Moving Average ((G)ARCH, AR(I)MA) models was deliberately not applied will be clarified. A classification of the results into higher theoretical issues, such as the question of efficient markets, is also discussed.

The stock market cycle, with bull and bear markets, is considered a quasi-natural element of the market in the literature. However, in empirical reality, market prices deviate far more from the postulated theoretical equilibrium than they actually approach it. This dissertation posits that mass psychological dynamics are a contributing factor to non-mean-reverting patterns in the stock market. These patterns can be observed in the overall behavior of individuals, and are driven by non-rational impulses perceived by market participants in complex and uncertain situations (Fenzl and Pelzmann 2012).

This thesis examines the discrepancy between traditional theory, which suggests that competition among rational investors who spread their investments leads to market balance, where the value of each share corresponds to the calculated rational anticipated cash flows, and the presence of specific risk premiums in equity markets and the overall risk premiums of equities.

Classical theory states that the correlation between returns and risks is solely determined by the cross-section of systematic risks. The CAPM is widely used as a foundation for multi-factor models and is utilized to calculate the cost of equity in uncertain conditions.

Recent research (as outlined in Chapter 3.2) suggests that investor sentiment can have an impact on risk factors and help explain variations in returns.

The linear regression analyses presented in the initial portion of the empirical study (see Chapter 4) demonstrate that investor sentiment can serve as an additional risk factor in understanding stock market yields in Germany.

The observations and the literature with empirical evidence from other countries partially support the assumption of Baker and Wurgler (2007), who find that investor sentiment has a larger effect on securities that are difficult to price with traditional asset pricing models.

In synopsis, investor sentiment factors significantly impact stock returns and can increase the explanatory power of three- and four-factor models. The findings of this thesis demonstrate that it is possible to convert the abstract concept of aggregate investor sentiment into a measurable risk factor, and that this approach is beneficial in terms of its explanatory contribution to returns and its ability to enhance the goodness of fit of multi-factor models. In light of the above, the $\bar{R}^2$ gain obtained from investor sentiment risk factors in the cross-sectional study is significantly different from zero but relatively modest. The significance of the Fama-French factors is reflected in their strong empirical relevance and partly due to the nature of the cross-sectional study, as the investor sentiment risk factor, in particular, is not stable over time and becomes more volatile, especially in times of crisis.

In addition to the findings from the formal hypotheses, the cross-sectional study documents four main results based on the monthly data of CDAX-listed companies. First, the examined PCA-based risk factors of investor sentiment, on aggregate, give reason to believe that investor sentiment contributes to improving the empirical model. However, the cost/benefit ratio in terms of effort is relatively high for a comparatively small improvement in the model.

Second, the studied investor sentiment risk factor is predominantly weakly correlated with the Carhart risk factors, making it generally suitable for integration into the existing, widely accepted multi-factor models of Fama-French and Carhart. However, this statement refers to the cross-section of the sample period. When using the risk factors in linear regression models, there is always a probability that the independent variables suddenly become highly correlated in times of crisis. The diversification effect may be lost just when it is most needed from a portfolio manager's perspective.

Third, it is evident that the Fama and French three-factor model (1993) provides a superior explanation for the variance of returns, with an $\bar{R}^2$ of 0.765, compared to the CAPM's $\bar{R}^2$ of 0.685.

Additionally, the inclusion of the momentum factor in Carhart's four-factor model slightly improves its explanatory power, resulting in an $\bar{R}^2$ of 0.774. Integrating investor sentiment risk factors improved model performance in the sample, both in the extended three-factor model ($\bar{R}^2 = 0.767$) and in the extended four-factor model ($\bar{R}^2 = 0.777$). The results show that especially the Fama-French factors in the cross-section explain a large part of the variance.

On the comparatively small terrain of unexplained variance, additional risk factors, such as momentum (WML) or investor sentiment ($SENT$), can only marginally improve an already very good initial target model.

Fourth, multi-factor models enhanced with investor sentiment factors perform better in terms of alpha range and keep insignificances stable. In the single-factor model (CAPM), for example, the autonomous return components a_i are largely unexplained by the factors and are statistically different from zero in 11 out of 16 Fama-French portfolios examined and exhibit a high range. Here, the strength of investor sentiment is shown virtually in the second row. By integrating the PCA-based investor sentiment factor, the alpha significance can be further reduced to a stable seven and, thus, substantially improved. The alpha range in the models extended by the PCA-based investor sentiment risk factor is the lowest among the multi-factor models examined.

The practical applicability of the findings in this study regarding the role of investor sentiment factors in the German stock market remains uncertain, and further research with higher resolution is needed to fully assess the potential benefit of these findings. The results of the LSTM-based study (see Chapter 5) of this thesis and also the results of the social investor sentiment study (see Chapter 6) show that in times of crisis, such as the COVID-19 period, a very good explanation of the stock market can be provided by investor sentiment, especially when applied into an artificial LSTM neuron-based network architecture.

Whether such good explainability can be achieved outside of extreme events remains to be seen. However, results from other countries provide empirical evidence for this assumption. In the cross-section, however, the indicator of investor sentiment with dimensionality reduction has shown that it does not have high explanatory power compared to the studies in the COVID-19 period.

In addition to scientific relevance, there is the question of what practical relevance the results have. However, it is immediately conceivable that robo-advisors and smart-beta strategies could benefit from it, for example, in the area of countercyclical portfolio management. However, it should be noted that care should be taken when incorporating multiple sentiment factors in a single model. Although not typically correlated, there may be instances of increased correlation during times of high market volatility, which could potentially decrease the intended diversification effects. This assumption needs to be verified in further research.

Backtesting procedures could validate trading strategies, while the LSTM and Twitter study signals would have been tradable already during the COVID-19 time window. However, further progress may be made in this context very soon. For example, research is underway on new forms of dimensionality reduction that are likely to be more suitable for sentiment-based research (Liang et al. 2022).

The question that may arise for the reader is why a generalized autoregressive conditional heteroskedasticity model for the conditional variance of the process or an autoregressive integrated moving average model for the conditional mean of the process was not applied instead of the LSTM-based study.

Traditionally, the methodology of the quantitative strategy involves the use of linear regressions, ARIMA models, and GARCH models to capture the characteristics of time series and to capture the stochasticity of volatility. For some time now, the quantitative trading industry has moved into the era of "deep learning", which is more commonly used nowadays.

Nevertheless, studies based on autoregressive models also exist in this investor sentiment-based research, some of which produce solid results. One of the advantages of LSTM over other simple autoregressive models is that it can use any number of input characteristics and is not bound by the constraints of autoregressive models. For example, the time series of intertemporally correlated sentiment indicators or even macroeconomic parameters can be included instead of relying only on an investor sentiment indicator (Zou and Qu 2020).

An important question that should still be discussed in this chapter is how to place the results in the context of overarching theoretical questions.

In Chapter 2.2, it was described that all research dealing with the predictive part of investor sentiment in stock markets implies a partial violation of the assumptions of the EMH.

This is because the market does not seem to have (fully) priced in the information extracted from investor sentiment, especially in times of crisis, so an LSTM-based model provides plausible trend predictions for lower- and higher-order statistics of the return distribution in the German stock market. The EMH states that security prices at any point in time contain all publicly available information about those securities. Therefore, the current market price is the best estimate of the future market price. The best estimate means that any other estimate is less likely to be correct. The best estimate does not mean that it must constantly prove to be correct.

A notable critical aspect of the EMH is the random walk theory, which states that historical price patterns are not predictive of future price patterns. If the market receives new information, the market price adjusts rapidly.

Publicly available information need not be facts, but naturally includes uncertain expectations and conjecture. This is probability-weighted information that is not fundamentally different from specific information in terms of its impact on price. Old information can be interpreted differently based on new evidence to trigger a price effect again or for the first time. Investor sentiment is also information. Thus, information efficiency does not mean that all information must be accurate in the future. Also, the EMH does not apply to every market, but only to established, well-developed, well-functioning capital markets, such as the well-regulated segments of the stock market. This is why researchers suggest that illiquid small companies, in particular, can be explained by investor sentiment (Kumar and Lee 2006; Barber and Odean 2008).

Active investors, that is, more than 90% of all market participants, obviously consider EMH to be at least partially wrong (Kommer 2018). Otherwise, the costly and tedious search for mispriced securities and bearing high risks that are not diversified would not make sense (Kommer 2018). On the other hand, the historical existence of market anomalies does not disprove EMH. A refutation of EMH would exist if market anomalies persisted permanently after their discovery.

However, this is not the case when risk, transaction costs, market structural arbitrage barriers, and taxes are taken into account. The EMH does not always seem correct, as regularly empirically confirmed by the empirical results of this and other studies.

Nevertheless, there is no better theory of price formation, so it cannot be replaced yet, and with each discovery of market anomalies or inefficiencies, the market eventually becomes more efficient, as they are "arbitrated away". This happens sooner or later with all other market anomalies that are made transparent. As soon as these strategies become public, their effectiveness fizzles out, as many market participants exploit the profit (McLean and Pontiff 2016).

9 LIMITATIONS AND FUTURE LINES OF RESEARCH

Precise return forecasts should be completely impossible based on the random walk hypothesis. However, several studies have produced empirical evidence that various risk factors influence future returns. This does not disprove the EMH, as shown in the previous discussion chapter. Nevertheless, it can be assumed that there are phases in which EMH does not seem to be particularly meaningful. This is specifically the case in times of crisis, when strong market fluctuations can be observed without any significant change in the fundamental data situation. Then, market psychology can be the driving force in stock markets, as shown in this dissertation using the example of COVID-19-induced market turbulence in the German stock market in an LSTM-based study and a Twitter study.

However, the out-of-sample tests performed on COVID-19 cannot necessarily be extrapolated to other yet unknown events in the future. The results of the LSTM-based study do show that artificial neural networks can give an excellent explanation of the stock market in times of crisis, such as the COVID-19 period.

Nevertheless, as mentioned above, whether such good explanatory power can be achieved outside extreme events, only broad empirical evidence in several countries, different maturities, and the consideration of various investor sentiment indicators will be able to show. As computational capacities are expected to become more affordable in the near future, even in the cloud domain, such models, which require much higher computational capacities than simple linear regression models, will become more important and more visible.

Based on the observations, further considerations should be made for analyses of the German stock market and other markets. The focus should be on which period investor sentiment significantly influences the return distribution's lower- and higher-order statistics. Another important aspect to consider is determining the optimal lag time between detecting investor sentiment and making investment decisions.

For example, Sul et al. (2017) show that Twitter sentiment is still significant 20 trading days after the investor sentiment detection.

The observations in the Twitter-based study in this thesis also suggest this. It would be surprising if the market were to actually take that long to price this information.

Further research is necessary at this point. This thesis examines only intraday investor sentiment and investor sentiment on a monthly basis. Various empirical works show that weekly surveys also have a significant impact on stock market returns. Of particular note in this regard is the work of Hilliard et al. (2016, 2020), which shows that weekly survey-based investor sentiment makes significant explanatory contributions to stock market returns.

In addition, it is crucial to investigate the extent to which investor sentiment might also influence higher statistical moments on a weekly or daily basis. News from outside the company also affects its stock prices, often more than news from the company itself, but only ever in the short and medium terms, not the long term (Sommer 2019).

Shen et al. (2018) show that daily Twitter sentiment affects the skewness of stock return distributions. Investor sentiment risk coefficients could also be estimated using ARCH or GARCH models in the context of time series analysis, but with higher independent variable requirements, as discussed in the chapter before.

In addition, panel regression could also be used to combine the advantages of cross-sectional and longitudinal analyses. It would also be conceivable to construct a sentiment index for the German stock market with subsequent evaluation using PCA that also includes investor sentiment from social media. Regardless of the results of this and further work for the German stock market, only broad cross-country validity will lead to a sustainable establishment of investor sentiment risk factors.

An additional sample based on weekly returns could have provided additional insights into the German stock market. Methodologically, there are also other ways to determine investor sentiment risk factors in cross-sectional analyses. Here, it is also advisable to conduct further research.

This thesis uses the 10% and 90% quantiles of returns to determine the investor sentiment risk factor in the cross-section, while other papers, such as Hilliard et al. (2016), use the 30% and 70% quantiles following Carhart's momentum factor. The background of this thesis approach is that a strong influence of investor sentiment is assumed, especially for stocks with a strong correlation to the respective source of investor sentiment.

Another notable feature of this thesis is the consideration of equally weighted returns in the cross-section. Consideration of market capitalization-weighted returns could also be helpful in future research. Forthcoming studies with market capitalization-weighted indices could provide even more meaningful results for large caps, as it can be assumed that DAX30/40 stocks, in particular, are the subject of public discussions in Germany, especially on social media.

A more substantial weighting within the Fama-French portfolios should, therefore, further increase the respective explanatory contributions. The equally weighted return approach conducted in this thesis, thus, represents a more conservative procedure than the analyses of the Fama and French multi-factor models.

Moreover, the impact of the negative risk-free investment opportunity available during the study period is worth discussing. This cannot be explained by risk factors, but only by monetary policy influences. An investment in a hypothetical portfolio with a CAPM beta factor of zero should be geared exclusively to capital preservation, regardless of general market developments, and should correspond to the negative risk-free interest rate level, which was partially present during the period under investigation.

De facto, if transaction and insurance costs are neglected, the (retail) investor can also hold cash for free and, thus, achieve an autonomous positive risk-free premium over the market. It can be assumed that the variance of stock market returns in the period under study was significantly influenced by the ECB's zero and negative interest rate policy.

Different maturity bands representing different monetary policy stances, including financial crises, should be examined for further clarification. Considering the numerous meaningful research results on Twitter (see Sections 3.2.3 and 6), especially for the U.S. market, further separate Twitter investor sentiment studies for the German stock market should be reviewed.

Social sentiment might be the future, as, for example, Bollen et al. (2011) show that survey-based sentiment is less suitable for integration into multi-factor models than social media sentiment. Furthermore, the question arises of which actors or events have a particular influence on investor sentiment. If the assumption that investor sentiment influences returns is confirmed, the question of the manipulability of investor sentiment inevitably arises. In particular, investor sentiment from social media, which fluctuates daily, could be affected by this.

If investor sentiment was increasingly incorporated into financial market models, actors could actively try to alter particular investor sentiments in social media (e.g. in the short term with bot networks) to indirectly manipulate the stock market by triggering buy or sell signals in these models.

Fake news and conspiracy myths are currently the subject of intense discussion since it was announced that Elon Musk would take over the short message service Twitter to reshape it according to his ideas.[31] One could see a gatekeeper problem here. Musk's understanding of freedom of expression is relatively permissive: If he had his way, factually untrue claims might be protected, and a Massachusetts Institute of Technology study shows what that can lead to.

The researchers examined Twitter records and found that fake news spreads significantly faster on the platform than real news and reaches more users because it comes as a surprise and attracts attention (Vosoughi et al. 2018).

[31] When private individuals with their own particular interests act as gatekeepers for public-relevant information infrastructure, this can lead to disadvantages with regard to the common good.

In conclusion, it is interesting to put the obtained results in the context of larger theoretical questions. Both the empirical research conducted in this thesis and the empirical evidence from other studies show, sometimes more, sometimes less, that, first, investor sentiment is able to explain both returns and other risk measures, such as variance and skewness, and second, that investor sentiment is able to predict returns and other risk measures, such as variance and skewness in trend.

The question is whether the EMH can be considered disproven by this empirical evidence. The possibly somewhat unexpected result is that it cannot be confirmed. In some respects, the opposite is true, as discussed in the previous chapter. The growing recognition that investor sentiment is a risk factor that is offset by market returns will potentially lead the market to become more efficient in the long run. This is because the absorption of the risk factor due to investor sentiment is likely to intensify, so the opportunity to exploit the risk factor that generates returns is likely to decrease.

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APPENDIX

ANNEX 1. List of investor sentiment indicators utilized in the multi-factor model-based study and the LSTM RNN based study.

Indicator	**Source/ TRDS Mnemonic**	**Reference**
BD CONSUMER CONFIDENCE INDICATOR - GERMANY SADJ (GFK)	BDCNFCONQ	Finter et al., 2012
BD G-MIND: GERMAN MARKET INDICATOR-STOCKS (RANGE -10 TO +10) NADJ (G-Mind)	BDGMSTCKR	Finter et al., 2012
SENTIX NEUTRAL 1 M -DAX INDEX - ECONOMIC SERIES	DAXINU1	Finter et al., 2012
DAX (XETRA) TURNOVER - TURNOVER BY VALUE	FFOVDAX	Finter et al., 2012
Delta of in- and outflows of German open end equity mutual funds	Deutsche Bundesbank, own calculations	Finter et al., 2012
Equity issuance to aggregated debt issuance, E/D-Ratio	Deutsche Bundesbank, own calculations	Finter et al., 2012
EUREX BOND OPTIONS PUT/CALL RATIO - PRICE INDEX	EUXBFPC	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
EUREX INDEX OPTIONS PUT/CALL RATIO - PRICE INDEX	EUXDIPC	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
EUREX STOCK OPTIONS PUT/CALL RATIO - PRICE INDEX	EUXCAPT	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
EUREX TOTAL OPTIONS PUT/CALL RATIO - PRICE INDEX	EUXTOTL	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY

Indicator	**Source/ TRDS Mnemonic**	**Reference**
BD IFO BUSINESS CLIMATE GERMANY: BUS CLIMATE, INDEX VOLA	BDCNFBUSQ	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD CPI: TOTAL NADJ	BDCONPRCF	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD NEW PASSENGER CAR REGISTRATIONS VOLN	BDCAR...P	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD UNEMPLOYMENT: \% CIVILIAN LABOUR(\% DEPENDENT LABOUR TO DEC 196	BDUN\%TOTR	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD CONSUMER CONFIDENCE INDICATOR - GERMANY SADJ	BDCNFCONQ	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD IFO BUSINESS CLIMATE GERMANY EXPECT IN 6MO, INDEX VOLA	BDCYLEADQ	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD INDL PROD: MANUFACTURING (CAL ADJ) VOLA	BDIPMAN.G	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD DAX SHARE PRICE INDEX, EP NADJ	BDSHRPRCF	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD INDL PROD: INDUSTRY INCL CNSTR (CAL ADJ) VOLA	BDIPTOT.G	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD CPI (\%YOY) NADJ	BDCONPR\%F	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY

Indicator	**Source/ TRDS Mnemonic**	**Reference**
BD MANUFACTURING ORDERS (CAL ADJ). VOLA	BDNEWORDG	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD CURRENT ACCOUNT BALANCE CURN	BDCURBALA	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD EMPLOYED PERSONS (RESIDENCE CONCEPT, ILO) VOLA	BDEMPTOTO	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD EXPORTS OF GOODS (FOB) CURA	BDEXPGDSB	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD HICP: TOTAL NADJ	BDCPHARMF	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD NEW ORDERS RECD: CNSTR - RESL CNSTR VOLA	BDHOUSE.G	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD RETAIL SALES EXCL CARS (CAL ADJ) X-12-ARIMA VOLA	BDRETTOTG	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD RETAIL SALES EXCLUDING CARS INDEX VOLN	BDRETTOTH	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD BOP CAPITAL \ FINANCIAL ACCOUNT BALANCE (PAN BD M0790) CURN	BDCAFBALA	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD BOP: VISIBLE TRADE BALANCE CURA	BDVISBOPB	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY

Indicator	**Source/ TRDS Mnemonic**	**Reference**
BD EMPLOYED PERSONS (RESIDENCE CONCEPT) ((\%YOY) VOLA	BDEMPTO\%O	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD EUROPACE HEDONIC HOUSE PRICE COMPOSITE INDEX NADJ	BDHOUPRCF	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD EXPORTS FOB (PAN BD M0790) (\%YOY) CURA	BDEXPBO\%B	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD FIBOR - 3 MONTH (MTH.AVG.) NADJ	BDINTER3	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD GERMAN MARKS TO US\$ (MTH.AVG.) NADJ	BDXRUSD.	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD IMPORTS OF GOODS (CIF) CURA	BDIMPGDSB	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD INSOLVENCIES - BUSINESS ENTERPRISES VOLN	BDBNKRPTP	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD INTERNATIONAL RESERVES CURN	BDRESERVA	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD LENDING TO ENTERPRISES \ INDIVIDUALS CURN	BDBANKLPA	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD MNY.SUPL-M3(CONTRIB TO EUR BASIS FM.M0195), FM M06 2010 EXC	BDM3....B	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY

Indicator	**Source/ TRDS Mnemonic**	**Reference**
BD MONEY SUPPLY-GERMAN CONTRIBUTION TO EURO M1(PAN BD M0790)	BDM1....A	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD MONEY SUPPLY- M2 (CONTRIBUTION TO EURO BASIS FROM M0195) CURA	BDM2....B	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD PPI: INDL. PRODUCTS, TOTAL, SOLD ON THE DOMESTIC MARKET NADJ	BDPROPRCF	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD PRODUCTIVITY: OUTPUT PER MAN-HOUR WORKED,M\Q\MFG SCT(B+C)	BDPRODVTQ	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD RETAIL SALES EXCL CARS (CAL ADJ) X-12-ARIMA SADJ	BDRETTOTE	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD TERMS OF TRADE (PAN BD FROM 1991) NADJ	BDTOTPRCF	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD TOTAL EXPORTS OF GOODS CURN	BDEXPBOPA	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD TOTAL IMPORTS OF GOODS CURN	BDIMPBOPA	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD UNEMPLOYMENT LEVEL (PAN BD FROM SEPT 1990) VOLN	BDUNPTOTP	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD VACANCIES (PAN BD FROM M0790) VOLN	BDVACTOTP	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY

Indicator	Source/ TRDS Mnemonic	Reference
BD VISIBLE TRADE BALANCE CURA	BDVISGDSB	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD WAGE \ SALARY, OVERALL ECONOMY-ON A MTHLY BASIS(PAN BD M0191)	BDWAGES.F	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
SALARY: ON HRLY. BASIS - PRDG. SECTOR (BDHRWAGEF) NADJ	BDWAGMANF	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD EXPORT PRICE INDEX NADJ	BDEXPPRCF	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD IMPORT PRICE INDEX NADJ	BDIMPPRCF	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD IMPORTS CIF (PAN BD M0790) (\%YOY) CURA	BDIMPBO\%B	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD MONEY SUPPLY M0 CURN	BDM0....A	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD WAGE \ SALARY, OVERALL ECONOMY - ON A MTHLY BASIS (\%YOY) NADJ	BDWAGES\%F	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD WAGE\ SALARY, HRLY.BASIS - PRDG. SECTOR (BDHRWAGEF)(\%YOY) NADJ	BDWAGMA\%F	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD INFLATION NADJ	BDCPANNL	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY

Indicator	**Source/ TRDS Mnemonic**	**Reference**
BD UNEMPLOYMENT REGISTERED (PAN BD FROM JAN 1992) (CAL ADJ) VOLA	BDUNPTOTO	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD CPI (CAL ADJ) SADJ	BDCONPRCE	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD MANUFACTURING ORDERS SADJ	BDNEWORDE	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD BOP: EXPORTS FOB CURA	BDEXPBOPB	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD BOP: IMPORTS CIF CURA	BDIMPBOPB	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD BUSINESS EXPECTATIONS (PAN GERMANY) (\%YOY) SADJ	BDCYLE\%D	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD COMPOSITE LEADING INDICATOR - TREND RESTORED SADJ	BDCYLEADT	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD EXPORT PRICE INDEX (CAL ADJ) SADJ	BDEXPPRCE	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD IMPORT PRICE INDEX (CAL ADJ) SADJ	BDIMPPRCE	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD TERMS OF TRADE (ON THE BASIS OF PRICE INDICES) (CAL ADJ) SADJ	BDTOTPRCE	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY

Indicator	Source/ TRDS Mnemonic	Reference
BD US \$ TO 1 EURO (DEUTSCHEMARK DERIVED HISTORY PRIOR 1999) NADJ	BDXRUSE.	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY
BD VACANCIES (DEC 1999 ONWARDS NEW DEFINITION) VOLA	BDVACTOTO	TRDS Key Indicator List for Germany Mnemonic: M\#BDKEY

Note. This Annex shows the list of investor sentiment indicators utilized in the multi-factor model-based study and the LSTM RNN based study.

[Source: Author's representation]

ANNEX 2. Linear Regressions of the Monthly Excess Returns of $i = 1, \ldots, 16$ Fama-French Portfolios (Lagged Investor Sentiment enhanced Empirical Three-factor Model).

$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + \psi_i \cdot SENT_{PCA_{t-1}} + \varepsilon_{it}$				
	Ratios of book value to market value			
	1 (Low)	2	3	4 (High)
Market Value	Approximated value for β_i (CAPM)			
1 (Small)	1.032***	0.934***	1.065***	1.311***
2	0.971***	0.966***	0.919***	1.063***
3	0.877***	1.052***	0.992***	1.144***
4 (Big)	0.904***	0.959***	1.051***	1.260***
	Approximated value for s_i (SMB)			
1 (Small)	1.081***	0.560***	0.275**	1.182***
2	0.098	-0.099	-0.100**	0.107
3	-0.350***	-0.360***	-0.324***	-0.296***
4 (Big)	-0.612***	-0.528***	-0.608***	-0.690***
	Approximated value for h_i (HML)			
1 (Small)	-0.887***	-0.394**	0.093	0.637***
2	-0.263***	-0.099	-0.059	0.122
3	-0.232***	0.060	0.047	0.097
4 (Big)	-0.212***	0.053	0.129***	0.278***
	Approximated value for ψ_i (SENT)			
1 (Small)	-0.001	0.001	-0.001	0.002**
2	-0.001	-0.001*	-0.001	0.000
3	0.000	0.000	-0.001**	-0.001**
4 (Big)	0.001***	0.000	0.000	-0.001*

	Corrected coefficient of determination R_k^2			
1 (Small)	0.785	0.415	0.480	0.734
2	0.694	0.786	0.821	0.727
3	0.797	0.910	0.902	0.779
4 (Big)	0.867	0.917	0.882	0.767
Significance levels: 10% (*), 5% (**), and 1% (***).				

Note. This Annex documents the estimates of the regression analysis of the Fama-French-compatible three-factor model enhanced by a lagged investor sentiment factor. In addition to the coefficients of the $RMRF$ risk factor, the size and book/market ratio-factor as well as the corrected coefficient of determination and the investor sentiment factor are shown.

[Source: Author's representation]

ANNEX 3. Linear Regressions of the Monthly Excess Returns of $i = 1, \ldots, 16$ Carhart Portfolios (Lagged Investor Sentiment Enhanced Empirical Four-factor Model).

$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot RMRF_t + s_i \cdot SMB_t + h_i \cdot HML_t + w_i \cdot WML_t + \psi_i \cdot SENT_{PCA_{t-1}} + \varepsilon_{it}$				
	Ratios of book value to market value			
	1 (Low)	2	3	4 (High)
Market Value	Approximated value for β_i (CAPM)			
1 (Small)	1.026***	1.090***	1.043***	1.152***
2	1.078***	1.029***	0.929***	0.978***
3	0.981***	1.122***	1.011***	1.083***
4 (Big)	0.980***	1.001***	1.070***	1.208***
	Approximated value for s_i (SMB)			
1 (Small)	1.077***	0.658***	0.262**	1.083***
2	0.165**	-0.059	-0.093**	0.053
3	-0.285***	-0.316***	-0.313***	-0.335***
4 (Big)	-0.564***	-0.501***	-0.596***	-0.723***
	Approximated value for h_i (HML)			
1 (Small)	-0.884***	-0.471***	0.103	0.715***
2	-0.316***	-0.131**	-0.064	0.164**
3	-0.283***	0.025	0.037	0.127*
4 (Big)	-0.249***	0.032	0.119**	0.304***
	Approximated value for w_i (WML)			
1 (Small)	-0.015	0.366***	-0.049	-0.371***
2	0.251***	0.149***	0.026	-0.201***
3	0.243***	0.164***	0.044	-0.145***
4 (Big)	0.179***	0.100***	0.046	-0.123*

	Approximated value for ψ_i (SENT)			
1 (Small)	-0.001	0.000	-0.001	0.002***
2	-0.001**	-0.001	0.000	0.001
3	0.000	0.000	0.000**	-0.001**
4 (Big)	0.001***	0.000	0.000	-0.001
	Corrected coefficient of determination R_k^2			
1 (Small)	0.784	0.438	0.479	0.757
2	0.716	0.795	0.820	0.740
3	0.827	0.921	0.902	0.785
4 (Big)	0.882	0.922	0.883	0.769
Significance levels: 10% (*), 5% (**), and 1% (***).				

Note. This annex documents the estimates of the regression analysis of the Carhart-compatible four-factor model enhanced by a lagged investor sentiment factor. In addition to the coefficients of the $RMRF$ risk factor, the size and book/market ratio-factor and the momentum factor as well as the corrected coefficient of determination is shown.

[Source: Author's representation]

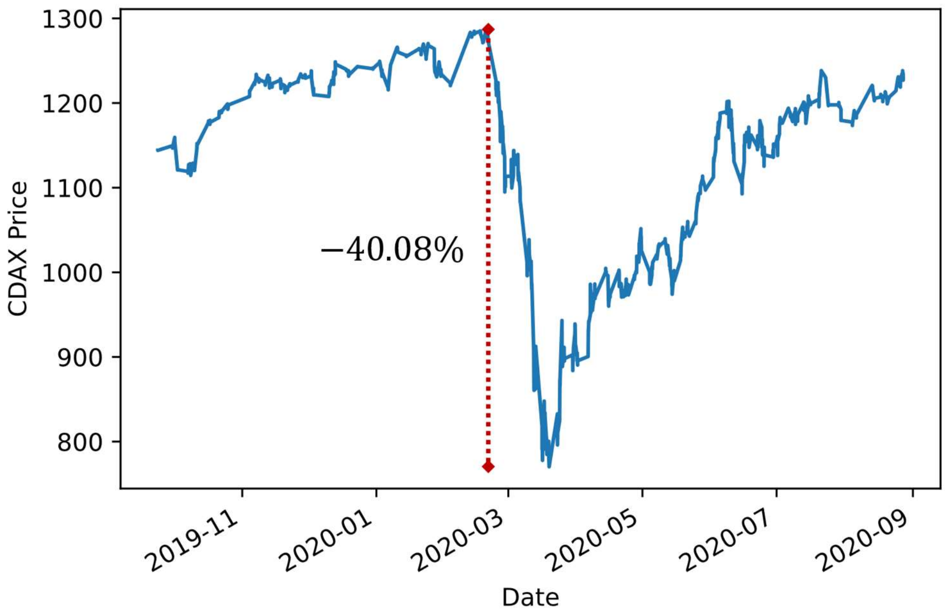
ANNEX 4. Equations Used for Determination of Return-Distribution Moments.

$$Variance = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

$$Skewness = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i - \bar{x}}{\sigma}\right)^3$$

$$Kurtosis = \frac{N(N+1)}{(N-1)(N-2)(N-3)}\sum_{i=1}^{N}\left(\frac{x_i - \bar{x}}{\sigma}\right)^4 - \frac{3(N-1)^2}{(N-2)(N-3)}$$

ANNEX 5. Price Decline of the CDAX Based on Minute-by-Minute Tick Data



Note. This annex documents the price decline of the CDAX during the COVID-19 event window, which is the research object in the LSTM based study and the Twitter study.

[Source: Author's representation]